Technical Details of:
US-Europe Differences in Technology-Driven Growth
Quantifying the Role of Education

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Abstract

In this companion document to our paper, "US-Europe Differences in Technology-Driven Growth: Quantifying the Role of Education" we present a complete analytical analysis of the Balanced Growth Path of our model.

1 Analytic Characterization of the Balanced Growth Path

Given the motivation of our paper, we are particularly interested in the comparative statics with respect to the education policy parameter, s, parameters intended to capture product and labor market frictions, C, f, and the speed of technological innovation λ.

Our strategy is to first solve the different stages of the adopting firm's problem for a schedule that relates the optimal productivity threshold $\bar{E}_h$, beyond which the firm chooses the adoption option, to a given measure η_g of households available for work in the high-tech sector. This can be viewed as a human capital "demand" schedule. We then derive a similar schedule from the households' education problem – the productivity threshold $\bar{E}_h$, with its implied wage differential, that is necessary to induce a given measure η_g of households to work in the high-tech sector. A high $\bar{E}_h$ is indicative of the high wages households need to expect before they are willing to supply a given η_g. For this reason the household condition, denoted by $\bar{E}_h^{HH}\left(\eta_g\right)$, can be viewed as a human capital "supply" schedule. Finally we combine these two schedules to solve for the balanced growth path education allocation and productivity threshold. All other BGP equilibrium values follow.

1.1 Firms

We first discuss the solution of the optimal technology adoption problem, *conditional* on an acceptable E_h draw and then analyze the determination of the optimal threshold $\bar{E}_h$.

1.1.1 The Technology Adoption Decision

Recall that $H_a = n_a E_h$ is the effective labor used for production. The adoption problem is

$$\max_{x \leq \lambda} x H_a^\theta - w_a H_a - \frac{1}{2}(x-1)^2,$$

with associated first order condition

$$(H_a)^\theta \begin{cases} = x-1 & \text{if } x < \lambda \text{ (interior growth)} \\ \geq \lambda - 1 & \text{if } x = \lambda \text{ (maximal growth).} \end{cases}$$

The labor market equilibrium implies $n_a = \eta_g$ and therefore $H_a = \eta_g E_h$.

Motivated by the above first-order condition, it is convenient to define, for an arbitrary η_g, the threshold $\widehat{E}_h$ beyond which $x = \lambda$ is optimal (i.e. the speed of technology adoption is maximal):

$$\widehat{E}_h\left(\eta_g\right) \equiv \min\left\{\max\left\{1,\ \frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g}\right\}, h\right\}.$$

In general, there exist cutoffs η_g^1 and η_g^3, such that, for $\eta_g \in \left[1, \eta_g^1\right]$ only interior growth occurs, for $\eta_g \in \left[\eta_g^1, \eta_g^3\right]$ either interior (when $E_h < \widehat{E}_h$) or maximal growth (when $E_h \geq \widehat{E}_h$) is possible, and for $\eta_g \in \left[\eta_g^3, 1\right]$ only maximal growth occurs.[1] These cutoffs are defined by:

$$\eta_g^1 \equiv \frac{(\lambda-1)^{\frac{1}{\theta}}}{h} < (\lambda-1)^{\frac{1}{\theta}} \equiv \eta_g^3. \tag{1}$$

Figure 1 depicts the maximal-growth cutoff function $\widehat{E}_h(\eta_g)$. The $\widehat{E}_h$ function and the η_g cutoffs depend only on the model parameters.

[1] It can be seen that $\widehat{E}_h\left(0\right) = h$; that is, as $\eta_g \to 0$, only interior growth is possible. A sufficient condition for only interior growth to occur, for *all* η_g is $(\lambda-1)^{\frac{1}{\theta}} \geq h$. When $\eta_g \to 1$, *if* $(\lambda-1)^{\frac{1}{\theta}} \leq 1$, $\widehat{E}_h(\eta_g) = 1$, and only maximal growth occurs; if $1 < (\lambda-1)^{\frac{1}{\theta}} < h$, then $\widehat{E}_h$ is interior and growth could either be interior (whenever the draw $E_h < \widehat{E}_h$) or maximal (whenever $E_h \geq \widehat{E}_h$).

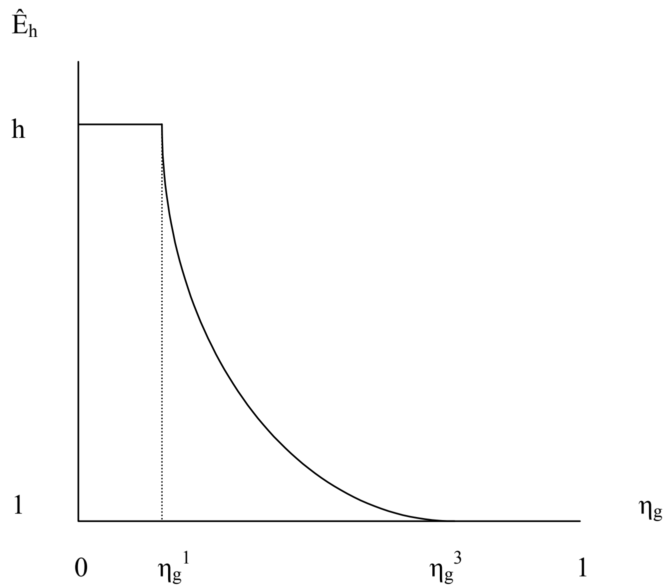

Figure 1: Cutoff $\widehat{E}_h\left(\eta_g\right)$ for maximal growth

Using the adopting firm's first order conditions we obtain, when $E_h < \widehat{E}_h$ (interior growth)

$$\begin{aligned} x &= 1 + (H_a)^{\theta} \\ w_a &= \theta x (H_a)^{\theta-1} = \theta (H_a)^{\theta-1} + \theta (H_a)^{2\theta-1} \\ \pi_i(E_h, \eta_g) &= (1-\theta)(H_a)^{\theta} + \left(\frac{1}{2} - \theta\right)(H_a)^{2\theta}. \end{aligned} \tag{2}$$

When $E_h \geq \widehat{E}_h$ (maximal growth) we obtain,

$$\begin{aligned} x &= \lambda \\ w_a &= \theta\lambda (H_a)^{\theta-1} \\ \pi_m(E_h, \eta_g) &= (1-\theta)\lambda (H_a)^{\theta} - \frac{1}{2}(\lambda-1)^2. \end{aligned} \tag{3}$$

In all these expressions, $H_a = \eta_g E_h$, is a random variable.

The wage *rate* in the adoption sector, w_a, as well as the actual wage for a worker, $w_a E_h$, is decreasing in the availability of adoption labor, η_g; the wage is increasing in the productivity draw, E_h. The profit function, $\pi(E_h, \eta_g)$, is an increasing function of E_h in both segments – it is subscripted with i for the interior region and m for the maximal region – with a kink at $\widehat{E}_h\left(\eta_g\right)$.[2] Profits are also increasing in η_g, which is relevant for the discussion of firm

[2] One can show, by evaluating both π's at $E_h = \widehat{E}_h(\eta_g)$, that $\pi(E_h, \eta_g)$ is continuous in E_h, and that the second part (when growth is maximal) is steeper than the first (when growth is interior).

behavior below. For low η_g, the firm relies on a high enough productivity draw to recoup the cost of adoption and to earn profits; for high η_g, the firm can afford to proceed with production for a lower productivity draw.

The optimality condition for the non-adopting firm in the main text, and the non-adoption labor market clearing condition implies that $w_n = \theta\beta\left(\left(1-\eta_g\right)h\right)^{\theta-1}$.

1.1.2 The Dynamic Problem

The high-tech firms' decision $x(E_h, \eta_g) = \frac{A'}{A}$ is a purely static one; given the complete spillover assumed, the adoption decision does not affect the technology that this firm has access to in the next period. However, its second stage problem of deciding which implementation route to follow to increase productivity is dynamic across sub-periods within a model period and is specified by

$$v(E_h, n_a) = \max_{\text{adopt, not adopt}} \left\{(1-C)\pi(E_h, n_a), -fn_aE_h + \int_1^h v(E_h', n_a)dF(E_h')\right\} \quad (4)$$

Profits π are an increasing function of E_h; the "dismiss" option is decreasing in E_h. We therefore anticipate a threshold $\bar{E}_h \in [1, h]$ such that for all $E_h < \bar{E}_h$ the firm fires and searches for a better workforce-technology match and for all $E_h \geq \bar{E}_h$ it proceeds with production using the current workforce. In this sub-section, we characterize the firm's BGP condition $\bar{E}_h(\eta_g)$.

Based on the productivity draw required for maximal growth, $\widehat{E}_h(\eta_g)$, one of two cases arises. In the first case, depicted in Figure 2, the threshold productivity $\bar{E}_h$ is less that the productivity needed for maximal growth. Therefore, some or all of the E_h draws that the firm decides to proceed with will not result in the maximal growth rate and thus the expected aggregate growth rate $E(x)$ is less than λ. Two sub-cases are possible. In case 1a, if $\eta_g < \eta_g^1$, the measure of generally-educated agents is low enough to cause $\widehat{E}_h = h$, which implies that *every* draw of E_h will result in non-maximal growth. For η_g greater than η_g^1, but less than a yet-to-be determined valued η_g^2, *some* draws of E_h will result in interior growth and some will result in maximal growth for that firm (case 1b).[3]

[3] Case 1b is listed only for completeness. It can be shown that this case does not arise on a BGP. This is the rationale for using λ, rather than an ever-increasing technological gap relative to the frontier, as the constraint on an individual firm's adoption problem. It can be shown that this is also true during the transition to a more rapid rate of technological change, λ', provided $(\lambda'-1)^{\frac{1}{\theta}} > h$. Detailed transitional analysis is available from the authors on request.

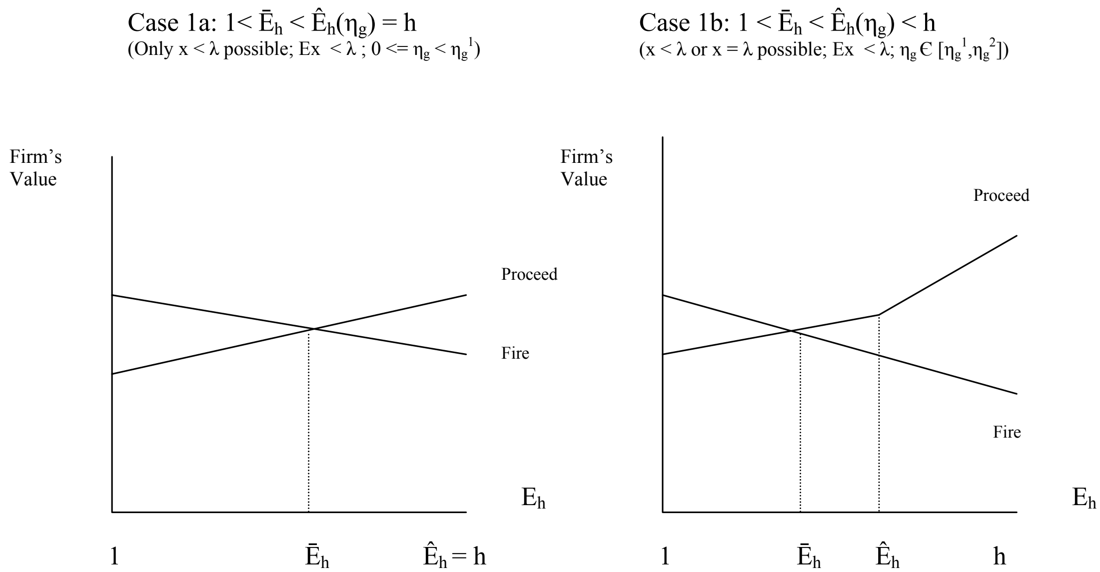

Figure 2: Firm's Bellman Equation when $\overline{E}_h < \widehat{E}_h$

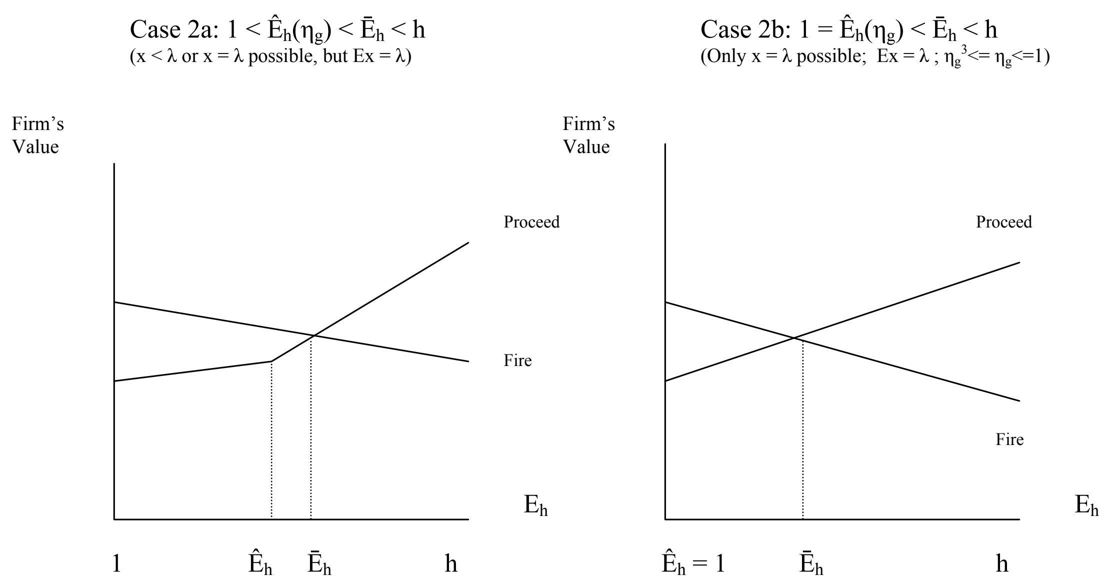

Figure 3: Firm's Bellman Equation when $\overline{E}_h \geq \widehat{E}_h$

In the second case, depicted in Figure 3, $\bar{E}_h$ is greater than or equal to the productivity needed for maximal growth. Every draw of E_h for which the "proceed" option is chosen by an individual firm will result in maximal growth; therefore, the expected aggregate growth rate $E\left(x\right)$ equals λ. Again, there are two sub-cases. If η_g is greater than or equal to η_g^2, but less than the η_g^3, some draws of E_h will not be compatible with maximal growth, but the firm will not choose to proceed with these (case 2a). In the second sub-case, for large enough η_g, that is, for $\eta_g \geq \eta_g^3$, only draws that are compatible with maximal growth will even occur (case 2b). The expression that determines the cutoff η_g^2 is derived as a by-product of the derivation of the firm's $\bar{E}_h\left(\eta_g\right)$ curve.

For ease of exposition, the above figures have been drawn assuming that the threshold $\bar{E}_h$ is interior for all η_g. We will consider the corner cases later. If the "proceed" option lies everywhere below the "fire" option, $\bar{E}_h = 1$, and if it lies everywhere above the "fire" option, $\bar{E}_h = h$.

1.1.3 The Firm Condition $\bar{E}_h\left(\eta_g\right)$

The solution for the Bellman equation depends on whether case 1 or case 2 is assumed. We first assume that case 1 applies and derive the $\bar{E}_h\left(\eta_g\right)$ function. In the appendix (section A.1), we show that the $\bar{E}_h\left(\eta_g\right)$-schedule is implicitly defined by

$$f\eta_g\left\{\left(1-F\left(\bar{E}_h\right)\right)\bar{E}_h + \int_1^{\bar{E}_h} E_h dF\left(E_h\right)\right\} \gtreqless$$
$$(1-C)\left[\int_{\bar{E}_h}^{\widehat{E}_h} \pi_i\left(E_h,\eta_g\right)dF\left(E_h\right) + \int_{\widehat{E}_h}^{h} \pi_m\left(E_h,\eta_g\right)dF\left(E_h\right) - \left(1-F\left(\bar{E}_h\right)\right)\pi_i\left(\bar{E}_h,\eta_g\right)\right]. \quad (5)$$

The left hand side is the marginal cost of firing the workforce and waiting for a better draw and the right hand side is the marginal benefit of such a draw in the form of extra expected net profits.[4] If (5) holds with $>$ at $\bar{E}_h = 1$, then the cost of firing is so high that the firm accepts any productivity draw, and $\bar{E}_h(\eta_g) = 1$. On the other hand, if (5) holds with $<$ at $\bar{E}_h = h$, the benefit of hiring is so high relative to the cost that the firm is willing to wait for the highest draw, and $\bar{E}_h(\eta_g) = h$. Otherwise an interior productivity threshold $\bar{E}_h(\eta_g)$ is uniquely determined, with (5) holding as an equality.

In the appendix (section A.2), we characterize the $\bar{E}_h\left(\eta_g\right)$ condition in detail. It is convenient to divide the above condition by η_g throughout; we will argue that as $\eta_g \to 0$, $\bar{E}_h \to h$, separately. In the following discussion, when we refer to condition (5), it is this modified condition we refer to.

For a given set of cost parameters, f, C, and labor availability, η_g, the left hand side of (5) is an increasing function of $\bar{E}_h$; waiting for a higher E_h's entails more rounds of firings and thus higher firing costs. The right hand side is decreasing in $\bar{E}_h$. While a higher $\bar{E}_h$ implies a higher profit for the adopting firm, it reduces the range of E_h over which actual production takes place for those profits to be realized. The decreasing right hand side evidently captures the diminishing returns inherent in the process of experimenting with E_h. Since the left hand side is increasing and the right hand side is decreasing in $\bar{E}_h$, an intersection in $[1, h]$ occurs. However, where the intersection occurs depends on η_g; we turn to this next.

[4] In the range of η_g in which $\widehat{E}_h = h$ (case 1a), the π_m term is absent.

The left hand side of (5) is independent of η_g. As argued in the appendix, one can find a sufficient condition to ensure that the right hand side is decreasing in η_g. Given these properties of (5) with respect to $\bar{E}_h$ and η_g, the intersection of the two sides, which gives the threshold $\bar{E}_h$, is decreasing in η_g.

We show in the appendix that the $\bar{E}_h\left(\eta_g\right)$ schedule starts at h (and potentially stays at h for a neighborhood of zero); for very low supply of adoption labor, the firing costs are negligible relative to the profits associated with the best possible draw so that the firm is willing to wait for it. It then decreases monotonically. The firm condition is depicted in Figure 4.[5]

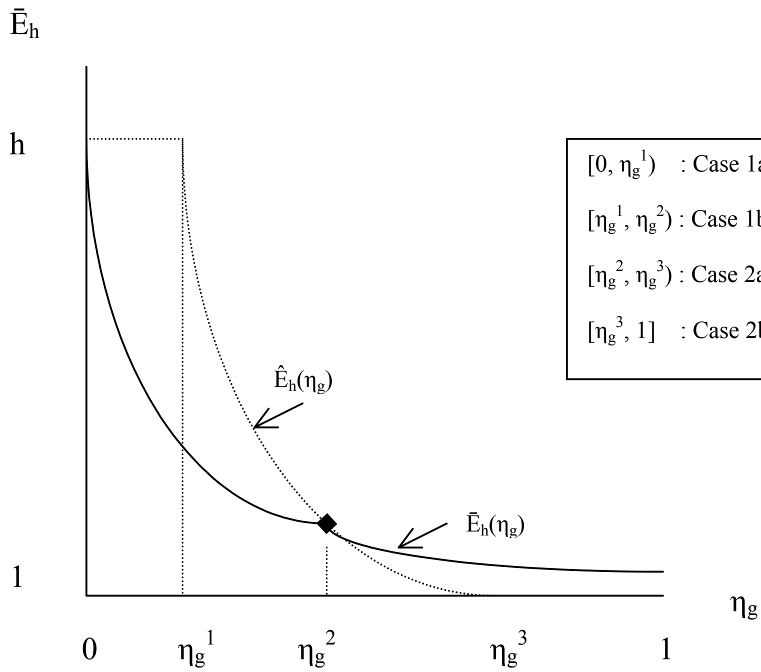

Figure 4: Firm Condition $\bar{E}_h\left(\eta_g\right)$

At some point the decreasing function $\bar{E}_h\left(\eta_g\right)$ crosses the $\widehat{E}_h\left(\eta_g\right)$ function, depicted as a dotted line in Figure 4. Beyond that point, case 2 applies. The η_g that corresponds to this switch-over, which we have labeled η_g^2, is implicitly defined by setting $\bar{E}_h = \widehat{E}_h = \frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}$ in (5). The condition, depicted in Figure 4, should therefore be interpreted as a composite of the two cases.

In the appendix (section A.3), we show that the $\bar{E}_h\left(\eta_g\right)$ condition for case 2 is implicitly

[5] As shown in the appendix, if $f < (1-C)(1-\theta)\lambda\left[\int_1^h (E_h)^\theta \, dF(E_h) - 1\right]$, we obtain $\bar{E}_h(1) > 1$.

given by:[6]

$$f\eta_g \left\{ \left(1 - F\left(\bar{E}_h\right)\right) \bar{E}_h + \int_1^{\bar{E}_h} E_h dF\left(E_h\right) \right\} =$$

$$(1 - C) \left[\int_{\bar{E}_h}^{h} \pi_m \left(E_h, \eta_g\right) dF\left(E_h\right) - \left(1 - F\left(\bar{E}_h\right)\right) \pi_m \left(\bar{E}_h, \eta_g\right) \right]. \tag{6}$$

The characterization of $\bar{E}_h\left(\eta_g\right)$ for this case parallels the one for the previous case. In the appendix (section A.4), we argue that the $\bar{E}_h\left(\eta_g\right)$ schedule is monotonically decreasing in η_g. It is depicted as the $\left[\eta_g^2, 1\right]$-portion of the $\bar{E}_h\left(\eta_g\right)$-curve in Figure 4. We also provide a condition on f and other parameters of the model to ensure $\bar{E}_h(1) > 1$.

Taken together, the decreasing $\bar{E}_h\left(\eta_g\right)$-curve is interpreted as follows.[7] As η_g increases, the firing cost increases linearly since it is paid for every person hired. Profits increase, but less than linearly, given the diminishing returns to H_a. Therefore the firm "experiments" less in adoption. Additionally, profits are high on account of high η_g, and therefore the firm does not have to set a high productivity threshold in order to realize high profits and meet adoption costs.

1.1.4 Dependence of $\bar{E}_h\left(\eta_g\right)$ on f

For a given η_g, an increase in the unit firing cost of labor, f, increases the marginal cost of drawing another E_h, and the adopting firm experiments less and accepts lower productivity matches for any given η_g; that is, $\bar{E}_h(\eta_g)$ decreases.[8] We graph the equilibrium implication of this shift in the firm condition once we have characterized the household condition below.

1.1.5 Dependence of $\bar{E}_h\left(\eta_g\right)$ on C

Unlike an increase in the firing cost, an increase in the entry or regulation costs decreases the benefit of redrawing instead of the cost. But the outcome is similar; the adopting firm experiments less and accepts lower productivity matches for any given η_g. We graph the equilibrium implication of this shift in the firm condition later. To study how the growth gap between the US and Europe changes with λ, we next examine the dependence of the adopting firm's condition on λ.

[6] We make assumptions in the appendix to guarantee that this condition holds as an equality.

[7] It is easy to see from (5) and (6), by setting $\pi_i = \pi_m$ at $\bar{E}_h = \widehat{E}_h$, that the schedule $\bar{E}_h\left(\eta_g\right)$ is continuous at η_g^2.

[8] In the appendix (section A.5), we provide analytical details on the dependence of $\bar{E}_h(\eta_g)$ on f, C, and λ.

1.1.6 Dependence of $\bar{E}_h\left(\eta_g\right)$ on λ

There are two effects of an increase in the growth rate of the available technology, λ. The minimum adoption labor supply, η_g, needed to induce the firm to adopt at the maximum possible rate increases. Since the fixed cost of adopting at the maximum rate, $\frac{1}{2}\left(\lambda-1\right)^2$, increases, the wage in the adoption sector has to be low enough to keep profits high, which is ensured by higher η_g. This effect is captured by the rightward shift of $\widehat{E}_h\left(\eta_g\right)$.

When λ increases, the expected profit from a higher draw increases, unless only interior draws occur. This increase in the marginal benefit of a redraw causes the firm to set a higher adoption threshold; the firm can afford the firing cost for lower productivity draws. This effect causes $\bar{E}_h(\eta_g)$ to increase. We graph this condition later.

1.1.7 Dependence of $\bar{E}_h\left(\eta_g\right)$ on parameters: Summary

In summary, the dependence of the firm condition on η_g and the various parameters is given by:

$$\begin{array}{rcl}\bar{E}_h & \equiv & \bar{E}_h\left(\eta_g;\ f,\ C,\ \lambda\right) \\ & & \qquad -;\ -,\ -,\ +\end{array}$$

1.2 Households

Workers take as given the adopting firm behavior, as characterized by $\bar{E}_h(\eta_g)$, and decide on their education, which will result in a BGP education allocation η_g.

In order to calculate the expected utilities, denote the value of a vocationally educated agent on the BGP by v_v and that of a generally educated agent by v_g. Conditional on being kept by the firm, the expected utility of a latter agent is $\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\int_{\bar{E}_h}^{h}\log\left(w_a E_h\right)dF\left(E_h\right)$. The likelihood of this outcome is $\left(1-F\left(\bar{E}_h\right)\right)$. With probability $F\left(\bar{E}_h\right)$, he gets fired, earns nothing in the current period, but can expect a utility v_g in the future. Therefore,

$$v_g = \left(1-F\left(\bar{E}_h\right)\right)\left[\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\int_{\bar{E}_h}^{h}\log\left(w_a E_h\right)dF\left(E_h\right)\right] + F\left(\bar{E}_h\right)v_g,$$

which implies

$$v_g = \frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\int_{\bar{E}_h}^{h}\log\left(w_a E_h\right)dF\left(E_h\right).$$

The condition that pins down the threshold ability for obtaining general education is given by

$$v_g - v_v \equiv \frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\int_{\bar{E}_h}^{h}\log\left(w_a E_h\right)dF\left(E_h\right) - \log\left(w_n h\right) = \log\left(e\left(a^*\right)\right) - \log\left(\frac{s_g}{s_v}\right).$$

Assuming a parametric form for the cost function, $e(a) = \frac{1}{a}$, noting that $a^* = 1 - \eta_g$, and recalling the definition of our education policy variable, $s = \frac{s_g}{s_v}$, the threshold condition can be written as

$$\frac{1}{\left(1 - F\left(\bar{E}_h\right)\right)} \int_{\bar{E}_h}^{h} \log\left(w_a E_h\right) dF\left(E_h\right) - \log\left(w_n h\right) = \log\left(\frac{1}{1-\eta_g}\right) - \log\left(s\right). \tag{7}$$

From the optimality condition of the non-adopting firm's problem, and the market clearing condition in the low-tech sector, it follows that

$$w_n h = \theta \beta h^{\theta} / \left(1 - \eta_g\right)^{1-\theta}. \tag{8}$$

From (2) and (3) we observe that wages in the high-tech sector depend on whether growth is interior maximal. In particular

$$w_a E_h = \begin{cases} \theta\left(\eta_g\right)^{\theta-1}\left(E_h\right)^{\theta} + \theta\left(\eta_g\right)^{2\theta-1}\left(E_h\right)^{2\theta} & \text{when } x < \lambda \\ \theta\lambda\left(\eta_g\right)^{\theta-1}\left(E_h\right)^{\theta} & \text{when } x = \lambda \end{cases} \tag{9}$$

The equilibrium household condition depends on which of these two cases occurs.

1.2.1 Household Condition $\bar{E}_h^{HH}\left(\eta_g\right)$

In the appendix (section A.6), we derive the expression that implicitly defines the household condition $\bar{E}_h^{HH}\left(\eta_g\right)$ by using (8) and (9) in (7). For the case of $x < \lambda$ this yields

$$\begin{aligned} &\frac{1}{\left(1 - F\left(\bar{E}_h\right)\right)} \left\{ \int_{\bar{E}_h}^{\widehat{E}_h} \log\left[\theta\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta} + \theta\left(E_h\right)^{2\theta}\right] dF\left(E_h\right) + \int_{\widehat{E}_h}^{h} \log\left[\theta\lambda\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}\right] dF\left(E_h\right) \right\} \\ \gtreqqless\ & \log\left[\theta\beta h^{\theta}\right] + (1-2\theta)\log\left(\eta_g\right) - (2-\theta)\log\left(1-\eta_g\right) - \log\left(s\right). \end{aligned} \tag{10}$$

The left hand side of this expression can be viewed as the benefit of general education in the form of higher expected wages, and the right hand side as the cost net of subsidy, which includes the foregone wages in the non-adoption sector, in addition to the disutility of general education. If the left hand side is greater at $\bar{E}_h = 1$ or the right hand side greater at $\bar{E}_h = h$, a corner solution obtains.

In the appendix (section A.7), we characterize the $\bar{E}_h^{HH}\left(\eta_g\right)$ schedule in detail. The right hand side of (10) is independent of $\bar{E}_h$, while the left hand side increases with $\bar{E}_h$: the benefit of general education is increasing in expected wages, which is in turn increasing in the productivity threshold. An intersection in $[1, h]$ occurs. However, as in the firm condition, where the intersection occurs depends on η_g.

It can directly be seen that the right hand side of (10) is increasing in η_g; this results from an increase in the foregone wages in the labor-scarce non-adoption sector, as well as the increased disutility of inducing inframarginal agents to acquire general education. It can be shown that the left hand side is decreasing in η_g: adoption wages decrease with the labor supplied.

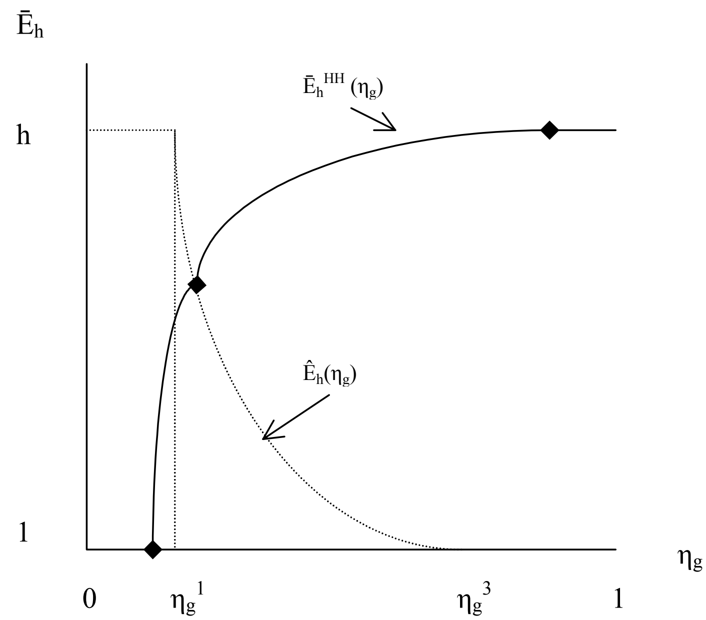

Figure 5: The Household Condition $\bar{E}_h^{HH}\left(\eta_g\right)$

Given these properties of (10) with respect to $\bar{E}_h$ and η_g, the intersection of the two sides, which gives the threshold $\bar{E}_h^{HH}(\eta_g)$, is increasing in η_g. In the appendix we show that when $\eta_g \to 0$, the wage premium is so high that even an $\bar{E}_h = 1$ is enough to attract entry into general education. The $\bar{E}_h^{HH}\left(\eta_g\right)$ curve starts at 1, (potentially) stays at 1 for an interval, and increases after that. This condition is depicted in Figure 5. As with the $\bar{E}_h(\eta_g)$ schedule case 1 applies only up to the intersection with the $\widehat{E}_h(\eta_g)$ curve; case 2 applies beyond that, and is discussed below. We denote by η_g^{HH} the point where this intersection occurs.

In the appendix (section A.8), we show that the expression that implicitly defines the household condition $\bar{E}_h^{HH}\left(\eta_g\right)$ for the case that $x = \lambda$ is:

$$\begin{aligned} & \frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\int_{\bar{E}_h}^{h} \log\left[\theta\lambda\left(E_h\right)^{\theta}\right] dF\left(E_h\right)\right\} \\ \leq \quad & \log\left[\theta\beta h^{\theta}\right] + (1-\theta)\log\left(\eta_g\right) - (2-\theta)\log\left(1-\eta_g\right) - \log\left(s\right). \end{aligned} \tag{11}$$

The analysis of this condition parallels that of the first case, and the details are given in the appendix (section A.9). The left hand side of (11) is strictly increasing in $\bar{E}_h$ and the right hand side is independent of $\bar{E}_h$; the left hand side is independent of η_g and the right hand side is strictly increasing with η_g. This indicates that the intersection of the two sides, which gives the threshold $\bar{E}_h$ necessary for any given labor supply η_g, is increasing in η_g. We can show that as $\eta_g \to 1$, $\bar{E}_h \to h$. When η_g is very high, even the lowest ability agents obtain general education. The disutility cost is so prohibitive that the $\bar{E}_h$, and thus the expected wage premium, has to be very high to induce entry. Therefore, the $\bar{E}_h^{HH}\left(\eta_g\right)$ curve increases up to h, and (potentially) stays at h for an interval. This interval is depicted as the $\left[\eta_g^{HH}, 1\right]$ portion of the $\bar{E}_h^{HH}\left(\eta_g\right)$ curve in Figure 5.

Taken together, the increasing $\bar{E}_h^{HH}\left(\eta_g\right)$ curve is interpreted as follows.[9] As η_g increases, a downward pressure on adoption wages arises, and the disutility of the marginal general education enrollee increases. To counter these, the productivity levels $E_h > \bar{E}_h$ with which the firm produces and pays wages have to be high.

1.2.2 Dependence of $\bar{E}_h^{HH}\left(\eta_g\right)$ on s

From both (10) and (11), we see that the right hand sides decrease with s. Given the increasing left hand sides, this means that the point of intersection, $\bar{E}_h$, if interior, shifts down, for a given η_g.[10] As the subsidy for general education increases, households are willing to supply a given η_g for lower thresholds $\bar{E}_h$ and thus lower wage premia. We will graph the equilibrium implication of this shift in the household condition induced by a change in s below.

1.2.3 Dependence of $\bar{E}_h^{HH}\left(\eta_g\right)$ on λ

As in the case of the analysis of firms, (1) implies that the $\widehat{E}_h\left(\eta_g\right)$ threshold curve shifts to the right when λ increases, and the thresholds η_g^1 and η_g^3 increase.

In both (10) and (11), the right hand sides – the cost of acquiring general education – do not depend on λ. A mere examination of the left hand side of (11) shows that it is increasing in λ. In the appendix (section A.10), we show this is also true for (10). Given the flat right hand sides, the threshold, $\bar{E}_h$, decreases.[11]

Expected adoption wages are increasing in λ; therefore, such an increase induces supply of a given η_g even for lower thresholds $\bar{E}_h$.

---

[9] As in the firm condition, it is easy to see from (10) and (11) by setting $\bar{E}_h = \widehat{E}_h$ that the schedule $\bar{E}_h^{HH}\left(\eta_g\right)$ is continuous at η_g^{HH}.

[10] The intervals for η_g for which corner solutions arise change, however. The corner $\bar{E}_h = 1$ occurs because the benefit of general education exceeds the cost for any $\bar{E}_h$. Since the cost of general education decreases with s, the benefit of general education exceeds the cost for a *larger* interval of η_g. Likewise, the corner $\bar{E}_h = h$ occurs because the cost of general education exceeds the benefit for any $\bar{E}_h$. Since the cost of general education has decreases in s, the cost of general education exceeds the benefit for a *smaller* interval of η_g.

[11] The behavior of the η_g intervals in which $\bar{E}_h = 1$ or $\bar{E}_h = h$ is very similar to the one discussed above for an increase in s; the only difference is that an increase in λ works by increasing the benefit while the increased s works by decreasing the cost of general education.

1.2.4 Dependence of $\bar{E}_h^{HH}\left(\eta_g\right)$ on parameters (summary)

To summarize, the household condition is given by

$$\bar{E}_h^{HH} \equiv \underset{+;\ -,\ -}{\bar{E}_h^{HH}\left(\eta_g;\ s\ \lambda\right)}.$$

1.3 Existence and Uniqueness of Balanced Growth Path Equilibrium

Given the strictly decreasing, continuous firm condition, $\bar{E}_h\left(\eta_g\right)$, and the strictly increasing, continuous household condition, $\bar{E}_h^{HH}\left(\eta_g\right)$, we obtain a unique BGP equilibrium. It is depicted in Figure 6; the intersection of the two conditions yield the equilibrium education allocation $\left(\eta_g\right)^*$ and productivity threshold $\left(\bar{E}_h\right)^*$.

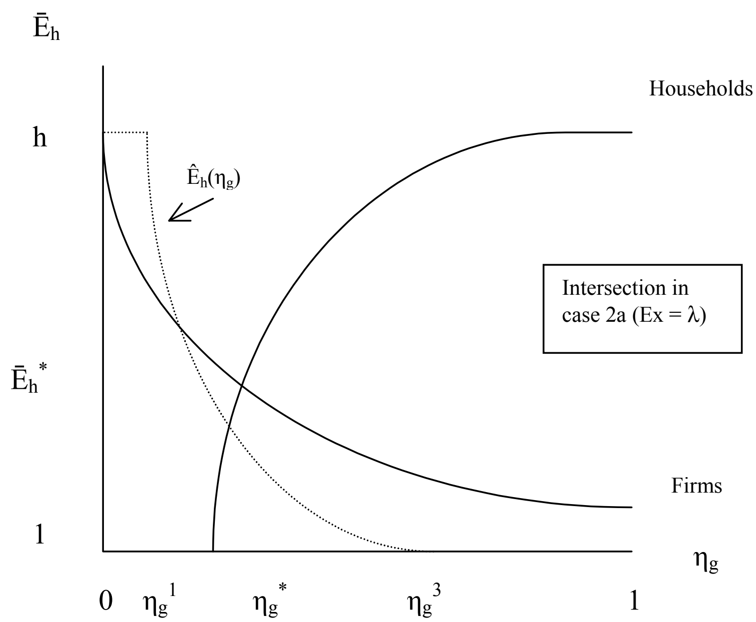

Figure 6: The BGP Equilibrium

In the example shown in Figure 6, the intersection between $\bar{E}_h\left(\eta_g\right)$ and $\bar{E}_h^{HH}\left(\eta_g\right)$ occurs in case 2a of the firm's condition where $\bar{E}_h \geq \hat{E}_h$. Therefore all firms always choose maximal growth. As discussed in section 1.1.2, if the intersection occurs in case 1, the expected growth rate of the economy satisfies $E\left(x\right) < \lambda$. Note that once $\left(\eta_g\right)^*$ and $\left(\bar{E}_h\right)^*$ are determined, all other variables, such as the expected growth rate, expected wages, and profits on the BGP can be determined. For instance, the average growth rate in the economy is given by

$$E\left(x\right) = \frac{1}{\left(1 - F\left(\left(\bar{E}_h\right)^*\right)\right)} \left\{ \int_{\left(\bar{E}_h\right)^*}^{\left(\hat{E}_h\right)^*} \left[1 + \left[\left(\eta_g\right)^* \left(E_h\right)^*\right]^{\theta}\right] dF\left(E_h\right) + \lambda\left(1 - F\left(\left(\widehat{E}_h\right)^*\right)\right)\right\}.$$

If $\left(\bar{E}_h\right)^* \geq \left(\hat{E}_h\right)^*$, this equation reduces to $E(x) = \lambda$.[12] The expected growth rate $E(x)$ is increasing in the equilibrium values of both η_g and $\bar{E}_h$.

1.4 Comparative Statics

The previous analysis of how the firm and household conditions depend on the parameters (f, C, s, λ) now makes it straightforward to characterize their influence on the BGP equilibrium.

1.4.1 Increase in Firing Cost, f

An increase in the firing cost parameter f (our proxy for labor market regulations) affects only the firm condition by shifting it down. As a result, the equilibrium threshold as well as the general education attainment and the expected growth rate decreases. Firms are willing to produce with lower E_h-draws rather than fire employees; this translates into lower wages in the adoption sector and a lower incentive to obtaining general education. Figure 7 depicts this situation.

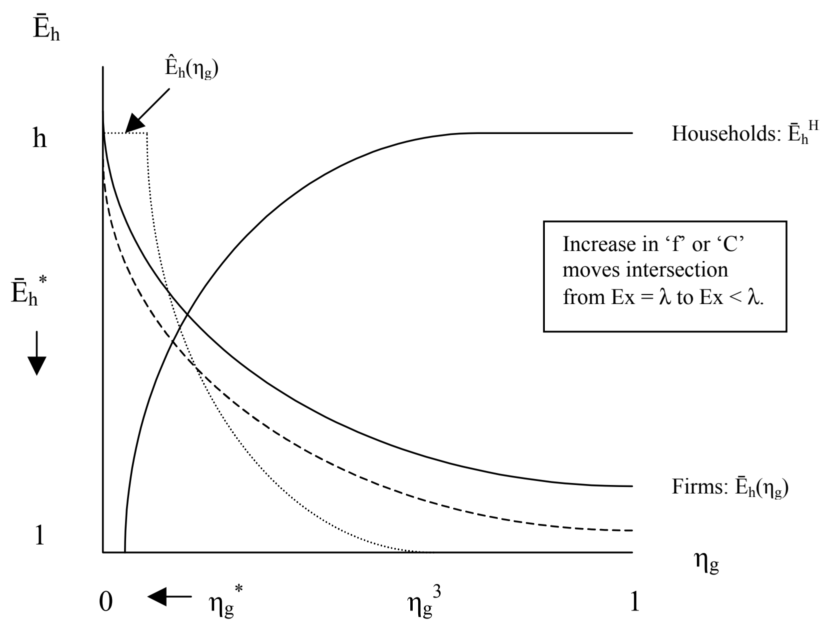

Figure 7: Effect of an increase in f, C

[12] $\left(\widehat{E}_h\right)^* = \frac{(\lambda-1)^{\frac{1}{\theta}}}{(\eta_g)^*}$.

1.4.2 Increase in Entry Cost, C

An increase of the entry cost parameter C (our proxy for product market regulations and bureaucratic costs) also only shifts the firm's condition downward, again decreasing the equilibrium threshold, general education attainment, and expected growth $E(x)$. Firms are willing to accept lower E_h-draws as the benefits of higher draws are diminished, which results in lower wages in the adoption sector and a lower incentive to obtain general education.

1.4.3 Increase in General Education Subsidy, s

The subsidy parameter s (our proxy for general education focus) only impacts the household condition; an increase in s shifts the household's condition down, reducing the equilibrium productivity threshold E_h, but increasing general education attainment η_g. If the latter effect dominates, expected growth $E(x)$ increases. When the effective cost of general education decreases, households require a lower threshold $\bar{E}_h$ and wage premium to choose general education. While a decrease in f or C or an increase in s will all result in higher general education attainment, the subsidy increase reduces $\bar{E}_h$ while the others increase it. This implies that the average number of draws before successful production, given by $1/\left(1 - F\left(\bar{E}_h\right)\right)$, declines with s (and increases with f, C).[13]

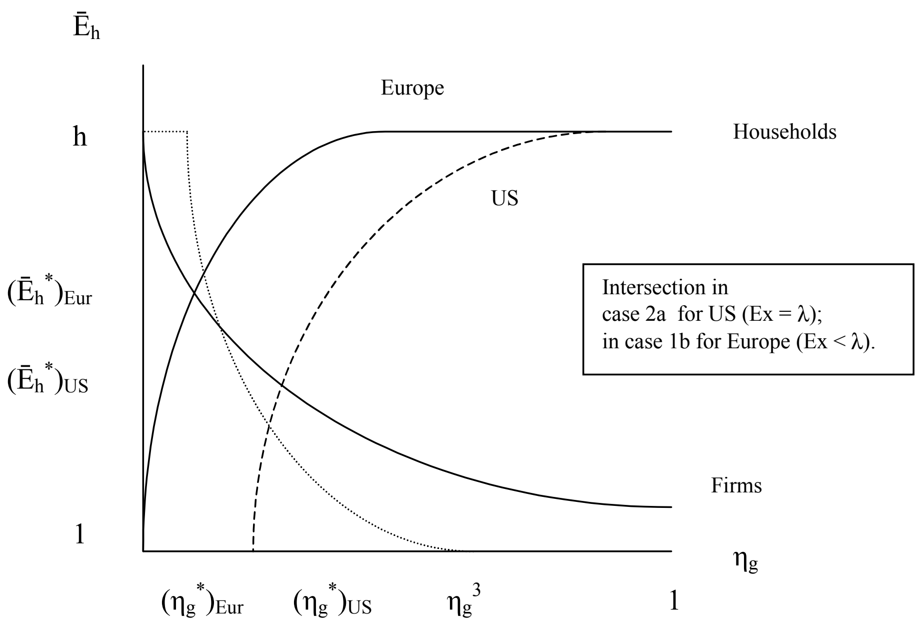

Figure 8: Effect of an increase in s

[13] The mean number of attempts is given by $\sum_{j=1}^{\infty} j\left[F\left(\bar{E}_h\right)\right]^{j-1}\left(1 - F\left(\bar{E}_h\right)\right) = \frac{1}{1-F\left(\bar{E}_h\right)}$. We have chosen to abstract from discounting in our setup for sake of simplicity; the advantage of the subsidy scheme is even more readily apparent if discounting is taken into account.

1.4.4 Dependence of BGP on λ

Analyzing the dependence of the BGP equilibrium on the speed of technology *availability,* λ, is more complicated because all three schedules – $\widehat{E}_h(\eta_g)$ (shifts right), $\bar{E}_h(\eta_g)$ (shifts up), and $\bar{E}_h^{HH}(\eta_g)$ (shifts right) are affected. The effect on the equilibrium productivity threshold $\bar{E}_h$ is ambiguous, whereas the equilibrium general education attainment increases unambiguously. The higher productivity threshold set by the firm, as well as the direct increase in the maximal growth rate, increase the expected wage in the adoption sector. Therefore the incentive to acquire general education increases, and the resulting η_g is higher. Figure 9 depicts this situation.[14]

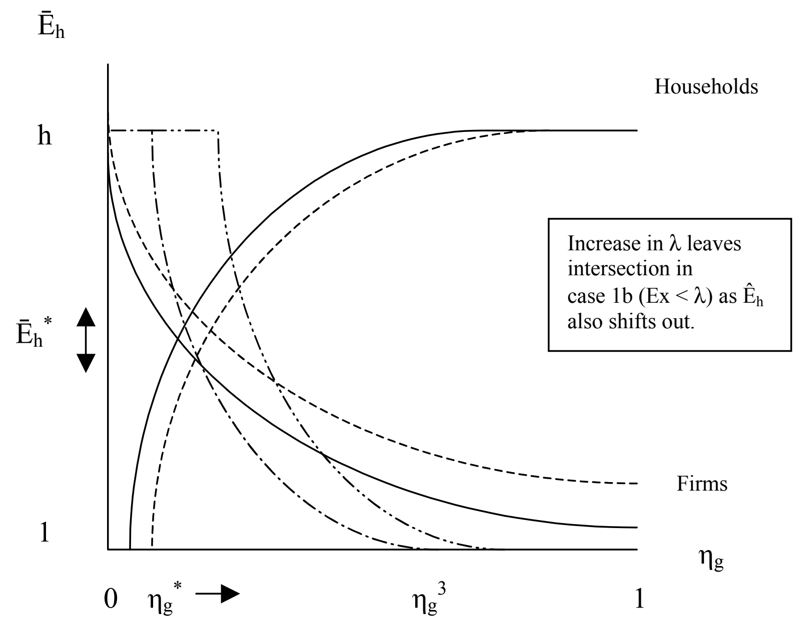

Figure 9: Effect of an increase in λ

What happens to the US-Europe (expected) growth gap when λ increases? The answer to this question is again complicated by the shifts of all three curves. In Figure 10, we hold the household curves at their old positions to achieve graphical tractability. The accompanying box explains how a growth gap, initially zero, can expand with an increase in λ. The behavior

[14] In the particular example shown in Figure 9, case 1b obtains both before and after the increase. However, if the shifting of the $\widehat{E}_h$ curve is more muted, which will happen if θ is not too low, it is possible that the new intersection is in case 2a, and maximal growth obtains.

For ease of drawing, the entire firm condition is shown as shifting upward. In the interval $[1, \eta_g^1]$, when $\widehat{E}_h = 1$, and only interior growth is possible, a change in λ has no effect. The argument made in section 1.1.6 is for a marginal change in λ that leaves the relevant case for $\bar{E}_h$ unchanged. As long as the intersection of the household and firm conditions does not correspond to case 1a (only interior growth draws), the figure will be an accurate representation of an increase in λ.

of the maximal growth cutoff function, $\widehat{E}_h(\eta_g)$, is important for this possibility. While the shifts of the firm and household conditions increase the equilibrium general education attainment for Europe, the productivity threshold corresponding to this increased attainment may still fall short of the *new* $\widehat{E}_h$ needed for maximum growth. Alternately, the wages in the adopting sector have to be low enough to induce the firm to adopt at the higher maximum speed. In the US, where general education subsidy and attainment are higher, η_g can exceed even the increased cutoff η_g^1. It may therefore continue to grow at the higher potential rate.

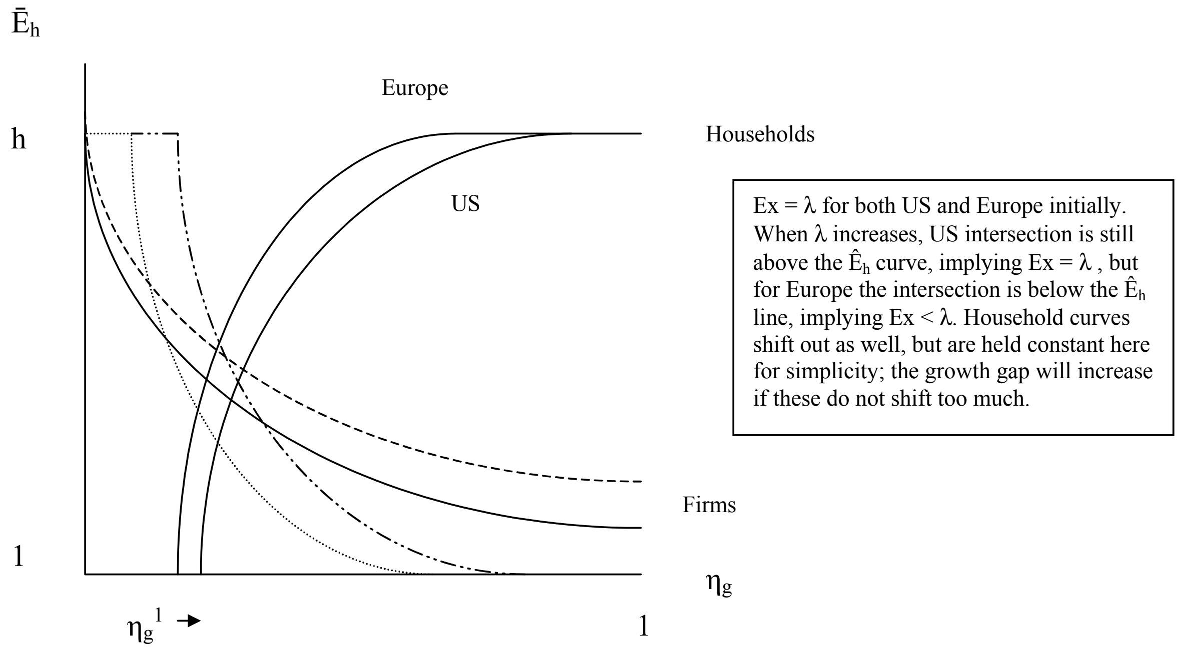

Figure 10: Dependence of the US-Europe growth gap on λ

A Appendix

A.1 Firm Condition for Case 1

Assume that case 1 is the relevant one, and equate the payoffs in (**??**) at the threshold to get:

$$Ev - f\eta_g \bar{E}_h \lesseqqgtr (1-C)\left[(1-\theta)\left(\eta_g \bar{E}_h\right)^{\theta} + \left(\frac{1}{2}-\theta\right)\left(\eta_g \bar{E}_h\right)^{2\theta}\right], \tag{12}$$

with equality if $\bar{E}_h \in (1,h)$. Given the distribution function F (with density $\widetilde{f}$) on E_h, integrating the appropriate v over the three segments, substituting for Ev from the threshold condition and simplifying we obtain the firm condition that implicitly defines $\bar{E}_h\left(\eta_g\right)$, in a form suitable for comparative statics:

$$f\left\{\left(1-F\left(\bar{E}_h\right)\right)\bar{E}_h + \int_1^{\bar{E}_h} E_h dF\left(E_h\right)\right\} \gtreqqless \frac{(1-C)(1-\theta)}{\left(\eta_g\right)^{1-\theta}}\left[\int_{\bar{E}_h}^{\widehat{E}_h}\left(E_h\right)^{\theta} dF\left(E_h\right) - \left(1-F\left(\bar{E}_h\right)\right)\left(\bar{E}_h\right)^{\theta}\right] +$$
$$\frac{(1-C)\left(\frac{1}{2}-\theta\right)}{\left(\eta_g\right)^{1-2\theta}}\left[\int_{\bar{E}_h}^{\widehat{E}_h}\left(E_h\right)^{2\theta} dF\left(E_h\right) - \left(1-F\left(\bar{E}_h\right)\right)\left(\bar{E}_h\right)^{2\theta}\right] + \frac{(1-C)}{\eta_g}\int_{\widehat{E}_h}^{h}\left[(1-\theta)\lambda\left(\eta_g\right)^{\theta}\left(E_h\right)^{\theta} - \frac{1}{2}(\lambda-1)^2\right] dF\left(E_h\right). \tag{13}$$

A more concise version is presented in the main text as (5).

A.2 Characterizing $\bar{E}_h\left(\eta_g\right)$ for Case 1

A.2.1 Dependence on $\bar{E}_h$

For a given η_g, f, C, the LHS of (13) is an increasing function of $\bar{E}_h$. Use Leibniz' formula for the term within curly braces to find that the derivative is $1-F\left(\bar{E}_h\right)-\widetilde{f}\left(\bar{E}_h\right)\bar{E}_h+\widetilde{f}\left(\bar{E}_h\right)\bar{E}_h>0$. The LHS starts out at f when $\bar{E}_h=1$ and increases to $fE\left(E_h\right)>f$ at $\bar{E}_h=h$, independent of η_g.

In the RHS of (13) the third term does not depend on $\bar{E}_h$. Again use Leibniz' formula for the terms within the first set of square brackets to find the derivative as $-\left(\bar{E}_h\right)^{\theta}\widetilde{f}\left(\bar{E}_h\right)-\theta\left(1-F\left(\bar{E}_h\right)\right)\left(\bar{E}_h\right)^{\theta-1}$ $+\left(\bar{E}_h\right)^{\theta}\widetilde{f}\left(\bar{E}_h\right)<0$. Similarly, the terms within the second set of square brackets are also decreasing in $\bar{E}_h$. So the RHS is decreasing in $\bar{E}_h$. This is true, independent of whether case 1a or case 1b is relevant, since $\widehat{E}_h$ does not enter the calculation. Since the LHS is increasing and the RHS decreasing in $\bar{E}_h$, an intersection is likely to occur. However, where the intersection occurs depends on η_g.

A.2.2 Dependence on η_g

The LHS of (13) does not change with η_g. We now argue that the RHS of (13) is decreasing in η_g. When $\widehat{E}_h=h$ (case 1a), the third term vanishes and the first two terms are directly seen to decrease in η_g. If $(\lambda-1)^{\frac{1}{\theta}}>1$, $\widehat{E}_h=1$ can be ruled out. The case of interior $\widehat{E}_h$ is more involved, as $\widehat{E}_h=\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g}$ directly depends on η_g. The derivative of the first three terms of the RHS of (13) with respect to η_g can be shown to reduce to:

$$
\begin{aligned}
&-\frac{(1-C)(1-\theta)^2}{(\eta_g)^{2-\theta}}\left[\int_{\bar{E}_h}^{\widehat{E}_h}(E_h)^{\theta}\,dF(E_h)-(1-F(\bar{E}_h))(\bar{E}_h)^{\theta}\right]\\
&-\frac{(1-C)\left(\frac{1}{2}-\theta\right)(1-2\theta)}{(\eta_g)^{2(1-\theta)}}\left[\int_{\bar{E}_h}^{\widehat{E}_h}(E_h)^{2\theta}\,dF(E_h)-(1-F(\bar{E}_h))(\bar{E}_h)^{2\theta}\right]\\
&-\frac{(1-C)}{(\eta_g)^2}\int_{\widehat{E}_h}^{h}\left[(1-\theta)^2\lambda(\eta_g)^{\theta}(E_h)^{\theta}-\frac{1}{2}(\lambda-1)^2\right]dF(E_h).
\end{aligned}
$$

Since the terms within the square brackets are all not necessarily positive, the entire derivative cannot be readily signed. Since $\widehat{E}_h$ decreases with η_g, thereby increasing the range over which maximal profits result, there is ambiguity regarding the dependence of the RHS of (13) on η_g. For the above derivative to decrease in η_g, intuitively it appears that the concavity of profits in η_g should be strong enough; that is, θ should be low enough.

Using (13), one can show that the above derivative is negative if:

$$
\begin{aligned}
&\frac{(1-\theta)f\eta_g}{(1-C)}\left\{(1-F(\bar{E}_h))\bar{E}_h+\int_{1}^{\bar{E}_h}E_h dF(E_h)\right\}>\\
&\theta\left(\frac{1}{2}-\theta\right)\left[\int_{\bar{E}_h}^{\widehat{E}_h}(\eta_g)^{2\theta}(E_h)^{2\theta}\,dF(E_h)-(1-F(\bar{E}_h))(\eta_g)^{2\theta}(\bar{E}_h)^{2\theta}\right]+\\
&\frac{\theta}{2}(\lambda-1)^2\left(1-F\left(\widehat{E}_h\right)\right).
\end{aligned}
$$

To find a sufficient condition we investigate a low value for the LHS and a high one for the RHS. Given the definition of $\widehat{E}_h$, we can write $(\lambda-1)^2=(\eta_g)^{2\theta}\left(\widehat{E}_h\right)^{2\theta}$. Using this, evaluating the integral in the RHS at $\widehat{E}_h$, and simplifying we find one high value for the RHS is:

$$
\begin{aligned}
&\frac{\theta}{2}(\lambda-1)^2-\theta^2(\eta_g)^{2\theta}\left(\widehat{E}_h\right)^{2\theta}F\left(\widehat{E}_h\right)\\
&-\theta\left(\frac{1}{2}-\theta\right)(\eta_g)^{2\theta}\left(\widehat{E}_h\right)^{2\theta}F(\bar{E}_h)-\theta\left(\frac{1}{2}-\theta\right)(1-F(\bar{E}_h))(\eta_g)^{2\theta}(\bar{E}_h)^{2\theta}.
\end{aligned}
$$

For case 1b that we are discussing, it is true that $1<\bar{E}_h<\widehat{E}_h<h$, and $\eta_g>\eta_g^1=\frac{(\lambda-1)^{\frac{1}{\theta}}}{h}$. Using these facts, the high value for the RHS can be made more stringent, and the sufficient condition reduces to:

$$
\frac{(1-\theta)f(\lambda-1)^{\frac{1}{\theta}}}{(1-C)h}\left\{(1-F(\bar{E}_h))\bar{E}_h+\int_{1}^{\bar{E}_h}E_h dF(E_h)\right\}+\frac{\theta(\lambda-1)^2}{h^{2\theta}}\left[\left(\frac{1}{2}-\theta\right)+\theta F(\bar{E}_h)\right]>\frac{\theta}{2}(\lambda-1)^2.
$$

Since the LHS is increasing in $\bar{E}_h$, setting it to its minimum value of 1 yields the sufficient condition we seek for the RHS of (13) to decrease in η_g :

$$
\frac{(1-\theta)f(\lambda-1)^{\frac{1}{\theta}-2}}{(1-C)h}>\theta\left[\frac{1}{2}-\frac{\left(\frac{1}{2}-\theta\right)}{h^{2\theta}}\right].
$$

This condition is satisfied for $\theta\to 0$; as conjectured earlier, enough concavity of profits in η_g will ensure that the sign of the derivative will be the one we seek. When θ is at its maximum value of $\frac{1}{2}$, the above condition reduces to the simpler, but more stringent condition of $f>\frac{1}{2}h$.

A.2.3 The Nature of $\bar{E}_h(\eta_g)$

The above analysis indicates that the LHS is strictly increasing and the RHS is strictly decreasing in $\bar{E}_h$; the LHS is invariant in η_g and the RHS is strictly decreasing with η_g. These observations indicate that the intersection of the two sides, which gives the threshold $\bar{E}_h$, is decreasing in η_g. Consider condition (13), multiplied by η_g so that we can analyze the case of $\eta_g \to 0$; $\widehat{E}_h \to h$, and maximal growth is not possible for any draw in this case. Both sides of this modified condition tend to zero as $\eta_g \to 0$, but the ratio of LHS to RHS tends to zero using L'Hospital's rule. Since the RHS, the marginal benefit term of waiting for a draw, dominates the LHS, the marginal cost over all possible $\bar{E}_h$, we get $\bar{E}_h(0) = h$. Indeed $\bar{E}_h(\eta_g) = h$, for η_g in a neighborhood of 0 for which the RHS of (13) exceeds $fE(E_h)$.

Consider the following expressions for the two sides of (13) when $\eta_g \to 1$:

$$LHS\left(\bar{E}_h = 1; \eta_g = 1\right) = f; \; LHS\left(\bar{E}_h = h; \eta_g = 1\right) = fE(E_h) > 0,$$

$$RHS\left(\bar{E}_h = 1; \eta_g = 1\right) = (1-C)(1-\theta)\left[\int_1^{(\lambda-1)^{\frac{1}{\theta}}} (E_h)^{\theta} dF(E_h) - 1\right] +$$

$$(1-C)\left(\frac{1}{2}-\theta\right)\left[\int_1^{(\lambda-1)^{\frac{1}{\theta}}} (E_h)^{2\theta} dF(E_h) - 1\right] + (1-C)\int_{(\lambda-1)^{\frac{1}{\theta}}}^{h}\left[(1-\theta)\lambda(E_h)^{\theta} - \frac{1}{2}(\lambda-1)^2\right] dF(E_h),$$

$$RHS\left(\bar{E}_h = h; \eta_g = 1\right) = 0.$$

If $f < RHS\left(\bar{E}_h = 1; \eta_g = 1\right)$, $\bar{E}_h > 1$. Since the RHS is decreasing in η_g, $\bar{E}_h > 1$ at $\eta_g = 1$ guarantees the same for all η_g. The condition says that the firing cost f cannot be high enough to cause the firm to make do with *any* E_h. We will assume this for now and update it when we discuss the firm condition for case 2.

Therefore $\bar{E}_h(\eta_g)$ starts at h (and potentially stays at h for a neighborhood of zero) and decreases monotonically to a value less than 1, if the above assumption is satisfied. At the point at which the decreasing function $\bar{E}_h(\eta_g)$ crosses $\widehat{E}_h(\eta_g)$, the assumption that we are in case 1 of the firm's Bellman equation ceases to be valid because $\bar{E}_h \geq \widehat{E}_h$. Define η_g^2 implicitly by setting $\bar{E}_h = \widehat{E}_h = \frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}$ in (13). The following condition determines η_g^2 :

$$f\left\{\left(1-F\left(\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}\right)\right)\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2} + \int_1^{\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}} E_h dF(E_h)\right\} = \frac{(1-C)}{\eta_g^2}\int_{\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}}^{h}\left[(1-\theta)\lambda\left(\eta_g^2\right)^{\theta}(E_h)^{\theta} - \frac{1}{2}(\lambda-1)^2\right] dF(E_h) \tag{14}$$

$$-\frac{(1-C)(1-\theta)}{\left(\eta_g^2\right)^{1-\theta}}\left(1-F\left(\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}\right)\right)\left(\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}\right)^{\theta} - \frac{(1-C)\left(\frac{1}{2}-\theta\right)}{\left(\eta_g^2\right)^{1-2\theta}}\left(1-F\left(\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}\right)\right)\left(\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^2}\right)^{2\theta}.$$

We assume that the parameters of the model are such that $\eta_g^2 < 1$; that is, when $\eta_g^2 = 1$, the LHS of (14) is lower than the RHS.

A.3 Firm Condition for Case 2

Given that case 2 is the relevant one, equating the payoffs in (**??**) at the threshold yields:

$$Ev - f\eta_g\bar{E}_h = (1-C)\left[(1-\theta)\lambda\left(\eta_g\bar{E}_h\right)^{\theta} - \frac{1}{2}(\lambda-1)^2\right].$$

We have assumed that $\bar{E}_h > 1$ as discussed above; if we are in this case, $\bar{E}_h < h$, and therefore we can write the threshold condition as an equality. Integrate v over the two relevant segments, substitute for Ev from the threshold condition, then simplify to obtain the firm's condition for this case as:

$$f\left\{\left(1-F\left(\bar{E}_h\right)\right)\bar{E}_h+\int_1^{\bar{E}_h}E_h dF\left(E_h\right)\right\}=\frac{(1-C)(1-\theta)\lambda}{\left(\eta_g\right)^{1-\theta}}\left[\int_{\bar{E}_h}^{h}\left(E_h\right)^{\theta}dF\left(E_h\right)-\left(1-F\left(\bar{E}_h\right)\right)\left(\bar{E}_h\right)^{\theta}\right]. \tag{15}$$

A more concise version is presented in the main text as (6).

A.4 Characterizing $\bar{E}_h\left(\eta_g\right)$ for Case 2

A.4.1 Dependence on $\bar{E}_h$

The LHS of (15) is unchanged from case 1; it starts out at f when $\bar{E}_h = 1$ and increases to $fE\left(E_h\right) > f$ at $\bar{E}_h = h$, independent of η_g. In the RHS, using Leibniz' formula for the term within square brackets we find the derivative as $-\left(\bar{E}_h\right)^{\theta}\widetilde{f}\left(\bar{E}_h\right)-\theta\left(1-F\left(\bar{E}_h\right)\right)\left(\bar{E}_h\right)^{\theta-1}+\left(\bar{E}_h\right)^{\theta}\widetilde{f}\left(\bar{E}_h\right)<0$. As in case 1, the RHS is decreasing in $\bar{E}_h$.

A.4.2 Dependence on η_g

Since the LHS of (15) is increasing and the RHS decreasing in $\bar{E}_h$, an intersection is likely to occur. However, where the intersection occurs depends on η_g. The LHS does not change with η_g. In case 2, unlike case 1, detailed derivatives are not needed to argue that the RHS is decreasing in η_g; mere examination of (15) reveals this. Since $\bar{E}_h > \widehat{E}_h$, $\widehat{E}_h$ does not even figure in the condition for case 2, and the ambiguity referred to in case 1 does not arise here.

A.4.3 The Nature of $\bar{E}_h\left(\eta_g\right)$

The above analysis indicates that the intersection of the two sides, which gives the threshold $\bar{E}_h$, is decreasing in η_g. Since both sides of the expression in (13) and (15) coincide at $\eta_g = \eta_g^2$, the $\bar{E}_h\left(\eta_g\right)$ schedule is continuous at η_g^2. To characterize the relationship in the interval $\left[\eta_g^2, 1\right]$, we compute:

$$\begin{aligned}
LHS\left(\bar{E}_h=1;\eta_g=1\right) &= f;\ LHS\left(\bar{E}_h=h;\eta_g=1\right)=fE\left(E_h\right)>0,\\
RHS\left(\bar{E}_h=1;\eta_g=1\right) &= (1-C)(1-\theta)\lambda\left[\int_1^h\left(E_h\right)^{\theta}dF\left(E_h\right)-1\right];\ RHS\left(\bar{E}_h=h;\eta_g=1\right)=0.
\end{aligned}$$

An intersection clearly exists. To ensure, $\bar{E}_h > 1$, we need $f < RHS\left(\bar{E}_h = 1; \eta_g = 1\right)$. Since we will be in case 2, rather than case 1, for this corner to be relevant we update the earlier assumption on f with the following assumption:

$$f<(1-C)(1-\theta)\lambda\left[\int_1^h\left(E_h\right)^{\theta}dF\left(E_h\right)-1\right].$$

Therefore, $\bar{E}_h\left(\eta_g\right)$ starts at the case 1 value for η_g^2 and decreases monotonically to a value larger than 1, if the above assumption is satisfied. The firm condition depicted in Figure 4 is a composite of the two cases discussed thus far.

A.5 Dependence of $\bar{E}_h\left(\eta_g\right)$ on f, C, λ

In both (5) and (6), an increase in f shifts the left hand side upward. Given that the right hand sides of these conditions are decreasing in $\bar{E}_h$ and independent of f, the points of intersection shift leftward; that is, $\bar{E}_h(\eta_g)$ decreases. In the neighborhood of $\eta_g = 0$, it still is the case that $\bar{E}_h(0) = h$; the firm schedule for $\bar{E}_h(\eta_g) < h$ shifts down with an increase in f.

For a given η_g, when the cost of entry, C, increases the right hand sides of both (5) and (6) shift downward. Given that the left hand sides are increasing in $\bar{E}_h$ and independent of C, the points of intersection shift leftward; that is, $\bar{E}_h(\eta_g)$ decreases. In the neighborhood of $\eta_g = 0$, it will still be the case that $\bar{E}_h(0) = h$; the firm schedule for $\bar{E}_h(\eta_g) < h$ shifts down when there is an increase in C.

We can see from (1) that the $\widehat{E}_h\left(\eta_g\right)$ threshold curve shifts right when λ increases, and the thresholds η_g^1 and η_g^3 increase.

In both (5) and (6), the left hand sides – the marginal cost of firing and redrawing – do not depend on λ. Below, we provide sufficient conditions to ensure that the right hand sides are increasing in λ for cases other than case 1a, where only interior growth rates are realized. Given the nature of the two sides discussed above, the points of intersection shift rightward; that is, $\bar{E}_h(\eta_g)$ increases. Thus $\bar{E}_h\left(\eta_g\right)$ shifts up for cases other than case 1a.

A mere examination of the RHS of (15) shows that it is increasing in λ. To see if this is true for the RHS of (13), we take the derivative with respect to λ. This derivative can be simplified to:

$$\frac{(1-C)}{\eta_g}\int_{\widehat{E}_h}^{h}\left[(1-\theta)\left(\eta_g\right)^{\theta}\left(E_h\right)^{\theta}-(\lambda-1)\right]dF\left(E_h\right)$$

This expression is not automatically positive; the fact that profits are positive when growth equals λ does not imply that its derivative is positive. We need to make distributional assumptions to unambiguously sign the derivative. Anticipating the calibration, we assume a uniform distribution for productivity draws: $\widetilde{f}\left(E_h\right) = \frac{1}{h-1}$. Carrying out the integration in the above derivative, and using the definition of $\widehat{E}_h$ we can show that the expression is positive if:

$$\frac{(1-\theta)}{(1+\theta)}\left(\eta_g\right)^{\theta}(h)^{\theta} > (\lambda-1) - \frac{2\theta}{(1+\theta)}\frac{(\lambda-1)^{1+\frac{1}{\theta}}}{\eta_g h}.$$

For this case (1b), $\widehat{E}_h \leq h$, implies that $\left(\eta_g\right)^{\theta}(h)^{\theta}$ in the LHS cannot be smaller than $(\lambda-1)$, and the RHS is largest at $\eta_g = 1$. Make these substitutions to get a stringent sufficient condition for the derivative to be positive as:

$$(\lambda-1)^{\frac{1}{\theta}} > h.$$

Consistent with the sufficient condition in section (A.2), a low θ and a low h are more likely to satisfy the above condition and cause the RHS of (13) to increase in λ. Given the characterization of $\bar{E}_h\left(\eta_g\right)$ discussed earlier, the firm condition increases (shifts upward) when λ increases.

A.6 Household Condition for Case 1

Substituting the relevant expressions for the wages in (7), we find:

$$\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\int_{\bar{E}_h}^{\widehat{E}_h}\log\left[\theta\left(\eta_g\right)^{\theta-1}\left(E_h\right)^{\theta}+\theta\left(\eta_g\right)^{2\theta-1}\left(E_h\right)^{2\theta}\right]dF\left(E_h\right)+\int_{\widehat{E}_h}^{h}\log\left[\theta\lambda\left(\eta_g\right)^{\theta-1}\left(E_h\right)^{\theta}\right]dF\left(E_h\right)\right\}$$
$$\gtreqless \log\left[\theta\beta h^{\theta}/\left(1-\eta_g\right)^{1-\theta}\right]-\log\left(1-\eta_g\right)-\log\left(s\right). \tag{16}$$

Factoring out $\left(\eta_g\right)^{2\theta-1}$ from the two integrals when $\eta_g > 0$ – we will discuss the $\eta_g = 0$ case separately – on the left hand side, transposing terms, and noting that $1-2\theta>0$, we obtain the expression that implicitly defines the household condition $\bar{E}_h^{HH}\left(\eta_g\right)$ as:

$$\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\int_{\bar{E}_h}^{\widehat{E}_h}\log\left[\theta\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}+\theta\left(E_h\right)^{2\theta}\right]dF\left(E_h\right)+\int_{\widehat{E}_h}^{h}\log\left[\theta\lambda\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}\right]dF\left(E_h\right)\right\}$$
$$\gtreqless \log\left[\theta\beta h^{\theta}\right]+\left(1-2\theta\right)\log\left(\eta_g\right)-\left(2-\theta\right)\log\left(1-\eta_g\right)-\log\left(s\right).$$

A.7 Characterizing $\bar{E}_h^{HH}\left(\eta_g\right)$ for Case 1

A.7.1 Dependence on $\bar{E}_h$

The RHS of (10) is independent of $\bar{E}_h$. Differentiate the LHS with respect to $\bar{E}_h$ to get:

$$\frac{\widetilde{f}\left(\bar{E}_h\right)}{\left(1-F\left(\bar{E}_h\right)\right)^2}\left\{\int_{\bar{E}_h}^{\widehat{E}_h}\log\left[\theta\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}+\theta\left(E_h\right)^{2\theta}\right]dF\left(E_h\right)+\int_{\widehat{E}_h}^{h}\log\left[\theta\lambda\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}\right]dF\left(E_h\right)\right\}$$
$$+\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{-\log\left[\theta\left(\eta_g\right)^{-\theta}\left(\bar{E}_h\right)^{\theta}+\theta\left(\bar{E}_h\right)^{2\theta}\right]\widetilde{f}\left(\bar{E}_h\right)\right\}.$$

Since wages are higher in the maximum growth region (at $E_h=\widehat{E}_h$, interior and maximal growth wages are the same, and for higher E_h, maximal growth wages are higher), the second term is larger than the first. Therefore the derivative is:

$$> \frac{\widetilde{f}\left(\bar{E}_h\right)}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\int_{\bar{E}_h}^{h}\log\left[\theta\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}+\theta\left(E_h\right)^{2\theta}\right]dF\left(E_h\right)-\log\left[\theta\left(\eta_g\right)^{-\theta}\left(\bar{E}_h\right)^{\theta}+\theta\left(\bar{E}_h\right)^{2\theta}\right]\right\}$$
$$> \frac{\widetilde{f}\left(\bar{E}_h\right)}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\log\left[\theta\left(\eta_g\right)^{-\theta}\left(\bar{E}_h\right)^{\theta}+\theta\left(\bar{E}_h\right)^{2\theta}\right]\left(1-F\left(\bar{E}_h\right)\right)-\log\left[\theta\left(\eta_g\right)^{-\theta}\left(\bar{E}_h\right)^{\theta}+\theta\left(\bar{E}_h\right)^{2\theta}\right]\right\}=0.$$

Therefore the LHS of (10) is increasing in $\bar{E}_h$. Taken together with the flat RHS, this means that an intersection is likely to occur. However, where the intersection occurs depends on η_g.

A.7.2 Dependence on η_g

It can directly be seen that the RHS of (10) is increasing in η_g. To study the dependence of the LHS of (10) on η_g, differentiate the terms within curly braces and simplify to obtain the derivative as:

$$-\int_{\bar{E}_h}^{\widehat{E}_h}\frac{\theta^2\left(\eta_g\right)^{-\theta-1}\left(E_h\right)^{\theta}}{\theta\left(\eta_g\right)^{-\theta}\left(E_h\right)^{\theta}+\theta\left(E_h\right)^{2\theta}}dF\left(E_h\right)-\int_{\widehat{E}_h}^{h}\theta\left(\eta_g\right)^{-1}dF\left(E_h\right)<0.$$

That is, the LHS of (10) is decreasing in η_g.

A.7.3 The Nature of $\bar{E}_h^{HH}\left(\eta_g\right)$

The above analysis indicates that the LHS of (10) is strictly increasing and the RHS is independent of $\bar{E}_h$; the LHS is strictly decreasing with η_g and the RHS is strictly increasing with η_g. These results indicate that the intersection of the two sides, which gives the threshold $\bar{E}_h$ necessary for any given labor supply η_g, is increasing in η_g.

First examine the case of $\eta_g \to 0$. Working with (16), it can be seen that both $LHS\left(\bar{E}_h = 1; \eta_g = 0\right)$ and $LHS\left(\bar{E}_h = h; \eta_g = 0\right)$ diverge to ∞. Examination of the RHS of (10) reveals that $RHS\left(\eta_g = 0\right) \to \log\left[\theta\beta h^{\theta}\right] - \log(s)$, a finite quantity. Since the benefit of general education exceeds the cost for any $\bar{E}_h$, the corner case of $\bar{E}_h = 1$ results. It is likely for a neighborhood of $\eta_g = 0$ that $LHS > RHS$ and $\bar{E}_h = 1$. When η_g is very low, relative wages are so high that even an $\bar{E}_h = 1$ is enough to attract entry into general education.

The situation is reversed for $\eta_g \to 1$. We have that the is LHS of (10) well-defined at both endpoints of $\bar{E}_h$:

$$\begin{aligned} LHS\left(\bar{E}_h = 1; \eta_g = 1\right) &= \left\{\int_1^{\widehat{E}_h} \log\left[\theta\left(E_h\right)^{\theta} + \theta\left(E_h\right)^{2\theta}\right] dF\left(E_h\right) + \int_{\widehat{E}_h}^{h} \log\left[\theta\lambda\left(E_h\right)^{\theta}\right] dF\left(E_h\right)\right\} \\ LHS\left(\bar{E}_h = h; \eta_g = 1\right) &= \log\left[\theta\left(h\right)^{\theta} + \theta\left(h\right)^{2\theta}\right]. \end{aligned}$$

When $\bar{E}_h = h$, given case 1 requires $\widehat{E}_h \geq \bar{E}_h$, we need to set $\widehat{E}_h = h$; L'Hospital's rule is needed to evaluate the LHS. However, $RHS\left(\eta_g = 1\right) \to \infty$. Since the cost exceeds the benefit for any $\bar{E}_h$, the corner case of $\bar{E}_h = h$ results. It is likely for a neighborhood of $\eta_g = 1$ that $LHS < RHS$ and $\bar{E}_h = h$. When η_g is very high, even the lowest ability agents have to obtain general education, and the disutility is so prohibitive that the $\bar{E}_h$, and thus the expected wage premium of generally educated agents, has to be very high to induce entry.

This analysis indicates that the $\bar{E}_h^{HH}\left(\eta_g\right)$ curve starts at 1, (potentially) stays at 1 for an interval, increases up to h, and (potentially) stays at h for an interval. At some $\eta_g = \eta_g^{HH}$ the $\bar{E}_h^{HH}\left(\eta_g\right)$ curve will intersect with $\widehat{E}_h\left(\eta_g\right)$. Define η_g^{HH} implicitly, by setting $\bar{E}_h = \widehat{E}_h = \frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^{HH}}$ in (10):

$$\begin{aligned} &\frac{1}{\left(1 - F\left(\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^{HH}}\right)\right)} \left\{\int_{\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g^{HH}}}^{h} \log\left[\theta\lambda\left(\eta_g^{HH}\right)^{-\theta}\left(E_h\right)^{\theta}\right] dF\left(E_h\right)\right\} \\ = \; &\log\left[\theta\beta h^{\theta}\right] + (1-2\theta)\log\left(\eta_g^{HH}\right) - (2-\theta)\log\left(1-\eta_g^{HH}\right) - \log(s). \end{aligned}$$

Case 2, which is discussed below, becomes relevant beyond η_g^{HH}.

A.8 Household Condition for Case 2

Substituting the relevant expressions for the wages in (7), factoring out $\left(\eta_g\right)^{\theta-1}$ from the integral on the left hand side, and transposing terms we obtain the expression that implicitly defines the household condition $\bar{E}_h^{HH}\left(\eta_g\right)$ in this case as:

$$\begin{aligned} &\frac{1}{\left(1 - F\left(\bar{E}_h\right)\right)} \left\{\int_{\bar{E}_h}^{h} \log\left[\theta\lambda\left(E_h\right)^{\theta}\right] dF\left(E_h\right)\right\} \\ \leq \; &\log\left[\theta\beta h^{\theta}\right] + (1-\theta)\log\left(\eta_g\right) - (2-\theta)\log\left(1-\eta_g\right) - \log(s). \end{aligned}$$

A.9 Characterizing $\bar{E}_h^{HH}\left(\eta_g\right)$ for Case 2

A.9.1 Dependence on $\bar{E}_h$

The RHS of (11) is independent of $\bar{E}_h$. The derivative of the LHS of (11) with respect to $\bar{E}_h$ is:

$$\begin{aligned} &= \frac{\widetilde{f}\left(\bar{E}_h\right)}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\int_{\bar{E}_h}^{h}\log\left[\theta\lambda\left(E_h\right)^{\theta}\right]dF\left(E_h\right)-\log\left[\theta\lambda\left(\bar{E}_h\right)^{\theta}\right]\right\} \\ &> \frac{\widetilde{f}\left(\bar{E}_h\right)}{\left(1-F\left(\bar{E}_h\right)\right)}\left\{\frac{1}{\left(1-F\left(\bar{E}_h\right)\right)}\log\left[\theta\lambda\left(\bar{E}_h\right)^{\theta}\right]\left(1-F\left(\bar{E}_h\right)\right)-\log\left[\theta\lambda\left(\bar{E}_h\right)^{\theta}\right]\right\}=0. \end{aligned}$$

A.9.2 Dependence on η_g

The LHS of (11) is independent of η_g. Mere examination reveals that the RHS is increasing in η_g.

A.9.3 The Nature of $\bar{E}_h^{HH}\left(\eta_g\right)$

The above analysis indicates that the LHS of (11) is strictly increasing and the RHS is independent of $\bar{E}_h$; the LHS is independent of η_g and the RHS is strictly increasing with η_g. These results indicate that the intersection of the two sides, which gives the threshold $\bar{E}_h$ necessary for any given labor supply η_g, is increasing in η_g.

Since both sides of the expression in (10) and (11) coincide at $\eta_g=\eta_g^{HH}$, the $\bar{E}_h^{HH}\left(\eta_g\right)$ schedule is continuous at η_g^{HH}. To characterize the relationship in the interval $\left[\eta_g^{HH},1\right]$, we compute for (11):

$$\begin{aligned} LHS\left(\bar{E}_h=\bar{E}_h^{HH}\right) &= \frac{1}{\left(1-F\left(\bar{E}_h^{HH}\right)\right)}\left\{\int_{\bar{E}_h^{HH}}^{h}\log\left[\theta\lambda\left(E_h\right)^{\theta}\right]dF\left(E_h\right)\right\}>\log\left[\theta\lambda\left(h\right)^{\theta}\right] \\ LHS\left(\bar{E}_h=h\right) &= \frac{-\log\left[\theta\lambda\left(h\right)^{\theta}\right]\widetilde{f}\left(h\right)}{-\widetilde{f}\left(h\right)}=\log\left[\theta\lambda\left(h\right)^{\theta}\right], \end{aligned}$$

where L'Hospital's rule has been used for the second evaluation. The LHS is independent of η_g and is an increasing function in $\bar{E}_h$, with finite endpoints.

However, the $RHS\left(\eta_g=1\right)$ diverges to ∞. Therefore, when $\eta_g=1$, the cost of general education exceeds the benefit for any $\bar{E}_h$, and $\bar{E}_h=h$ results; this is likely to be true for a neighborhood of $\eta_g=1$.

A.10 Dependence of $\bar{E}_h^{HH}\left(\eta_g\right)$ on λ

Take the derivative of the terms in curly braces in the LHS of (10), noting $\widehat{E}_h=\frac{(\lambda-1)^{\frac{1}{\theta}}}{\eta_g}$, with respect to λ. This derivative is $\int_{\widehat{E}_h}^{h}\frac{1}{\lambda}dF\left(E_h\right)>0$.