A. Separate On-line Appendix

This a separate on-line appendix for 'When is Market Incompleteness Irrelevant for the Price of Aggregate Risk (and When is it not)?'. This appendix defines an equilibrium in the Arrow model and lists the first-order conditions for optimality in the detrended Arrow model in section A.1. Section A.2 provides a detailed explanation of the environment in the Bond economy, the definition of equilibrium as well as the first-order conditions for optimality. Section A.3 briefly discusses the representative agent economy. Section A.4 describes a single-input production economy in which the irrelevance result holds. Section A.5 analyzes the case of counter-cyclical labor income shares. Finally, section A.6 analyzes the case of non-expected utility.

A.1. Arrow Model

The definition of an equilibrium in the Arrow model is standard. Each household is assigned a label that consists of its initial financial wealth θ_0 and its initial state $s_0 = (y_0, z_0)$. A household of type (θ_0, s_0) then chooses consumption allocations $\{c_t(\theta_0, s^t)\}$, trading strategies for Arrow securities $\{a_t(\theta_0, s^t, z_{t+1})\}$ and shares $\{\sigma_t(\theta_0, s^t)\}$ to maximize her expected utility (1), subject to the budget constraints (10) and subject to solvency constraints (12) or (13).

Definition A.1. *For initial aggregate state z_0 and distribution Θ_0 over (θ_0, y_0), a competitive equilibrium for the Arrow model consists of household allocations $\{a_t(\theta_0, s^t, z_{t+1})\}$, $\{\sigma_t(\theta_0, s^t)\}$, $\{c_t(\theta_0, s^t)\}$ and prices $\{q_t(z^t, z_{t+1})\}$, $\{v_t(z^t)\}$ such that*

1. *Given prices, household allocations solve the household maximization problem*
2. *The goods market clears for all z^t,*

$$\int \sum_{y^t} \frac{\pi(y^t, z^t | y_0, z_0)}{\pi(z^t | z_0)} c_t(\theta_0, s^t) d\Theta_0 = e_t(z^t)$$

3. *The asset markets clear for all z^t*

$$\begin{aligned}
\int \sum_{y^t} \frac{\pi(y^t, z^t | y_0, z_0)}{\pi(z^t | z_0)} \sigma_t(\theta_0, s^t) d\Theta_0 &= 1 \\
\int \sum_{y^t} \frac{\pi(y^t, z^t | y_0, z_0)}{\pi(z^t | z_0)} a_t(\theta_0, s^t, z_{t+1}) d\Theta_0 &= 0 \text{ for all } z_{t+1} \in Z
\end{aligned}$$

A.1.1. Optimality Conditions for De-trended Arrow Model

Define the Lagrange multiplier

$$\hat{\beta}(s^t)\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\hat{\mu}(s^t) \geq 0$$

for the constraint in (15) and

$$\hat{\beta}(s^t)\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\hat{\kappa}_t(s^t, z_{t+1}) \geq 0$$

for the constraint in (16). The Euler equations of the de-trended Arrow model are given by:

$$\begin{aligned}
1 &= \frac{\hat{\beta}(s_t)}{\hat{q}_t(z^t, z_{t+1})} \sum_{s^{t+1}|s^t, z_{t+1}} \hat{\pi}(s_{t+1}|s_t) \frac{u'(\hat{c}_{t+1}(s^t, s_{t+1}))}{u'(\hat{c}_t(s^t))} \\
&\quad + \hat{\mu}_t(s^t) + \frac{\hat{\kappa}_t(s^t, z_{t+1})}{\hat{q}_t(z^t, z_{t+1})} \; \forall z_{t+1}. \qquad (44) \\
1 &= \hat{\beta}(s_t) \sum_{s^{t+1}|s^t} \hat{\pi}(s_{t+1}|s_t) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right] \frac{u'(\hat{c}_{t+1}(s^t, s_{t+1}))}{u'(\hat{c}_t(s^t))} \\
&\quad + \hat{\mu}_t(s^t) + \sum_{z_{t+1}} \hat{\kappa}_t(s^t, z_{t+1}) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right]. \qquad (45)
\end{aligned}$$

Only one of the two Lagrange multipliers enters the equations, depending on which version of the solvency constraint we consider. The complementary slackness conditions for the Lagrange multipliers are given by

$$\begin{aligned}
\hat{\mu}_t(s^t) \left[\sum_{z_{t+1}} \hat{a}_t(s^t, z_{t+1}) \hat{q}_t(z^t, z_{t+1}) + \hat{\sigma}_t(s^t) \hat{v}_t(z^t) - \hat{K}_t(y^t)\right] &= 0 \\
\hat{\kappa}_t(s^t, z_{t+1}) \left[\hat{a}_t(s^t, z_{t+1}) + \hat{\sigma}_t(s^t) \left[\hat{v}_{t+1}(z^{t+1}) + \alpha\right] - \hat{M}_t(y^t)\right] &= 0.
\end{aligned}$$

The appropriate transversality conditions read as

$$\begin{aligned}
\lim_{t\to\infty} \sum_{s^t} \hat{\beta}(s^{t-1}) \hat{\pi}(s^t|s_0) u'(\hat{c}_t(s^t)) [\hat{a}_{t-1}(s^{t-1}, z_t) - \hat{M}_{t-1}(y^{t-1})] &= 0. \\
\lim_{t\to\infty} \sum_{s^t} \hat{\beta}(s^{t-1}) \hat{\pi}(s^t|s_0) u'(\hat{c}_t(s^t)) [\hat{\sigma}_{t-1}(s^{t-1})(\hat{v}_t(z^t) + \alpha) - \hat{M}_{t-1}(y^{t-1})] &= 0,
\end{aligned}$$

and

$$\begin{aligned}
\lim_{t\to\infty} \sum_{s^t} \hat{\beta}(s^{t-1}) \hat{\pi}(s^t|s_0) u'(\hat{c}_t(s^t)) \left[\sum_{z_{t+1}} \hat{a}_t(s^t, z_{t+1}) \hat{q}_t(z^t, z_{t+1}) - \hat{K}_t(y^t)\right] &= 0. \\
\lim_{t\to\infty} \sum_{s^t} \hat{\beta}(s^{t-1}) \hat{\pi}(s^t|s_0) u'(\hat{c}_t(s^t)) \left[\hat{\sigma}_t(s^t) \hat{v}_t(z^t) - \hat{K}_t(y^t)\right] &= 0.
\end{aligned}$$

Since the household optimization has a concave objective function and a convex constraint set the first order conditions and complementary slackness conditions, together with the transversality condition, are necessary and sufficient conditions for optimality of household allocation choices.

A.2. The Bond Model

We now turn our attention to the main model of interest, namely the model with a stock and a single one-period discount bond $b_t(\theta_0, s^t)$; we shut down the contingent claims market. However, in the de-trended Arrow model, we demonstrated that contingent claims positions were in fact uncontingent: $\hat{a}_t(\theta_0, s^t, z_{t+1}) = \hat{a}_t(\theta_0, y^t)$ and equal to the Bond position in the Bewley equilibrium. We also argued in the Bewley model that we could focus, without loss of generality, on equilibria in which $\hat{a}_t(\theta_0, y^t) \equiv 0$. That immediately implies that we can implement the Arrow equilibrium allocations in the Bond model. In this equilibrium, there is no trade in the non-contingent bond market: $b_t(\theta_0, s^t) = 0$.

This section establishes the equivalence of equilibria in the Bond model and the Bewley model by showing that the optimality conditions in the de-trended Arrow and the de-trended Bond model are identical.

A.2.1. Market Structure

In the Bond model, agents only trade a one-period discount bond and a stock. An agent who starts period t with initial wealth composed of his stock holdings, bond and stock payouts, and labor income, buys consumption commodities in the spot market and trades a one-period bond and the stock, subject to the budget constraint:

$$c_t(s^t)+\frac{b_t(s^t)}{R_t(z^t)}+\sigma_t(s^t)v_t(z^t) \leq (1-\alpha)\eta(y_t)e_t(z_t)+b_{t-1}(s^{t-1})+\sigma_{t-1}(s^{t-1})\left[v_t(z^t)+\alpha e_t(z_t)\right]. \tag{46}$$

Here, $b_t(s^t)$ denotes the amount of bonds purchased and $R_t(z^t)$ is the gross interest rate from period t to $t+1$. As was the case in the Arrow model, short-sales of the bond and the stock are constrained by a lower bound on the value of the portfolio today,

$$\frac{b_t(s^t)}{R_t(z^t)}+\sigma_t(s^t)v_t(z^t) \geq K_t(s^t), \tag{47}$$

or a state-by-state constraint on the value of the portfolio tomorrow,

$$b_t(s^t)+\sigma_t(s^t)\left[v_{t+1}(z^{t+1})+\alpha e_{t+1}(z_{t+1})\right] \geq M_t(s^t, z_{t+1}) \text{ for all } z_{t+1}. \tag{48}$$

Since $b_t(s^t)$ and $\sigma_t(s^t)$ are chosen before z_{t+1} is realized, at most one of the constraints (48) will be binding at a given time.

Definition A.2. *For an initial aggregate state z_0 and distribution Θ_0 over (θ_0, y_0), a competitive equilibrium for the Bond model consists of household allocations $\{b_t(\theta_0, s^t)\}$, $\{c_t(\theta_0, s^t)\}$, $\{\sigma_t(\theta_0, s^t)\}$, and interest rates $\{R_t(z^t)\}$ and share prices $\{v_t(z^t)\}$ such that*

1. *Given prices, allocations solve the household maximization problem.*
2. *The goods market clears for all z^t:*

$$\int \sum_{y^t} \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} c_t(\theta_0, s^t) d\Theta_0 = e_t(z^t).$$

3. *The asset markets clear for all z^t:*

$$\int \sum_{y^t} \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} \sigma_t(\theta_0, s^t) d\Theta_0 = 1.$$

$$\int \sum_{y^t} \frac{\pi(y^t, z^t|y_0, z_0)}{\pi(z^t|z_0)} b_t(\theta_0, s^t) d\Theta_0 = 0.$$

The optimality conditions for the Bond Model are standard. Define the Lagrange multiplier

$$\hat{\beta}(s^t)\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\hat{\mu}(s^t) \geq 0$$

for the constraint in (52) and

$$\hat{\beta}(s^t)\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\hat{\kappa}_t(s^t, z_{t+1}) \geq 0$$

for the constraint in (53). In the detrended Bond model the Euler equations read as

$$\begin{aligned} 1 &= \hat{\beta}(s_t) \sum_{s^{t+1}|s^t} \hat{\pi}(s_{t+1}|s_t) \left[\frac{R_t(z^t)}{\lambda(z_{t+1})}\right] \frac{u'(\hat{c}_{t+1}(s^t, s_{t+1}))}{u'(\hat{c}_t(s^t))} \\ &\quad + \hat{\mu}_t(s^t) + \sum_{z_{t+1}} \hat{\kappa}_t(s^t, z_{t+1}) \left[\frac{R_t(z^t)}{\lambda(z_{t+1})}\right]. \end{aligned} \tag{49}$$

$$\begin{aligned} 1 &= \hat{\beta}(s_t) \sum_{s^{t+1}|s^t} \hat{\pi}(s_{t+1}|s_t) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right] \frac{u'(\hat{c}_{t+1}(s^t, s_{t+1}))}{u'(\hat{c}_t(s^t))}. \\ &\quad + \hat{\mu}_t(s^t) + \sum_{z_{t+1}} \hat{\kappa}_t(s^t, z_{t+1}) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right], \end{aligned} \tag{50}$$

with complementary slackness conditions given by:

$$\hat{\mu}_t(s^t)\left[\frac{\hat{b}_t(s^t)}{R_t(z^t)} + \hat{\sigma}_t(s^t)\hat{v}_t(z^t) - \hat{K}_t(y^t)\right] = 0$$

$$\hat{\kappa}_t(s^t, z_{t+1})\left[\frac{\hat{b}_t(s^t)}{\lambda(z_{t+1})} + \hat{\sigma}_t(s^t)\left[\hat{v}_{t+1}(z^{t+1}) + \alpha\right] - \hat{M}_t(y^t)\right] = 0.$$

The transversality conditions are given by

$$\lim_{t\to\infty} \sum_{s^t} \hat{\beta}(s^{t-1})\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\left[\frac{\hat{b}_{t-1}(s^{t-1})}{\lambda(z_t)} - \hat{M}_{t-1}(y^{t-1})\right] = 0.$$

$$\lim_{t\to\infty} \sum_{s^t} \hat{\beta}(s^{t-1})\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))[\hat{\sigma}_{t-1}(s^{t-1})(\hat{v}_t(z^t) + \alpha) - \hat{M}_{t-1}(y^{t-1})] = 0$$

and

$$\lim_{t\to\infty}\sum_{s^t}\hat{\beta}(s^{t-1})\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\left[\frac{\hat{b}_t(s^t)}{R_t(z^t)}-\hat{K}_t(y^t)\right] = 0.$$

$$\lim_{t\to\infty}\sum_{s^t}\hat{\beta}(s^{t-1})\hat{\pi}(s^t|s_0)u'(\hat{c}_t(s^t))\left[\hat{\sigma}_t(s^t)\hat{v}_t(z^t)-\hat{K}_t(y^t)\right] = 0.$$

We now show that the equilibria in the Arrow and the Bond model coincide. As a corollary, it follows that the asset pricing implications of both models are identical. In order to do so, we first transform the model with growth into a stationary, de-trended model.

A.2.2. Equilibrium in the De-trended Bond Model

Dividing the budget constraint (46) by $e_t(z^t)$ we obtain

$$\hat{c}_t(s^t)+\frac{\hat{b}_t(s^t)}{R_t(z^t)}+\sigma_t(s^t)\hat{v}_t(z^t)\leq(1-\alpha)\eta(y_t)+\frac{\hat{b}_{t-1}(s^{t-1})}{\lambda(z_t)}+\sigma_{t-1}(s^{t-1})\left[\hat{v}_t(z^t)+\alpha\right], \quad (51)$$

where we define the deflated bond position as $\hat{b}_t(s^t)=\frac{b_t(s^t)}{e_t(z^t)}$. Using condition (4.4), the solvency constraints in the de-trended model are simply:

$$\frac{\hat{b}_t(s^t)}{R_t(z^t)}+\sigma_t(s^t)\hat{v}_t(z^t) \geq \hat{K}_t(y^t), \text{ or} \quad (52)$$

$$\frac{\hat{b}_t(s^t)}{\lambda(z_{t+1})}+\sigma_t(s^t)\left[\hat{v}_{t+1}(z^{t+1})+\alpha\right] \geq \hat{M}_t(y^t) \text{ for all } z_{t+1}. \quad (53)$$

The definition of equilibrium in the de-trended Bond model is straightforward and hence omitted. We now show that equilibrium consumption allocations in the de-trended Bond model coincide with those of the Arrow model.

A.2.3. Equivalence Results

As for the Arrow model, we can show that the Bewley equilibrium allocations and prices constitute, after appropriate scaling by endowment (growth) factors, an equilibrium of the Bond model with growth.

Theorem A.1. *An equilibrium of a stationary Bewley model, given by trading strategies* $\{\hat{c}_t(\theta_0,y^t),\hat{a}_t(\theta_0,y^t),\hat{\sigma}_t(\theta_0,y^t)\}$ *and prices* $\{\hat{R}_t,\hat{v}_t\}$*, can be made into an equilibrium for the Bond model with growth,* $\{b_t(\theta_0,s^t)\}$, $\{c_t(\theta_0,s^t)\}$, $\{\sigma_t^B(\theta_0,s^t)\}$ *and* $\{R_t(z^t)\}$ *and* $\{v_t(z^t)\}$ *where*

$$\begin{aligned}
c_t(\theta_0,s^t) &= \hat{c}_t(\theta_0,y^t)e_t(z^t)\\
\sigma_t^B(\theta_0,s^t) &= \frac{\hat{a}_t(\theta_0,y^t)}{[\hat{v}_{t+1}+\alpha]}+\hat{\sigma}_t(\theta_0,y^t)\\
v_t(z^t) &= \hat{v}_te_t(z^t)\\
R_t(z^t) &= \hat{R}_t*\frac{\sum_{z_{t+1}}\phi(z_{t+1})\lambda(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1})\lambda(z_{t+1})^{-\gamma}}
\end{aligned}$$

and bond holdings given by $b_t(\theta_0, s^t) = 0$.

Proof of Theorem A.1:

Proof. As in the Arrow model, the crucial part of the proof is to argue that Bewley equilibrium allocations and prices can be made into an equilibrium for the de-trended Bond model. The Euler equations of the Bewley model where given in (31) and (32).

The corresponding Euler equations for the de-trended Bond model, evaluated at the Bewley equilibrium allocations and Lagrange multipliers $\hat{\mu}(y^t)$ and $\hat{\kappa}_t(y^t)\hat{\phi}(z_{t+1})$, read as (see (49) and (50))

$$\begin{aligned} 1 &= \hat{\beta}(s_t) \sum_{s^{t+1}|s^t} \hat{\pi}(s_{t+1}|s_t) \left[\frac{R_t(z^t)}{\lambda(z_{t+1})}\right] \frac{u'(\hat{c}_{t+1}(y^t, y_{t+1}))}{u'(\hat{c}_t(y^t))} \\ &\quad + \hat{\mu}(y^t) + \hat{\kappa}_t(y^t) \sum_{z_{t+1}} \hat{\phi}(z_{t+1}) \left[\frac{R_t(z^t)}{\lambda(z_{t+1})}\right]. \\ 1 &= \hat{\beta}(s_t) \sum_{s^{t+1}|s^t} \hat{\pi}(s_{t+1}|s_t) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right] \frac{u'(\hat{c}_{t+1}(y^t, y_{t+1}))}{u'(\hat{c}_t(y^t))} \\ &\quad + \hat{\mu}(y^t) + \hat{\kappa}_t(y^t) \sum_{z_{t+1}} \hat{\phi}(z_{t+1}) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right]. \end{aligned}$$

Using the conjectured prices, we note that

$$\begin{aligned} \hat{v}_t(z^t) &= \hat{v}_t. \\ R_t(z^t) &= \hat{R}_t * \frac{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{-\gamma}}. \end{aligned}$$

Now we use the independence and i.i.d. assumptions, which imply

$$\begin{aligned} \hat{\pi}(s_{t+1}|s_t) &= \varphi(y_{t+1}|y_t)\hat{\phi}(z_{t+1}) \\ \hat{\beta}(s_t) &= \hat{\beta}. \end{aligned}$$

Furthermore by definition of $\hat{\phi}(z_{t+1})$,

$$R_t(z^t) \sum_{z_{t+1}} \frac{\hat{\phi}(z_{t+1})}{\lambda(z_{t+1})} = R_t(z^t) \frac{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{-\gamma}}{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{1-\gamma}} = \hat{R}_t$$

and thus the Euler equations can be restated as:

$$1 = \hat{\beta}\hat{R}_t \sum_{y^{t+1}|y^t} \varphi(y_{t+1}|y_t) \frac{u'(\hat{c}_{t+1}(y^t, y_{t+1}))}{u'(\hat{c}_t(y^t))} + \hat{\mu}(y^t) + \hat{\kappa}_t(y^t)\hat{R}_t. \tag{54}$$

$$\begin{aligned} 1 &= \hat{\beta} \sum_{y^{t+1}|y^t} \varphi(y_{t+1}|y_t) \left[\frac{\hat{v}_{t+1}+\alpha}{\hat{v}_t}\right] \frac{u'(\hat{c}_{t+1}(y^t, y_{t+1}))}{u'(\hat{c}_t(y^t))} \sum_{z_{t+1}} \hat{\phi}(z_{t+1}) \\ &+ \hat{\mu}(y^t) + \hat{\kappa}_t(y^t) \left[\frac{\hat{v}_{t+1}+\alpha}{\hat{v}_t}\right] \sum_{z_{t+1}} \hat{\phi}(z_{t+1}), \end{aligned} \tag{55}$$

which again are, given that $\sum_{z_{t+1}} \hat{\phi}(z_{t+1}) = 1$, exactly the Euler conditions (31) and (32) of the Bewley model and hence satisfied by the Bewley equilibrium allocations. For the Bewley model, the complementary slackness conditions were given in (37) and (38), and for the de-trended Bond model, evaluated at the proposed allocations in the theorem (which had bond holdings equal to zero) these equations are given by:

$$\begin{aligned} \hat{\mu}_t(y^t)\left[\hat{\sigma}_t^B(y^t)\hat{v}_t - \hat{K}_t(y^t)\right] &= \hat{\mu}_t(y^t)\left[\left(\frac{\hat{a}_t(y^t)}{[\hat{v}_{t+1}+\alpha]} + \hat{\sigma}_t(y^t)\right)\hat{v}_t - \hat{K}_t(y^t)\right] \\ &= \hat{\mu}_t(y^t)\left[\frac{\hat{a}_t(y^t)}{\hat{R}_t} + \hat{\sigma}_t(y^t)\hat{v}_t - \hat{K}_t(y^t)\right] = 0, \end{aligned}$$

and

$$\begin{aligned} \hat{\kappa}_t(y^t)\left[\hat{\sigma}_t^B(y^t)\left[\hat{v}_{t+1}+\alpha\right] - \hat{M}_t(y^t)\right] &= \hat{\kappa}_t(y^t)\left[\left(\frac{\hat{a}_t(y^t)}{[\hat{v}_{t+1}+\alpha]} + \hat{\sigma}_t(y^t)\right)\left[\hat{v}_{t+1}+\alpha\right] - \hat{M}_t(y^t)\right] \\ &= \hat{\kappa}_t(y^t)\left[\hat{a}_t(y^t) + \hat{\sigma}_t(y^t)\left[\hat{v}_{t+1}+\alpha\right] - \hat{M}_t(y^t)\right] = 0/\hat{\phi}(z_{t+1}), \end{aligned}$$

where we use the fact that the Bewley equilibrium prices and interest rates satisfy

$$\hat{R}_t = \frac{\hat{v}_{t+1}+\alpha}{\hat{v}_t}.$$

These complementary slackness conditions are satisfied since the Bewley equilibrium allocations satisfy the complementary slackness conditions in the Bewley model. The argument is exactly identical for the transversality conditions. Finally, we have to check whether the allocations proposed in the theorem satisfy the de-trended Bond model budget constraints. Plugging these into the de-trended Bond model budget constraint yields

$$\begin{aligned} \hat{c}_t(y^t) + \frac{\hat{b}_t(y^t)}{R_t} + \hat{\sigma}_t^B(y^t)\hat{v}_t &\leq (1-\alpha)\eta(y_t) + \frac{\hat{b}_{t-1}(y^{t-1})}{\lambda(z_t)} + \hat{\sigma}_{t-1}^B(y^{t-1})\left[\hat{v}_t+\alpha\right] \\ \hat{c}_t(y^t) + \left[\frac{\hat{a}_t(y^t)}{[\hat{v}_{t+1}+\alpha]} + \hat{\sigma}_t(y^t)\right]\hat{v}_t &\leq (1-\alpha)\eta(y_t) + \left[\frac{\hat{a}_{t-1}(y^{t-1})}{[\hat{v}_t+\alpha]} + \hat{\sigma}_{t-1}(y^{t-1})\right]\left[\hat{v}_t+\alpha\right] \\ \hat{c}_t(y^t) + \frac{\hat{a}_t(y^t)}{\hat{R}_t} + \hat{\sigma}_t(y^t)\hat{v}_t &\leq (1-\alpha)\eta(y_t) + \hat{a}_{t-1}(y^{t-1}) + \hat{\sigma}_{t-1}(y^{t-1})\left[\hat{v}_t+\alpha\right], \end{aligned} \tag{56}$$

which is exactly the budget constraint in the Bewley model. Thus, given the conjectured prices the allocations proposed in the theorem are optimal household choices in the de-trended Bond model. Equation (56) shows why, in contrast to the Arrow model, in the Bond model bond positions have to be zero. Nothing in this equation depends in the aggregate shock z_t except for the term $\frac{\hat{b}_{t-1}(y^{t-1})}{\lambda(z_t)}$. Therefore the budget constraint can only be satisfied if $\hat{b}_{t-1}(y^{t-1}) = 0$. In the model with growth, households want to keep wealth at the beginning of the period proportional to the aggregate endowment in the economy, but, since bond positions are chosen in the previous period, and thus cannot depend on the realization of the aggregate shock today, bond positions have to be zero to achieve proportionally of wealth and the aggregate endowment. The market clearing conditions for bonds in the de-trended Bond model is trivially satisfied because bond positions are identically equal to zero. The goods market clearing condition is identical to that of the Bewley model and thus satisfied by the Bewley equilibrium consumption allocations. It remains to be shown that the stock market clears. We know that:

$$
\begin{aligned}
& \int \sum_{y^t} \varphi(y^t|y_0) \hat{\sigma}_t^B(\theta_0, y^t) d\Theta_0 \\
= \quad & \int \sum_{y^t} \varphi(y^t|y_0) \left[\frac{\hat{a}_t(\theta_0, y^t)}{[\hat{v}_{t+1} + \alpha]} + \hat{\sigma}_t(\theta_0, y^t) \right] d\Theta_0 \\
= \quad & \frac{1}{[\hat{v}_{t+1} + \alpha]} \int \sum_{y^t} \varphi(y^t|y_0) \hat{a}_t(\theta_0, y^t) d\Theta_0 + \int \sum_{y^t} \varphi(y^t|y_0) \hat{\sigma}_t(\theta_0, y^t) d\Theta_0 \\
= \quad & 0 + 1,
\end{aligned}
$$

where the last line follows from the fact that the bond and stock market clears in the Bewley equilibrium. Thus, we conclude that the allocations and prices proposed in the theorem indeed are an equilibrium in the de-trended Bond model, and, after appropriate scaling, in the original Bond model. ☐

The crucial step of the proof shows that Bewley allocations, given the prices proposed in the theorem, satisfy the necessary and sufficient conditions for household optimality and all market clearing conditions in the de-trended Bond model.

This theorem again has several important consequences. First, equilibrium risk-free rates in the Arrow and in the Bond model coincide, despite the fact that the set of assets agents can trade to insure consumption risk differs in the two models. Second, in equilibrium of the Bond model, the bond market is inoperative: $b_{t-1}(s^{t-1}) = \hat{b}_{t-1}(s^{t-1}) = 0$ for all s^{t-1}. Therefore all consumption smoothing is done by trading stocks, and agents keep their net wealth proportional to the level of the aggregate endowment.[34] From the

[34]There is a subtle difference between this result and the corresponding result for the Arrow model. In the Arrow model we demonstrated that contingent claims positions were in fact uncontingent: $\hat{a}_t(\theta_0, s^t, z_{t+1}) = \hat{a}_t(\theta_0, y^t)$ and equal to the Bond position in the Bewley equilibrium, but not necessarily equal to zero. In the Bond model bond positions *have to be* zero. But since bonds in the Bewley equilibrium are a redundant asset, one can restrict attention to the situation where $\hat{a}_t(\theta_0, y^t) = 0$, although this is not necessary for our results.

deflated budget constraint in (51):

$$\hat{c}_t(y^t) + \frac{\hat{b}_t(y^t)}{R_t} + \sigma_t(y^t)\hat{v}_t \leq (1-\alpha)\eta(y_t) + \frac{\hat{b}_{t-1}(y^{t-1})}{\lambda(z_t)} + \sigma_{t-1}(y^{t-1})\left[\hat{v}_t + \alpha\right],$$

it is clear that bond holdings $b_t(y^{t-1}) = 0$ need to be zero for all y^{t-1}, if all the consumption, portfolio choices and prices, are independent of the aggregate history z^t, simply because the bond return in the deflated economy depends on the aggregate shock z_t through $\lambda(z_t)$. This demonstrates the irrelevance of the history of idiosyncratic shocks y^t for portfolio choice. There is no link between financial wealth and the share of this wealth being held in equity.

In summary, our results show that one can solve for equilibria in a standard Bewley model and then map this equilibrium into an equilibrium for both the Arrow model and the Bond model *with* aggregate risk. The risk-free interest rate and the price of the Lucas tree are the same in the stochastic Arrow and Bond models. Finally, without loss of generality, we can restrict attention to equilibria in which bonds are not traded; consequently transaction costs in the bond market would not change our results. Transaction costs in the stock market of course would (see section (5)). In addition, this implies that our result is robust to the introduction of short-sale constraints imposed on stocks and bonds separately, because agents choose not to trade bonds in equilibrium, as long as these short-sale constraints are not tighter than the solvency constraints.

Agents only trade a single bond a single stock. Wealth tomorrow in state $s^{t+1} = (s^t, y_{t+1}, z_{t+1})$ is given by

$$\theta_{t+1}(s^{t+1}) = (1-\alpha)\eta(y_{t+1})e_{t+1}(z_{t+1}) + b_t(s^t) + \sigma_t(s^t)\left[v_{t+1}(z^{t+1}) + \alpha e_{t+1}(z_{t+1})\right].$$

A.3. Representative Agent Model

The budget constraint of the representative agent who consumes aggregate consumption $c_t(z^t)$ reads as

$$\begin{aligned} & c_t(z^t) + \sum_{z_{t+1}} a_t(z^t, z_{t+1})q_t(z^t, z_{t+1}) + \sigma_t(z^t)v_t(z^t) \\ \leq \quad & e_t(z^t) + a_{t-1}(z^{t-1}, z_t) + \sigma_{t-1}(z^{t-1})\left[v_t(z^t) + \alpha e_t(z_t)\right] \end{aligned}$$

After deflating by the aggregate endowment $e_t(z^t)$, the budget constraint reads as

$$\begin{aligned} & \hat{c}_t(z^t) + \sum_{z_{t+1}} \hat{a}_t(z^t, z_{t+1})\hat{q}_t(z^t, z_{t+1}) + \sigma_t(z^t)\hat{v}_t(z^t) \\ \leq \quad & 1 + \hat{a}_{t-1}(z^{t-1}, z_t) + \sigma_{t-1}(z^{t-1})\left[\hat{v}_t(z^t) + \alpha\right], \end{aligned}$$

where $\hat{a}_t(z^t, z_{t+1}) = \frac{a_t(z^t, z_{t+1})}{e_{t+1}(z^{t+1})}$ and $\hat{q}_t(z^t, z_{t+1}) = q_t(z^t, z_{t+1})\lambda(z_{t+1})$ as well as $\hat{v}_t(z^t) = \frac{v_t(z^t)}{e_t(z^t)}$, precisely as in the Arrow model. Obviously, in an equilibrium of this model the representative agent consumes the aggregate endowment. The next (well-known) lemma follows directly from the household Euler equations and the definitions of $\hat{\beta}, \hat{\phi}$, $\hat{q}$ and $\hat{v}$.

Lemma A.1. *Equilibrium asset prices are given by*

$$\begin{aligned}\hat{q}_t(z^t, z_{t+1}) &= \hat{\beta}\hat{\phi}(z_{t+1}) = \hat{q}(z_{t+1}) \text{ for all } z_{t+1}.\\ \hat{v}_t(z^t) &= \hat{\beta}\sum_{z_{t+1}} \hat{\phi}(z_{t+1})\left[\hat{v}_{t+1}(z^{t+1}) + \alpha\right].\end{aligned}$$

A.4. Production Model

We consider a production economy[35] in which output is produced using labor L as the only input in production:

$$Y_t(z^t) = e_t(z_t)L_t(z^t)^{1-\alpha}$$

where $e_t(z^t)$ now denotes Total Factor Productivity, but evolves stochastically as before. Total labor input is given by

$$L_t(z^t) = \int \sum_{s^t} \pi_t(s^t|s_0)\eta(y_t)l_t(\theta_0, s^t)d\Phi_0$$

where $\eta(y_t)$ is the idiosyncratic labor productivity shock; so far this was the idiosyncratic income shock. Firms hire efficiency units of labor in a competitive labor market at a wage

$$w_t(z^t) = (1-\alpha)Y_t(z^t)/L_t(z^t)$$

Dividends that accrue to the holder of the shares in the company are given by:

$$d_t(z^t) = (1-\alpha)Y_t(z^t)$$

and the labor income share is $(1-\alpha)$ as before. Household labor income is given by:

$$w_t(z^t)\eta(y_t)l_t(s^t);$$

otherwise the household budget constraint remains unchanged. We assume that the period utility function is given by:

$$u(c,l) = \frac{\left[c_t(s^t)^{\gamma}(1-l_t(s^t))^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$$

so that the intratemporal optimality condition becomes

$$\frac{c_t(s^t)}{1-l_t(s^t)} = \frac{\gamma}{1-\gamma}w_t(z^t)\eta(y_t) \tag{57}$$

This suggests that the entire aggregate shock will be absorbed by consumption, and labor supply is only affected by the idiosyncratic shock. We can apply the same detrending procedure as before, and then solve the Bewley model with labor productivity shocks and endogenous labor supply. Denote the resulting allocation in the Bewley model by a hat.

[35] We thank Per Krusell for useful discussions leading to this subsection.

The corresponding equilibrium allocations in the growing economy satisfy: $l_t(s^t) = \hat{l}_t(y^t)$, and thus $L_t(z^t) = \hat{L}_t$, where $\hat{L}_t$ is the nonstochastic aggregate labor supply in the Bewley model. In addition, consumption and wages in the growing economy can be obtained as:

$$\begin{aligned} c_t(s^t) &= e_t(z_t)\hat{c}_t(y^t) \\ w_t(s^t) &= e_t(z_t)\hat{w}_t \end{aligned}$$

and the mapping of assets between the Bewley model and the stochastically growing model is exactly the same as before. Note that these allocations satisfy the intratemporal optimality condition (57). The other optimality conditions remain unchanged from the case with exogenous labor supply.

A.5. Counter-cyclical aggregate labor income share

With this endowment specification in hand, we can show that the irrelevance result survives the counter-cyclicality of the capital income share. We start in the de-trended Arrow economy. As before, dividing the budget constraint by $e_t(z^t)$ yields the deflated budget constraint:

$$\begin{aligned} & \hat{c}_t(s^t) + \sum_{z_{t+1}} \hat{a}_t(s^t, z_{t+1})\hat{q}_t(z^t, z_{t+1}) + \sigma_t(s^t)\hat{v}_t(z^t) \\ \leq \quad & \alpha - \alpha(z_t) + (1-\alpha)\eta(y) + \hat{a}_{t-1}(s^{t-1}, z_t) + \sigma_{t-1}(s^{t-1})\left[\hat{v}_t(z^t) + \alpha(z_t)\right], \end{aligned} \quad (58)$$

where we have defined the deflated Arrow positions as $\hat{a}_t(s^t, z_{t+1}) = \frac{a_t(s^t, z_{t+1})}{e_{t+1}(z^{t+1})}$ and prices as $\hat{q}_t(z^t, z_{t+1}) = q_t(z^t, z_{t+1})\lambda(z_{t+1})$. The deflated stock price is given by $\hat{v}_t(z^t) = \frac{v_t(z^t)}{e_t(z^t)}$. Similarly, by deflating the solvency constraints we get:

$$\sum_{z_{t+1}} \hat{a}_t(s^t, z_{t+1})\hat{q}_t(z^t, z_{t+1}) + \sigma_t(s^t)\hat{v}_t(z^t) \quad \geq \quad \hat{K}_t(y^t). \quad (59)$$

$$\hat{a}_t(s^t, z_{t+1}) + \sigma_t(s^t)\left[\hat{v}_{t+1}(z^{t+1}) + \alpha\right] \quad \geq \quad \hat{M}_t(y^t) \text{ for all } z_{t+1}. \quad (60)$$

Finally, the goods market clearing condition is given by:

$$\int \sum_{y^t} \phi(y^t|y_0)\hat{c}_t(\theta_0, s^t)d\Theta_0 = 1. \quad (61)$$

In this case,it is easy to show that we can still implement the Bewley equilibrium derived in section 4.1.

Theorem A.2. *An equilibrium of the Bewley model* $\{\hat{c}_t(\theta_0, y^t), \hat{a}_t(\theta_0, y^t), \hat{\sigma}_t(\theta_0, y^t)\}$ *and* $\{\hat{R}_t, \hat{v}_t\}$ *can be made into an equilibrium for the Arrow model with growth,* $\{a_t(\theta_0, s^t, z_{t+1})\}$,

$\{\sigma_t(\theta_0, s^t)\}$, $\{c_t(\theta_0, s^t)\}$ *and* $\{q_t(z^t, z_{t+1})\}$, $\{v_t(z^t)\}$, *with*

$$
\begin{aligned}
c_t(\theta_0, s^t) &= \hat{c}_t(\theta_0, y^t)e_t(z^t) \\
\sigma_t(\theta_0, s^t) &= \hat{\sigma}_t(\theta_0, y^t) \\
a_t(\theta_0, s^t, z_{t+1}) &= \hat{a}_t(\theta_0, y^t)e_{t+1}(z^{t+1}) - (\alpha - \alpha(z_{t+1})) + \hat{\sigma}_t(\theta_0, y^t)(\alpha - \alpha(z_{t+1})) \\
v_t(z^t) &= \hat{v}_t e_t(z^t) \\
q_t(z^t, z_{t+1}) &= \frac{1}{\hat{R}_t} * \frac{\hat{\phi}(z_{t+1})}{\lambda(z_{t+1})} = \frac{1}{\hat{R}_t} * \frac{\phi(z_{t+1})\lambda(z_{t+1})^{-\gamma}}{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{1-\gamma}} \qquad (62)
\end{aligned}
$$

Proof. The proof is simple. With these contingent bond positions $a_t(\theta_0, s^t, z_{t+1})$ defined above, the budget constraint in equation (58) is identical to the one in the economy with constant capital shares. Also, note that these contingent bond positions average to zero by construction:

$$\int \sum_{y^t} \varphi(y^t|y_0)\hat{a}_t(\theta_0, s^t, z_{t+1})d\Theta_0 \quad = \quad 0 \text{ for all } z_{t+1} \in Z,$$

because we can set the Bewley economy bond position to zero $\hat{a}_t(\theta_0, y^t) = 0$ without loss of generality. Since the stock market clears in the Bewley economy

$$\int \sum_{y^t} \varphi(y^t|y_0)\hat{\sigma}_t(\theta_0, y^t)d\Theta_0 = 1,$$

the contingent bond market clears as well. The rest of the argument is identical to the one in the standard case. □

A.6. *Recursive Utility*

We consider the class of preferences due to Epstein and Zin (1989) . Let $V(c^i)$ denote the utility derived from consuming c^i :

$$V(c^i) = \left[(1-\beta)c_t^{1-\rho} + \beta(\mathcal{R}_t V_1)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where the risk-adjusted expectation operator is defined as:

$$\mathcal{R}_t V_{t+1} = \left(E_t V_{t+1}^{1-\alpha}\right)^{1/1-\alpha}.$$

α governs risk aversion and ρ governs the willingness to substitute consumption intertemporally. These preferences impute a concern for the timing of the resolution of uncertainty to agents. In the special case where $\rho = \frac{1}{\alpha}$, these preferences collapse to standard power utility preferences with CRRA coefficient α. As before, we can define *growth-adjusted* probabilities and the growth-adjusted discount factor as:

$$\hat{\pi}(s_{t+1}|s_t) = \frac{\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}}{\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}} .$$

$$\text{and } \hat{\beta}(s_t) = \beta\left(\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}} .$$

As before, $\hat{\beta}(s_t)$ is stochastic as long as the original Markov process is not *iid* over time. Note that the adjustment of the discount rate is affected by both ρ and α. If $\rho = \frac{1}{\alpha}$, this transformation reduces to the case we discussed in section (2).

Finally, let $\hat{V}_t(\hat{c})(s^t)$ denote the lifetime expected continuation utility in node s^t, under the new transition probabilities and discount factor, defined over consumption shares $\{\hat{c}_t(s^t)\}$:

$$\hat{V}_t(\hat{c})(s^t) = \left[(1-\beta)\hat{c}_t^{1-\rho} + \hat{\beta}(s_t)(\hat{\mathcal{R}}_t\hat{V}_{t+1}(s^{t+1}))^{1-\rho}\right]^{\frac{1}{1-\rho}} ,$$

where $\mathcal{R}$ denotes the following operator:

$$\hat{\mathcal{R}}_t V_{t+1} = \left(\hat{E}_t\hat{V}_{t+1}^{1-\alpha}\right)^{1/1-\alpha} .$$

and $\hat{E}$ denotes the expectation operator under the hatted measure $\hat{\pi}$.

Proposition A.1. *Households rank consumption share allocations in the de-trended model in exactly the same way as they rank the corresponding consumption allocations in the original growing model: for any s^t and any two consumption allocations c, c'*

$$V(c)(s^t) \geq V(c')(s^t) \Longleftrightarrow \hat{V}(\hat{c})(s^t) \geq \hat{V}(\hat{c}')(s^t),$$

where the transformation of consumption into consumption shares is given by (5).

Proof of Proposition A.1:

Proof. First, we divided through by $e_t(z^t)$ on both sides in equation (A.6):

$$\frac{V_t(s^t)}{e_t(z^t)} = \left[(1-\beta)\frac{c_t^{1-\rho}}{e_t^{1-\rho}} + \beta\frac{(\mathcal{R}_t V_{t+1})^{1-\rho}}{e_t^{1-\rho}}\right]^{\frac{1}{1-\rho}}$$

$$\hat{V}_t(s^t) = \left[(1-\beta)\hat{c}_t^{1-\rho} + \beta(\frac{\mathcal{R}_t V_{t+1}}{e_t})^{1-\rho}\right]^{\frac{1}{1-\rho}} . \quad (63)$$

Note that the risk-adjusted continuation utility can be stated as:

$$\begin{aligned}\frac{\mathcal{R}_t V_{t+1}}{e_t(z^t)} &= \left(E_t\left(\frac{e_{t+1}}{e_t}\right)^{1-\alpha}\frac{V_{t+1}^{1-\alpha}}{e_{t+1}^{1-\alpha}}\right)^{1/1-\alpha} \\ &= \left(\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\hat{V}_{t+1}^{1-\alpha}(s_{t+1})\right)^{1/1-\alpha}\end{aligned}$$

Next, we define growth-adjusted probabilities and the growth-adjusted discount factor as:

$$\hat{\pi}(s_{t+1}|s_t) = \frac{\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}}{\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}} \text{ and } \hat{\beta}(s_t) = \beta\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}.$$

and note that:

$$\begin{aligned}\frac{\mathcal{R}_t V_{t+1}}{e_t(z^t)} &= \left(\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\hat{V}_{t+1}^{1-\alpha}(s_{t+1})\right)^{1/1-\alpha} \\ &= \left(\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\right)^{1/1-\alpha}\hat{\mathcal{R}}_t\hat{V}_{t+1}(s_{t+1})\end{aligned}$$

Using the definition of $\hat{\beta}(s_t)$:

$$\hat{\beta}(s_t) = \beta\left(\sum_{s_{t+1}}\pi(s_{t+1}|s_t)\lambda(z_{t+1})^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}},$$

we finally obtain the desired result:

$$\hat{V}_t(s^t) = \left[(1-\beta)\hat{c}_t^{1-\rho} + \hat{\beta}(s_t)(\hat{\mathcal{R}}_t\hat{V}_{t+1}(s^{t+1}))^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

As before, if the z shocks are i.i.d, then $\hat{\beta}$ is constant. □

Detrended Arrow Model We proceed as before, by conjecturing that the equilibrium consumption shares only depend on y^t. Our first result states that if the consumption shares in the de-trended model do not depend on the aggregate history z^t, then it follows that the interest rates in this model are deterministic.

Proposition A.2. *In the de-trended Arrow model, if there exists a competitive equilibrium with equilibrium consumption allocations* $\{\hat{c}_t(\theta_0, y^t)\}$ *, then there is a deterministic* *interest rate process* $\left\{\hat{R}_t^A\right\}$ *and equilibrium prices* $\{\hat{q}_t(z^t, z_{t+1})\}$*, that satisfy:*

$$\hat{q}_t(z^t, z_{t+1}) = \frac{\hat{\phi}(z_{t+1})}{\hat{R}_t^A}. \tag{64}$$

Proof of Proposition A.2:

Proof. First, we suppose the borrowing constraints are not binding, which is the easiest case. Assume the equilibrium allocations only depend on y^t, not on z^t. Then conditions 4.1 and 4.2 imply that the Euler equations of the Arrow economy, for the contingent claim and the stock respectively, read as follows:

$$\begin{aligned}
1 &= \frac{\hat{\beta}\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t, z_{t+1})} \sum_{y_{t+1}} \varphi(y_{t+1}|y_t) \left(\frac{\hat{c}_{t+1}(y^t, y_{t+1})}{\hat{c}_t(y^t)}\right)^{-\rho} \left(\frac{\hat{V}_{t+1}(y^{t+1})}{\hat{V}_t(y^t)}\right)^{\rho-\alpha} \quad \forall z_{t+1} \\
1 &= \hat{\beta} \sum_{z_{t+1}} \hat{\phi}(z_{t+1}) \left[\frac{\hat{v}_{t+1}(z^{t+1}) + \alpha}{\hat{v}_t(z^t)}\right] && (65) \\
& * \sum_{y_{t+1}} \varphi(y_{t+1}|y_t) \left(\frac{\hat{c}_{t+1}(y^t, y_{t+1})}{\hat{c}_t(y^t)}\right)^{-\rho} \left(\frac{\hat{V}_{t+1}(y^{t+1})}{\hat{V}_t(y^t)}\right)^{\rho-\alpha} forall\, z_{t+1}. && (66)
\end{aligned}$$

In the first Euler equation, the only part that depends on z_{t+1} is $\frac{\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t, z_{t+1})}$ which therefore implies that $\frac{\hat{\phi}(z_{t+1})}{\hat{q}_t(z^t, z_{t+1})}$ cannot depend on z_{t+1} : $\hat{q}_t(z^t, z_{t+1})$ is proportional to $\hat{\phi}(z_{t+1})$. Thus define $\hat{R}_t^A(z^t)$ by

$$\hat{q}_t(z^t, z_{t+1}) = \frac{\hat{\phi}(z_{t+1})}{\hat{R}_t^A(z^t)} \tag{67}$$

as the risk-free interest rate in the stationary Arrow economy. Using this condition, the Euler equation simplifies to the following expression:

$$\begin{aligned}
1 &= \hat{\beta}\hat{R}_t^A(z^t) \sum_{y_{t+1}} \varphi(y_{t+1}|y_t) \left(\frac{\hat{c}_{t+1}(y^t, y_{t+1})}{\hat{c}_t(y^t)}\right)^{-\rho} && (68) \\
& \left(\frac{\hat{V}_{t+1}(y^{t+1})}{\hat{V}_t(y^t)}\right)^{\rho-\alpha} && (69)
\end{aligned}$$

Apart from $\hat{R}_t^A(z^t)$ noting in this condition depends on z^t, so we can choose $\hat{R}_t^A(z^t) = \hat{R}_t^A$. □

All the results basically go through. We can map an equilibrium of the Bewley model into an equilibrium of the detrended Arrow model. The proof of the next result follows directly from this proposition and uses exactly the same steps as the proofs of Theorems 4.1 and 5.1.

Theorem A.3. *An equilibrium of the Bewley model* $\{\hat{c}_t(\theta_0, y^t), \hat{a}_t(\theta_0, y^t), \hat{\sigma}_t(\theta_0, y^t)\}$ *and* $\{\hat{R}_t, \hat{v}_t\}$ *can be made into an equilibrium for the Arrow model with growth,* $\{a_t(\theta_0, s^t, z_{t+1})\}$,

$\{\sigma_t(\theta_0, s^t)\}$, $\{c_t(\theta_0, s^t)\}$ *and* $\{q_t(z^t, z_{t+1})\}$, $\{v_t(z^t)\}$, *with*

$$\begin{aligned}
c_t(\theta_0, s^t) &= \hat{c}_t(\theta_0, y^t)e_t(z^t) \\
\sigma_t(\theta_0, s^t) &= \hat{\sigma}_t(\theta_0, y^t) \\
a_t(\theta_0, s^t, z_{t+1}) &= \hat{a}_t(\theta_0, y^t)e_{t+1}(z^{t+1}) \\
v_t(z^t) &= \hat{v}_t e_t(z^t) \\
q_t(z^t, z_{t+1}) &= \frac{1}{\hat{R}_t} * \frac{\phi(z_{t+1})\lambda(z_{t+1})^{-\alpha}}{\sum_{z_{t+1}} \phi(z_{t+1})\lambda(z_{t+1})^{1-\alpha}}
\end{aligned}$$

As a result, even for a model with agents who have these Epstein-Zin preferences, the risk premium is not affected.