The Effects of Demographic Changes on Aggregate Savings: Some Implications from the Life Cycle Model*

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Abstract

In this essay I review the basic life cycle theory of saving to obtain predictions for aggregate savings dynamics in societies that undergo an aging process like the one predicted for all major industrialized countries in the near future. The data indicates that the phenomenon of population aging is driven both by longer life expectancy as well as lower birth rates (and thus lower population growth rates). The life cycle model is then used to deduce the likely effect on aggregate savings from both longer life expectancy and lower population growth rates. While longer expected life, ceteris paribus, increases individual and thus aggregate savings, a lower population growth rate may increase per capita saving in the short run, but reduces it in the long run.

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1 Introduction: Motivation and Some Data

We are getting older. This fact not only becomes apparent every morning when trying to get out of bed. When plotting measures of the age distribution of industrialized countries over time, one observes a substantial increase in the share of the population of old age in the last decade, a trend that is predicted to persist and even accelerate in the near future.

To make things precise, let's define the *Old Age Dependency Ratio* as the ratio of the total population of a country or region 65 years and older to the total population of ages 15 to 64, or

$$\text{Old Age Dependency Ratio} = \frac{65+}{15-64}.$$

Roughly speaking, this ratio measures how many people of retirement age a country has per person of working age. For countries with a public pension system it also is a good approximation as to how many pensions have to be supported per working age adult.

Figure 1, taken from Brooks (2003) plots Old Age Dependency Ratios for eight major regions of the world from 1950 onwards, and predicts them into the future.

[Figure 1 {Brooks Old Age Dependency Ratio} about here]

While the forecasted data beyond 2050 rely heavily on the assumption that population growth rates will converge across regions at that date, the data before 2050 show several intriguing facts. First, population aging is predicted to occur for all world regions, albeit at different degree and speed. Japan's and Europe's aging process has already set in, and is predicted to continue until about 2035, at which old age dependency ratios of close to 50% are reached. At that time, assuming all working age individuals actually work and no change in retirement ages (and disregarding early retirement), one working person has to support the pension of one pensioner. Taking into account labor market non-participation, the ratio is likely to be substantially higher. The figure also shows that population aging in North America, driven mainly by the US, is a phenomenon mostly of the future, and is predicted to occur slower than in Europe and Japan. The difference in timing of population aging is also an important determinant of the direction and size of international capital flows likely to occur in the near future..

Before discussing the basic theory underlying the changes in aggregate savings, it is instructive to look one step deeper into the causes of the phenomenon of the aging population within the US, Europe and elsewhere. There are two basic reasons for this trend. First, individuals live longer and longer. Second, households have fewer and fewer children. Table 1, taken from Börsch-Supan and Lusardi (2002), demonstrates that, conditional on surviving to the age of 65, a typical male in Germany is expected to live another 15 years, a typical woman can count on living another 19 years. The numbers for other European countries and the US are similar, Japanese fare even better, with residual life expectancies at 17 and 22 years for males and females, respectively.

[Table 1 {Börsch-Supan, Lusardi Table} about here]

In Table 2 we show how longevity has changed over time. Using data for the US we demonstrate how live expectancies have changed over the last hundred years.[1] The increases are quite astonishing. Around the turn of the 19-th century a typical male could expect to live 48 years; in the last hundred years roughly 26 years of extra live have been added to that expectation, due to improvements in medical technology, basic hygiene, improved nutrition and other important factors. Women even gained 29 extra years on average, or more than a 50% longer life now than a hundred years ago. Despite all the negative connotations the discussion of the phenomenon of population aging usually has[2], we should not forget the basic fact that adding fruitful and enjoyable years to the average person's life is a very positive development (at least according to the most commonly used specifications of individual's preferences).

Table 2: US Life Expectancy at Birth, Over Time

Group	2000	90	80	70	60	50	40	30	20	10	1900
Pop.	77	75	74	71	70	68	64	59	56	52	49
Female	80	79	78	75	73	71	66	61	57	53	51
Male	74	72	70	67	67	66	62	58	56	50	48

[1] See the website of the National Center for Health Statistics. The data used stem from http://www.cdc.gov/nchs/data/hus/tables/2003/03hus027.pdf

[2] For a refreshing exception, see the essay by E. Niejahr in *Die Zeit*, 43, 2004.

The second major factor responsible for the aging of the population in industrialized countries is the decline in the number of children households have, that is, in fertility rates. Define as the *Total Fertility Rate* the average number of children a woman has during her reproductive years. Absent migration a total fertility rate of about 2.1 is needed to keep a population stable over time. Tables 3 and 4 summarize total fertility rates (past and predicted) for rich and poor regions, as well as current fertility rates in Europe and North America.[3] We observe from Table 3 that a) rich countries have lower fertility rates than poor countries, b) rich countries have, and are predicted to have in the future, fertility rates consistent with a shrinking population (gain absent immigration from poor countries).

Table 3: Past and Future Total Fertility Rates

Region	1990	2000	2010	2025
World	3.4	2.8	2.5	2.3
Poor Countries	4.7	3.1	2.7	2.4
Rich Countries	1.9	1.6	1.7	1.7

Finally Table 4 summarizes the extremely low total fertility rates in Europe, especially Germany and Southern Europe, whereas the US has a total fertility rate that just suffices to keep the US population size stable even without immigration (and thus expanding if one takes into account substantial legal and illegal immigration into the US).

Table 4: Current Total Fertility Rates

Country	Germany	Italy	France	UK	Spain	Canada	US
TFR (1998)	1.3	1.2	1.6	1.6	1.2	1.7	2.1

The previous two facts (longer lifetimes, lower fertility rates) motivate performing two experiments with the basic life cycle model of consumption and saving to be constructed next. We want to investigate how aggregate saving is expected to change over time as households live longer, and as the size of newborn cohorts, relative to existing cohorts, decreases (that is population growth rates decline), due to a decline in total fertility rates.

[3] See McDevitt (1999), p. A-39. Alternatively, the data can be found at http://www.overpopulation.com/faq/basic_information/total_fertility_rate/.

2 The Basic Life Cycle Model

The basic life cycle model developed by Franco Modigliani and Richard Brumberg (1954), and, with slightly different focus, by Milton Friedman (1957) envisions a single individual living for J years (in practice, J may equal to 60, from 21, the age at which the individual becomes economically independent, to age 80, at which she dies. She enjoys consumption in all her living years, denoted by $(c_1, c_2, \ldots, c_J)$. The lifetime utility the individual derives from consumption is given by the discounted sum of period by period utility

$$\sum_{j=1}^{J} \beta^{j-1} u(c_j)$$

where β is the time discount factor, a number typically assumed to lie between 0 and 1 and measuring the degree of impatience of the household (the smaller β, the more impatient). The period utility function is increasing and strictly concave, which simply means that the individual likes more consumption better than less, but that each additional amount of consumption gives less and less additional utility.

We assume that households work for jr years (in practice jr may equal 45 years), earning income. After age jr no labor income (or pension income) is being earned. For simplicity it is assumed that as time progresses the real income of the household increases at a constant growth rate g, reflecting both general technological progress as well as individual learning on the job. If we let $y_1 = y$ denote real income of the individual in her first working year, this assumption implies that income in the second year equals $y_2 = (1+g)y$, and in general

$$y_j = (1+g)^{j-1} y$$

for all years $j = 1, \ldots, jr$. For technical reasons we make the empirically plausible assumption that $g < r$.

In the simplest possible version of the model households are assumed to be able to borrow and lend money at a fixed common real net interest rate r. Abstracting from bequests from parents, the individual solves the following maximization problem: by choosing $(c_1, c_2, \ldots, c_J)$,

$$\max \sum_{j=1}^{J} \beta^{j-1} u(c_j)$$

subject to the intertemporal budget constraint

$$\sum_{j=1}^{J} \frac{c_j}{(1+r)^{j-1}} = \sum_{j=1}^{jr} \frac{y_j}{(1+r)^{j-1}} \tag{1}$$

which simply states that the present discounted expenses for consumption has to equal the present discounted value of income. The right hand side of this equation can be simplified, using the assumption of constant income growth, to[4]

$$\sum_{j=1}^{jr} \frac{y_j}{(1+r)^{j-1}} = \sum_{j=1}^{jr} \frac{(1+g)^{j-1}y}{(1+r)^{j-1}} = \frac{y(1+r)}{r-g} * \left(1 - \left[\frac{1+g}{1+r}\right]^{jr}\right).$$

The optimal consumption life cycle profile an individual chooses depends on the relative size of the individual's impatience and the market interest rate. For now we assume that these two forces exactly balance, so that $\beta(1+r) = 1$. Then the assumption that the individual values additional consumption at a decreasing rate (strict concavity of u) immediately imply that it is optimal for the individual to equalize consumption in all periods of life[5], or

$$c_1 = c_2 = \ldots = c_J$$

Let this common consumption level be denoted by c.

Using the budget constraint and some tedious algebra the optimal consumption level is given by

$$c = y * \frac{r}{r-g} * \frac{1 - \left[\frac{1+g}{1+r}\right]^{jr}}{1 - [1+r]^{-J}} \tag{2}$$

Several implications immediately arise from the optimal consumption rule (2) :

[4] In general, for any number $a \in (0,1)$ we have the formula

$$\sum_{j=0}^{J} a^j = \frac{1-a^{J+1}}{1-a}.$$

[5] If $\beta(1+r) < 1$ it is optimal to have consumption to decline over time, and if $\beta(1+r) > 1$ consumption should increase over time. The explicit solution of the problem cannot be characerized without making specific assumptions on the form of the utility function, unless $\beta(1+r) = 1$, as assumed in the text.

- An increase in income y increases consumption in all periods of the individual's life.

- The higher the growth rate of income g (for a given level y), the higher is consumption in each period of a person's life.

- For given retirement age jr and given income, increasing a person's life expectancy J decreases consumption in all periods.

- For given life expectancy and income, increasing the retirement age increases consumption in all periods.

- A change in the real interest rate has ambiguous effects on lifetime consumption.

Our main interest in the life cycle model arises from its predictions for savings behavior and asset accumulation. The lifetime budget constraint (1) obscures savings behavior over an individual's lifetime. The period by period budget constraint of the individual reads as

$$c_j + s_j = y_j + (1+r)s_{j-1} \tag{3}$$

for each period j of the individual's life. Here s_j are financial assets (or debt, if negative) carried from period j to $j+1$. Casually this is often called savings, but this term should be reserved for

$$\begin{aligned} sav_j &= s_j - s_{j-1} \\ &= y_j + rs_{j-1} - c_j \end{aligned}$$

that is, for the change in an individual's asset position, equal to total income (labor income y_j plus capital rs_{j-1} minus consumption c_j). Since the individual starts her life with no assets, $s_0 = 0$. Furthermore we require $s_J \geq 0$, that is, individuals cannot die in debt. Therefore it is optimal to set $s_J = 0$, since, absent any altruism, it never makes sense to carry assets into the grave, given the preferences specified above.[6]

[6] It is easy to show that if one consolidates the period by period budget constraints (3) into a single constraint, one obtains back equation (1). We also implicitly assume that individuals cannot go deeper in debt than the amount they can repay with their future income (the borrowing constraints arising from this consideration are often called solvency constraints).

Since we know that $y_j = y(1+g)^{j-1}$ and have already solved for the optimal consumption level in (2), we can use (3) to obtain the optimal asset levels over an individual's lifetime. The precise formula changes once a person retires, and is given by

$$s_j = \frac{y}{r-g}\left(1-\left[\frac{1+g}{1+r}\right]^{jr}\right) * \left(\frac{1-(1+r)^{j-J}}{1-(1+r)^{-J}}\right)$$

for a retired person of age $j \geq jr$, and

$$s_j = y\sum_{\tau=0}^{j-1}(1+r)^{\tau}(1+g)^{j-1-\tau} - c\sum_{\tau=0}^{j-1}(1+r)^{\tau}$$

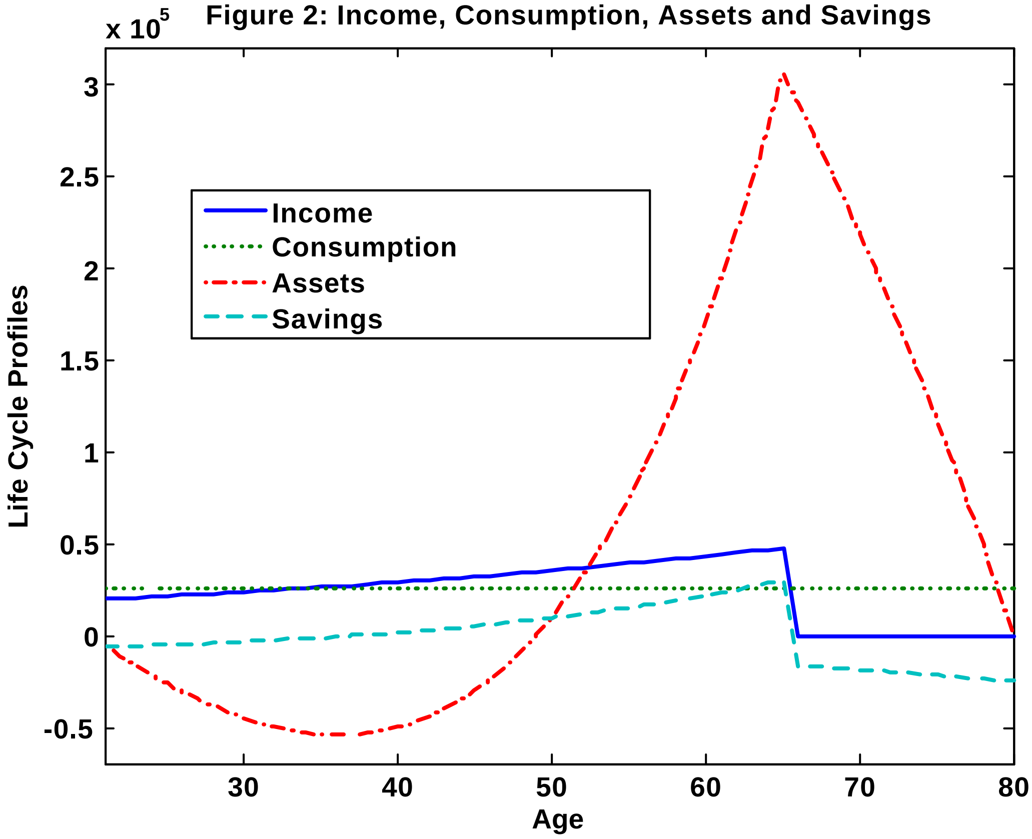
where c is given by (2). Thus for a retired person savings equal

$$\begin{aligned} sav_j &= s_j - s_{j-1} \\ &= \frac{y}{r-g}\left(1-\left[\frac{1+g}{1+r}\right]^{jr}\right) * \left(\frac{-r(1+r)^{j-1-J}}{1-(1+r)^{-J}}\right) \end{aligned} \tag{4}$$

which is negative (and increasingly so). For a nonretired person we obtain

$$sav_j = (y-c)(1+r)^{j-1} + g\sum_{\tau=0}^{j-2}(1+r)^{\tau}(1+g)^{j-2-\tau}. \tag{5}$$

Unfortunately we can't say whether $y > c$ from (2). Thus there are two possibilities. Either $y \geq c$, in which case savings sav_j are always positive, increasing over time until retirement, at which point they turn negative as individuals dissave to finance retirement consumption. Or, if $y < c$, savings are initially negative (and possibly at first become more negative over time), but at some point during the individual's working life start to increase and turn increasingly positive, until retirement. Then, as before, households, decumulate assets to finance retirement consumption, so that savings turn negative again after retirement. Figure 2 shows stylized life cycle profiles of labor income, consumption and savings.



2.1 Extensions of the Basic Model

Several simplifying assumptions were made to derive analytical solutions of the model above. First, individuals were allowed to freely borrow (subject to the constraint of having to pay back the loan with certainty in the future. In practice, to borrow against future labor income may be difficult, in particular if a country's personal bankruptcy code provides for only mild punishments for defaulting on noncollateralized loans. Second, individuals in the real world face important uncertainties that the model abstracted from, such as labor income uncertainty, uncertainty about health status and medical expensive, as well as uncertainty about the exact time of death. Third, while we modelled income as an exogenous manna from heaven, in reality households have to provide labor services to command labor income. If leisure and consumption are not separable in the utility function, the analysis above has to be altered to incorporate the interaction between consumption and labor supply choices. An important abstraction was the absence of government policy in the analysis, in particular income taxation and social security. As long as these policies do not distort the labor leisure decision or affect the after tax real interest rate, our analysis can proceed as above, with income streams adjusted for taxes. Finally, we have so far considered a household in isolation, taking the interest rate as exogenously given. More complex models like the ones discussed below determine the interest rate as a general equilibrium outcome, either to equilibrate domestic saving and investment, or, if an open economy is considered, to equilibrate international investment demand and saving supply.

Starting with the seminal work of Auerbach and Kotlikoff (1987), models incorporating the features described above have been solved on computers, with the basic features from the simple model remaining intact: individuals accumulate assets during their working years to finance consumption in retirement. Figure 3, taken from Conesa and Krueger (1999) shows the asset accumulation path over the life cycle derived from a model with borrowing constraints, income and mortality uncertainty, endogenous labor supply, a government pension system and interest rates determined in general equilibrium. Here one of the profiles refers to a model with a Pay As You Go social security system, and one to a model with a private accounts-based pension system.

Figure 3 about here

Figure 3: Life Cycle Asset Accumulation in an Extended Life Cycle Model

3 Model Predictions for Aggregate Savings of Population Aging

3.1 An Increase in Life Expectancy J

If people live longer, for a given income profile and retirement age, the implications of the model are clear-cut. From equation (2) we see that per-period consumption has to decline, and from (4) and (5) we see (after a little algebra) that savings increase in all periods of an individual's life. This is perfectly intuitive, as a given income now has to be stretched over a longer life time horizon, thus requiring bigger savings for retirement. It is, of course, possible that individuals will respond with an increase in the number of years worked, in which case the total effect on savings of an increase in life expectancy is ambiguous. To the extent that current public pension systems provide strong incentives to retire at the normal retirement age, baring reforms in the pension system the standard life cycle model predicts an increase in individual, and thus aggregate asset accumulation and savings in response to an increase in life expectancy.

3.2 A Decline in Fertility Rates

So far it was sufficient to analyze an individual in isolation. In order to study the impact of a decline in fertility and thus population growth rates we now have to consider the economy as a whole, aggregating over all individuals, each of which is assumed to behave according to the standard life cycle model. Formally, suppose that the population grows at a rate n per year, where n may be negative. This means that there are $(1+n)$ times as many individuals of age 25 as individuals of age 26. If we let μ_j denote the fraction of the population of age j, a population growth rate of n implies that

$$\mu_j = \frac{\mu_{j-1}}{1+n}$$

and, since all fractions have to sum to 1,

$$\sum_{j=1}^{J} \mu_j = 1,$$

so that one can compute the population fractions as

$$\mu_j = n\frac{(1+n)^{-j}}{1-(1+n)^{-J}}$$

Per capita asset holdings are then given as

$$\bar{S} = \sum_{j=1}^{J} \mu_j s_j$$

and per capita savings as

$$\overline{SAV} = \sum_{j=1}^{J} \mu_j sav_j.$$

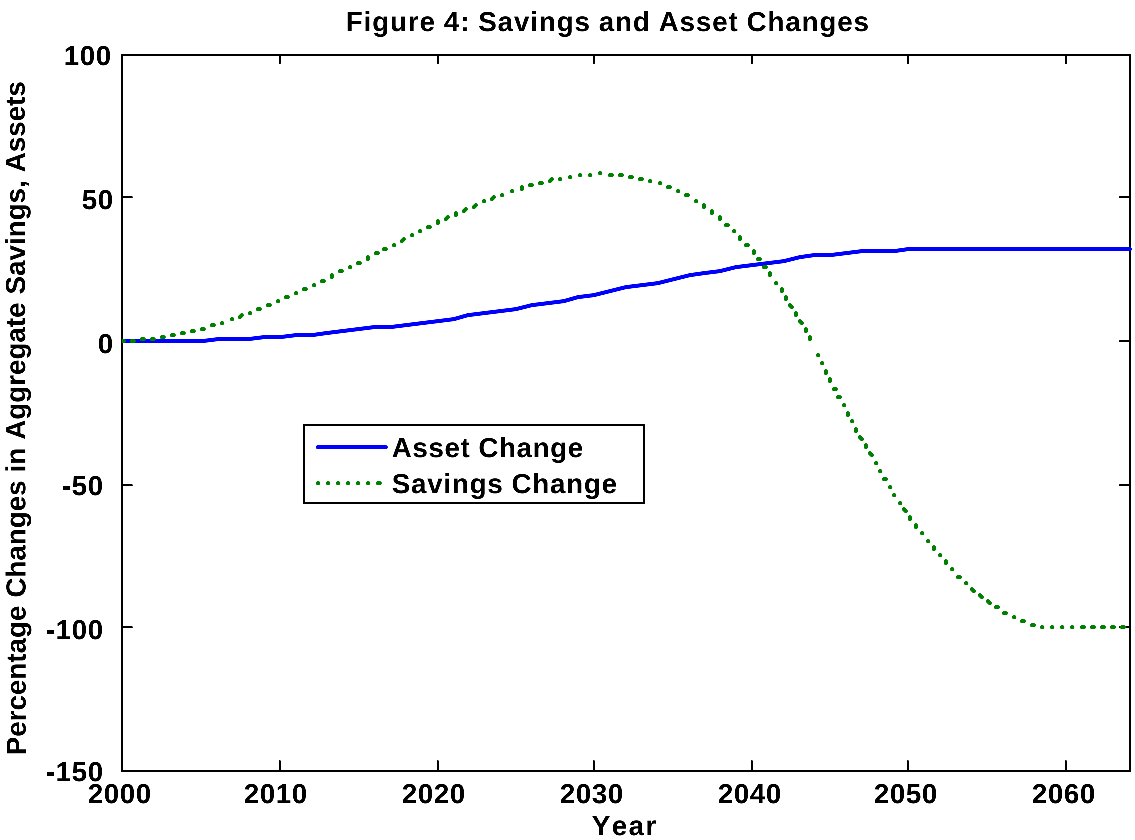
We are interested in the short and long run effect of declines in the population growth rate n on per capita asset holdings and per capita savings. The long-run effects can basically be read off Figure 2. A lower population growth rate, in the long run, yields a larger fraction of the population in older cohorts. Since older cohorts, as Figure 2 shows, tend to have more assets and save less (at least if they are already retired), we would expect per capita asset holding to increase and per capita savings to decline as a long-run consequence to a (permanent) decline in the population growth rate.

The short run consequences of a sudden and permanent decline in the population growth rate are less clear. The decline in n gradually makes the population older. As the large cohorts arising from the previous, higher population growth rate, age, they move from being borrowers to being savers (during the high earning years from 45-65) to being dissavers in retirement. This suggests that asset accumulation is increasing over time, while average savings should first increase, but then decline, as the big cohorts enter their retirement age.

Since this cannot be shown theoretically for all parameter values, the paper now presents a simple simulation analysis. This is meant to be a thought experiment demonstrating the economic forces at work, rather than an exact quantitative predictions about future asset accumulation and savings. For the simulation we assume the following parameter values, summarized in Table 5.

Table 5: Parameter Values		
Economic Birth		21
Retirement Age	jr	65
Life Expectancy	J	80
Interest Rate	r	3%
Time Discount Factor	β	$\frac{1}{1.03}$
Income Growth Rate	g	2%
Initial Pop. Growth Rate	n	1%

We carry out the following thought experiment: until the year 2000 the population growth rate is assumed to be 1%; then it falls permanently to 0%. In Figure 4 we plot how, over time, per capita asset holdings and savings change, in response to the decline in the population growth rate.



First, we observe that changes in aggregate savings and asset accumulation terminate after sixty years. This is due to our assumption that individuals live for 60 years, plus the assumption that the change in the population growth rate is permanent. This implies that all adjustments in the demographic structure of the economy are completed after 60 years. Since individual behavior does not change as the population growth rate changes, after 60 years the economy has reached its new long run equilibrium. The dynamics of per capita assets and savings is as conjectured above. While asset holdings increase monotonically over time as the population ages, per capita savings first increase (as the big cohorts reach their prime earnings years) and then decline as more and more people retire and dissave. In the long run, savings declines, compared to the initial situation with a higher population growth rate.

This completes our discussion of the predictions the standard life cycle model has for aggregate savings dynamics in the light of an aging population, driven both by increased life expectancy and reductions in fertility and thus population growth rates.

4 Conclusion

The population is aging, and, as I have argued above, theory predicts that this has profound consequences for individual and aggregate savings dynamics. Stepping beyond the partial equilibrium life cycle model is beyond this essay, but analyses of the consequences of population aging for the future of social security, the direction and magnitude of international capital flows in a world where regions age at different paces and other applied questions usually start from the individual optimization problem discussed here. Recently, the papers by De Nardi, Imrohoroglu and Sargent (1999), Brooks (2003) and Börsch-Supan, Ludwig and Winter (2004) have analyzed the issues of international capital flows and/or public pension reforms in dynamic general equilibrium models that have the life cycle model as its basic building block and demographic changes as exogenous driving force. Since the paper by Börsch-Supan, Ludwig and Winter (2004) is contained in this volume, it is not useful to give an execute summary here. But by providing the partial equilibrium analysis for an analytically solvable version of the life cycle model this paper has hopefully provided some additional intuition for the quantitative results obtained by the papers cited above.

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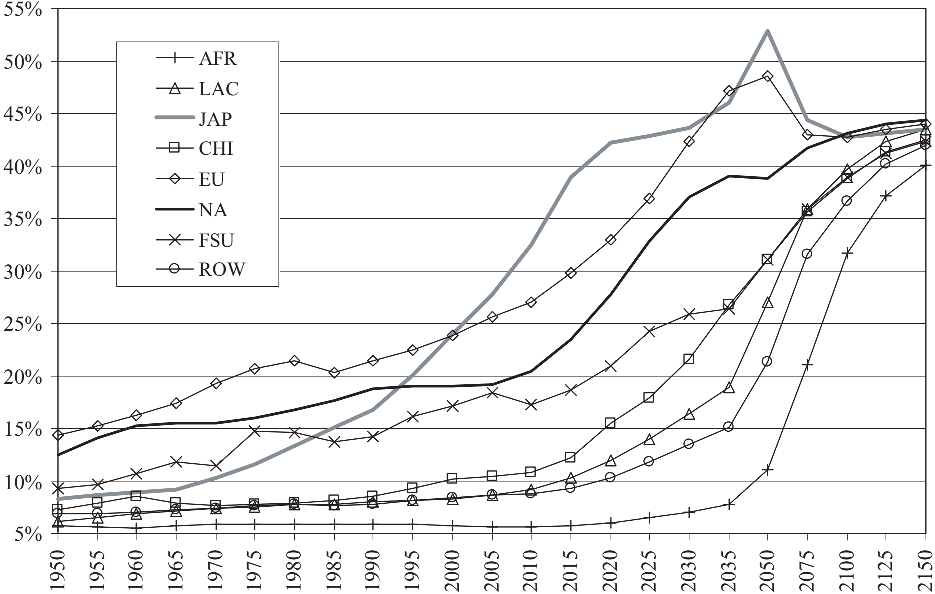
Figure 1: Old-Age Dependency Ratios, 65+ / (15–64)

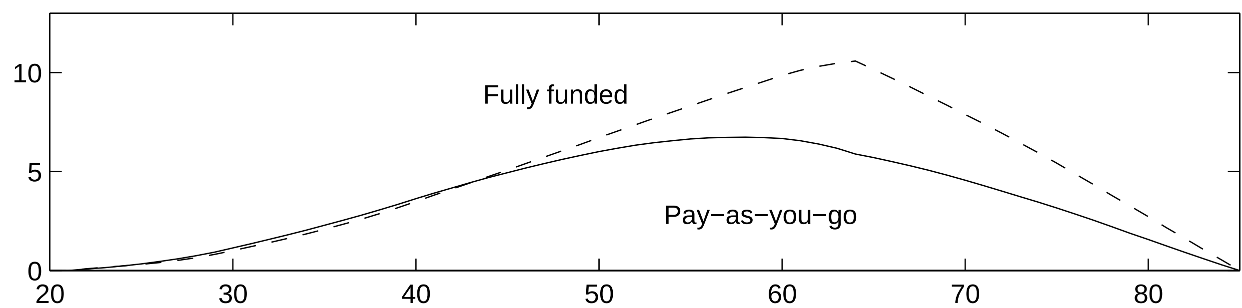


Notes: AFR = Africa; LAC = Latin American countries; JAP = Japan; CHI = People's Republic of China; EU = European Union region; NA = North America; FSU = countries of the former Soviet Union; and ROW = the rest of the world.

Table 1: Demographic features

	Germany	Italy	Japan	Netherlands	UK	USA
Life expectancy at birth (male/female; 1998)	74.5/80.5	75.3/81.6	77.2/84.0	75.2/80.7	74.8/79.7	73.9/79.4
Life expectancy at 65 (male/female; 1998)	15.3/19.0	15.8/20.2	17.1/22.0	14.7/18.8	15.0/18.5	16.0/19.1
Share of population aged 65 and over (1998)	16.4	18.2	17.1	13.8	16.0	12.5

Sources: OECD Health Data (2001).



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