Intergenerational Risk Sharing via Social Security when Financial Markets are Incomplete*

By Dirk Krueger and Felix Kubler

The role that an unfunded social security system may play in facilitating the allocation of aggregate risk among generations was a key consideration in its introduction and is an important aspect in its current reform debate (see Henry Aaron, Alan Blinder, Alicia Munnell and Peter Orzag (2001) and Gary Burtless (2000)). Despite this fact academic research on the role of social security for intergenerational risk sharing in models with aggregate uncertainty remains limited. In particular, it is unknown under which conditions a pay-as-you-go social security system that endows retired households with a claim to labor income may serve as a tool to share aggregate risk between generations.

In general equilibrium models with overlapping generations, the possibilities for Pareto-improving risk sharing policies are very limited when financial markets are sequentially complete and when an ex interim welfare criterion is used (i.e. an economic agent is not only identified by the time of birth, but also by the node of the event tree at which he is born).

In these models competitive equilibria are ex interim Pareto efficient if there is an asset which pays a non-negligible fraction of aggregate endowments at all date-events (Subir Chattopadhyay and Piero Gottardi (1999) or Gabrielle Demange (2002)). In most model specifications land constitutes such an asset. An introduction of a pay-as-you-go social security system might help the current old, but will hurt some or all future generations.

In this paper we argue that the crucial assumption underlying this result is the one of complete financial markets. Via a numerical example we demonstrate that when these markets are incomplete the introduction of social security into the competitive economy can lead to a welfare-improving consumption allocation in the Pareto sense. This Pareto improvement is due to enhanced intergenerational risk sharing of imperfectly correlated shocks to individual endowments, which is only possible because the

social security transfers cannot be replicated by transferring existing assets between generations.

In the example an introduction of social security is a simple Pareto-improving policy reform. Sequentially completing the markets through the introduction of new securities, on the other hand, is *not* Pareto-improving: it leads to a collapse of the price of land and a welfare loss of the current old who do not benefit from additional financial securities.

I. Overlapping Generations and Social Security

We consider an infinite horizon pure exchange economy which is populated by three overlapping generations. At each point of time there is a single, perishable consumption good. The only source of uncertainty are stochastic shocks to individual endowments of this good. The shocks, $(z_t)_{t=0}^{\infty}$, are assumed to follow a Markov chain with finite support. A date-event z^s is a history of shocks $(z_0, \ldots, z_s)$.

There is no heterogeneity within each generation and at each date-event a single 'representative' agent is born. An agent born at z^s has non-negative, stochastic individual endowment of the consumption good over her life cycle which depends only on the age of the agent and the current shock. We write $((e^1(z_s), e^2(z_{s+1}), e^3(z_{s+2}))$ for the endowment profile.

The only productive asset is land, an infinitely lived asset in unit net supply that pays a dividend of one unit of the consumption good in each period. Agents born at $t \geq 0$ have no endowments of land. We assume that at $t = 0$ there is an old and a middle aged agent who initially own the land.

In addition to land there may be a set of one-period Arrow securities in zero net supply available for trade between generations. We say that markets are sequentially complete if there is an Arrow security for each possible realization of the exogenous shock.

Let by $c^{z^s}(z^t)$ denote the consumption of an agent born at z^s in period $t \geq s$. Define as expected utility of an agent born into event history z^s

$$U(z^s, c) = E_{z^s} \sum_{t=s}^{s+2} \beta^{t-s} u(c^{z^s}(z^t)) \tag{1}$$

An allocation (c) is feasible if total consumption across generations does not exceed total endowments plus the dividend from land. A feasible allocation is ex interim efficient if there is no other feasible allocation $(\hat{c})$ such that $U(z^t;\hat{c}) \geq U(z^t;c)$ for all z^t and $U(z^t;\hat{c}) > U(z^t;c)$ for some z^t.

In each period agents trade in the available assets to maximize their utility. Competitive equilibrium is defined by market clearing in the goods, financial asset and land market as well as agents' optimality. We want to examine under which conditions it will Pareto-improving to introduce unfunded social security, because it facilitates intergenerational risk sharing.

We model social security as a defined contribution pay-as-you-go system that adheres to period by period budget balance, with size characterized by the payroll tax rate τ. A household born at z^s pays a tax $\tau e^{1+n}(z_{s+n})$ at date-event z^{s+n}, $n = 0, 1$ and receives benefits $b(z_{s+2})$ when old. Taxes and benefits satisfy

$$\tau(e^1(z_t) + e^2(z_t)) = b(z_t) \tag{2}$$

In this paper we perform the following comparative statics exercise: Suppose that in an equilibrium of the economy for a payroll tax rate $\tau = 0$ at some date-event z^t, there is an unanticipated increase of τ. What are the welfare effects for all individuals alive at z^t and for all individuals born at successor nodes?

II. Theoretical Results on Efficiency

Ever since Paul Samuelson's (1958) seminal paper it is well known that overlapping generation models can exhibit Pareto suboptimal equilibria. If the equilibrium allocation is *dynamically inefficient* transfers from young to old each period can make everybody better off. However, if there exists an infinitely lived asset, such as land, that promises to pay a non-negligible fraction of aggregate consumption at each state of the world in the future and markets are sequentially complete this cannot happen. When markets are *complete*, prices of commodities at future dates are uniquely defined. The current price of land is the sum of its payoffs discounted with these

prices. It can only have a finite price today if the value of commodities in the infinite future tend to zero and the allocation is dynamically efficient (see Chattopadhyay and Gottardi (1999)). Thus, with complete markets, for the introduction of an unfunded social security system to provide a Pareto improvement the empirically implausible assumption that land and other long-lived assets are absent from the economy must be made.

When agents only live for two periods and there is no intragenerational heterogeneity, land always ensures Pareto efficiency. However, if financial markets are incomplete *and* agents live for more than 2 periods allocations will generally not be Pareto efficient. In this paper we therefore focus on economies where markets are not sequentially complete. While it is well-known that equilibria will generally not be Pareto efficient, it is unknown whether one-sided intergenerational transfers (like social security) can possibly lead to Pareto improvements in these models.

We present a simple example where the allocation is dynamically efficient but where the introduction of a social security system is Pareto-improving.

III. A Simple Numerical Example

There are two shocks. Shock $z = 1$ has probability 0.9 and $e^1(1) = e^3(1) = 10$, $e^2(1) = 12$. Shock $z = 2$ has probability 0.1 and $e^1(2) = e^2(2) = 8$, $e^3(2) = 0$. Agents have constant relative risk aversion utility with coefficient of relative risk aversion of 4 and they do not discount the future, i.e. $\beta = 1$. The competitive equilibrium allocation is not Pareto efficient. Using the algorithm developed by Dirk Krueger and Felix Kubler (2001a) we compute a competitive equilibrium without social security. While prices and allocations depend on the distribution of land between middle aged and old, it turns out that in the computed equilibrium this dependency is very small. If the shock is 1, the current young buy approximately 0.26 units of land, the price of land lies around 8.7. If the shock is 2 the price of land drops to 4.5 and the current young buy around 0.3 units of land. Evidently, land is a terrible asset for the old generation since its payoff drops substantially exactly in the state in which this generation has little endowment.

The introduction of a pay-as-you-go social security system with a tax of 10 percent may be, depending on aggregate economic conditions, Pareto-improving. Even though social security payments, as with the price of land, are still lower in the bad state, they are much more even: 2.2 in the good state and 1.6 in the bad state. Thus the social security system enables the old generation to share some of their endowment risk with younger generations.

Through the introduction of social security all future generations born into shock 1 gain approximately 2 percent in terms of consumption equivalent variation. However, the introduction of the system in the good state is *not* Pareto-improving, since the currently old generation suffers welfare loses of around 2 percent because the price of the land they own collapses (the middle-aged gain around 3 percent).

If the system is introduced in the bad aggregate state of the world (say, the Great Depression) then the reform *is* Pareto-improving. Generations born into shock 2 gain approximately 0.2 percent, less than in the good state.[1] But now the current old gain 3.5 percent and the current middle aged gain 1.5 percent, because, even though the price of land declines with the introduction of social security, the received social security benefits more than compensate this negative shock to the value of these generations' asset portfolio. This example indicates that the timing of the reform matters: introducing social security in the Great Depression may have constituted a Pareto improvement while the same reform may hurt (some generations) when current aggregate conditions are good.

In this example the introduction of social security has two effects, a direct effect through the intergenerational transfers and a price effect through the decline in the price of land. While the change in prices is substantial it remains true that land has bad risk-characteristics for the old as it pays little when they are poor. The dominant effect is the direct effect. Social security enables transfers between generations which do not lie in the span of payoffs of existing assets (the marketed subspan). As Demange (2002) shows, in this model with land the direct effect of one-sided transfers between generations can never be Pareto-improving if the payoffs could be replicated with *existing* assets. The fact that land is the only asset in this economy and that

there are no Arrow securities available for trade is thus crucial for the result obtained above.

IV. Financial Innovations Versus Social Security

With two Arrow securities the competitive equilibrium in the above example would be Pareto efficient and without social security welfare for future generations is higher than with social security. However, an unanticipated *introduction* of Arrow securities into the incomplete markets economy does not result in a Pareto-improvement. On the contrary, the current old will lose because the price of land collapses with such financial innovation.

V. Concluding Remarks

The current US pay-as-you go social security system was introduced in 1935, partly as a response to the impoverishment of an entire generation during the Great Depression, the biggest negative aggregate shock the US economy has experienced so far. Our example shows that in a situation like in 1935, with a large adverse macroeconomic shock, it is possible to justify the introduction of a pay-as-you go social security system on the normative grounds that it provides a welfare improvement, in the sense of ex interim Pareto efficiency.

However, as Krueger and Kubler (2001b) argue, in a realistically calibrated *closed* economy with production the risk sharing benefits of an unfunded social security system tend to be dominated by its negative effects on capital accumulation and hence mean aggregate consumption. Future research has to show whether the same qualitative conclusions apply in an economy where social security also provides partial insurance against intragenerational idiosyncratic income and lifetime uncertainty.

References

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Footnotes

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[1]In general, people like to be born into the bad state, because in that state land is very cheap, compared to the good state, which overcompensates their bad endowment shock. The introduction of social security reduces the price dispersion of land between states, and thus is less beneficial for newborn agents in the bad state.