

Discussion of
“Asset Pricing Implications of Pareto Optimality with Private Information”

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Motivation

- Standard Models have unsatisfactory asset pricing implications
- This paper:
 1. deduce asset pricing implications of a dynamic private information model
 2. test these implications empirically
 3. compare results to implications of two standard full information models

Outline

- Model Structure, Theoretical Results and Asset Pricing Implications
- Comments on the Theory
- Empirical Findings
- Comments on the Empirical Results
- Where to Go from Here?

The Model: Stochastic Structure

- Aggregate shocks $z^t = (z_1, \dots, z_t)$, public information.
- Idiosyncratic shocks $\theta^t = (\theta_1, \dots, \theta_t)$, potentially private information.
- Idiosyncratic shocks are *iid* across agents, with density $\mu(\theta^t)$. Law of large numbers applies. Otherwise stochastic process is arbitrary.
- Shocks affect labor productivity $\phi(z_t, \theta_t)$.

The Model: Preferences and Technology _____

- Continuum of ex-ante identical individuals that live for T periods.
- Preferences $E_0 \sum_{t=1}^T \beta^{t-1} \{u(c_t) - v(l_t)\}$
- Efficiency units of labor $y_t = \phi(z_t, \theta_t)l_t$, where y_t is observable, but $\phi(z_t, \theta_t)$ and l_t are not.
- Aggregate production function $F(z_t, K_t(z^{t-1}), L_t(z^t))$.

The Model: Constrained-Efficient Allocations _____

- Social planner observes z^t , asks individuals to report θ^t . Allocates consumption $c_t(z^t, \theta^t)$, efficiency units $y_t(z^t, \theta^t)$ according to reports.
- An allocation $\{c_t, y_t\}_{t=1}^T$ is incentive compatible if households weakly prefer to report their productivity shocks truthfully.
- An allocation is incentive feasible if it is incentive compatible and satisfies the aggregate resource constraint.
- A constrained efficient allocation maximizes the ex-ante lifetime utility of the households within the set of incentive feasible allocations.

Model: Key Result

- Similar to Rogerson (1985) one obtains the Euler equation for this private information economy as

$$u'(c_t(z^t, \theta^t)) = \beta E_{z^{t+1}} \left\{ \frac{1 - \delta + F_K(z^{t+1})}{E_{\theta^{t+1}} \left\{ \frac{1}{u'(c_{t+1}(z^{t+1}, \theta^{t+1}))} \middle| \theta^t, z^{t+1} \right\}} \middle| z^t \right\}$$

- Denote by $\lambda(z^{t+1})$ the *Social Stochastic Discount Factor*:

$$\lambda(z^{t+1}) = \beta \frac{\frac{1}{u'(c_t(z^t, \theta^t))}}{E_{\theta^{t+1}} \left\{ \frac{1}{u'(c_{t+1}(z^{t+1}, \theta^{t+1}))} \middle| \theta^t, z^{t+1} \right\}}$$

Model: Key Assumptions

- Crucial assumption 1: Social Stochastic Discount Factor is a valid SDF for assets traded in private markets.
- SDF involves expectation over function of individual future consumption. Need to know stochastic consumption process. Not good for empirical work.
- Crucial assumption 2: Law of large numbers: $\mu(\theta^{t+1})$ is individual probability of θ^{t+1} and deterministic fraction of population with history θ^{t+1} . Thus can interpret $E_{\theta^{t+1}}$ as cross-sectional moment of the population consumption distribution. Good for empirical work.

Implementation

- Assume $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

- Can rewrite SDF as

$$\beta c_t(z^t, \theta^t)^\gamma = \lambda(z^{t+1}) E_{\theta^{t+1}} \left\{ c_{t+1}(z^{t+1}, \theta^{t+1})^\gamma \mid \theta^t, z^{t+1} \right\}$$

- Integrating both sides with respect to θ^t yields

$$\begin{aligned} \beta E_{\theta^t} \{ c_t(z^t, \theta^t)^\gamma \mid z^t \} &= \lambda(z^{t+1}) E_{\theta^{t+1}} \{ c_{t+1}(z^{t+1}, \theta^{t+1})^\gamma \mid z^{t+1} \} \\ \lambda(z^{t+1}) &= \beta \frac{E_{\theta^t} \{ c_t(z^t, \theta^t)^\gamma \mid z^t \}}{E_{\theta^{t+1}} \{ c_{t+1}(z^{t+1}, \theta^{t+1})^\gamma \mid z^{t+1} \}} \\ &= \beta \frac{C_{\gamma,t}(z^t)}{C_{\gamma,t+1}(z^{t+1})} \end{aligned}$$

where $C_{\gamma t}(z^t)$, $C_{\gamma t+1}(z^{t+1})$ can be interpreted (using the LLN) as the population averages of individual consumption, raised to the power γ

- Only need cross-sectional data to construct $C_{\gamma t}(z^t)$, $C_{\gamma t+1}(z^{t+1})$.

3 Models, 3 Stochastic Discount Factors _____

Model	PI	CM	SIM
SDF $\lambda(z^{t+1})$	$\beta \frac{C_{\gamma,t}(z^t)}{C_{\gamma,t+1}(z^{t+1})}$	$\beta \frac{C_{1,t+1}(z^{t+1})^{-\gamma}}{C_{1,t}(z^t)^{-\gamma}}$	$\beta \frac{C_{-\gamma,t+1}(z^{t+1})}{C_{-\gamma,t}(z^t)}$

- Here

$$C_{\gamma,t}(z^t) = E_{\theta^t} \{ c_t(z^t, \theta^t)^\gamma \mid z^t \} = \int c_t(z^t, \theta^t)^\gamma \mu(\theta^t) d\theta^t$$

is the γ -th moment of the cross-sectional consumption distribution.

Discussion of Theoretical Results

- Nice Theorems
- Key ingredient: Law of Large numbers allows one to replace individual expectation with cross sectional expectations
- Decentralization?
Kocherlakota (2005) derives optimal tax system which implements PI allocations. However, as he writes, “In the real world, there is no reason to expect politically determined tax systems to be the same as the socially optimal one described in this paper.”

Discussion of Theoretical Results: An Aggregate SDF

- Evidently, for each individual the necessary conditions for optimality must hold and could be tested with appropriate data.
- Obviously, this would be a bad test for many reasons. Aggregation mitigates many of the problems.
- In both PI and SIM, one can derive many alternative stochastic discount factors, which are all valid in the absence of measurement errors and 'individual mistakes'

- For example for SIM, Brav et al. (2002) use

$$\beta \int_{\theta^{t+1}} \frac{c(z^{t+1}, \theta^{t+1})^{-\gamma}}{c(z^t, \theta^t)^{-\gamma}}$$

- Similarly, for PI one would obtain

$$\frac{\beta}{\int_{\theta^{t+1}} \frac{c(z^{t+1}, \theta^{t+1})^\gamma}{c(z^t, \theta^t)^\gamma}}$$

- Narayana and Luigi give nice conditions on measurement errors under which their approach is advantageous, but there is little empirical evidence on what are good assumptions on measurement error.

Discussion of Theoretical Results: Aggregation —

- Integrating across agents has disadvantage that one has to deal with unobserved heterogeneity.
- Effect of differences across agents in discounting and risk aversion?

In Rescue of Standard Incomplete Markets? _____

- Constantinides and Duffie (1996) and Krebs (2004) show that the theory places few restrictions on moments of the cross-sectional distribution of consumption growth and prices.
- Personal disaster process more realistic than aggregate disaster state (Rietz, Mehra and Prescott, 1988). But would show up in cross-section.
- It is true that in the data the left tail of the consumption seems to play less of a role.

Asset Pricing Tests

- Equity Premium

$$E \left\{ \lambda(z^{t+1}) \left[R_{t+1}^{stock}(z^{t+1}) - R_{t+1}^{bond}(z^{t+1}) \right] \right\} = 0$$

- Bond Returns

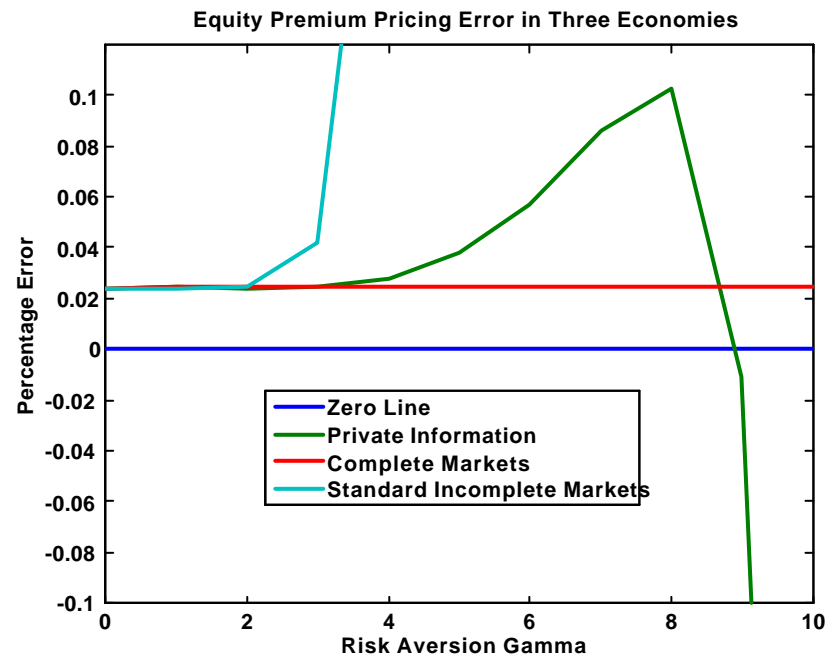
$$E \left\{ \lambda(z^{t+1}) R_{t+1}^{bond}(z^{t+1}) \right\} = 1$$

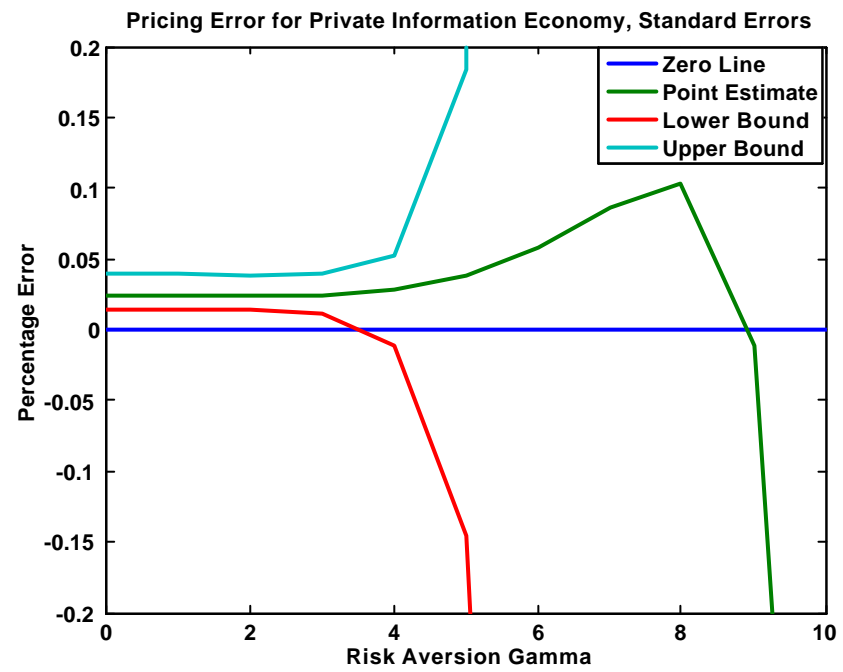
- Autocorrelation of Bond Returns

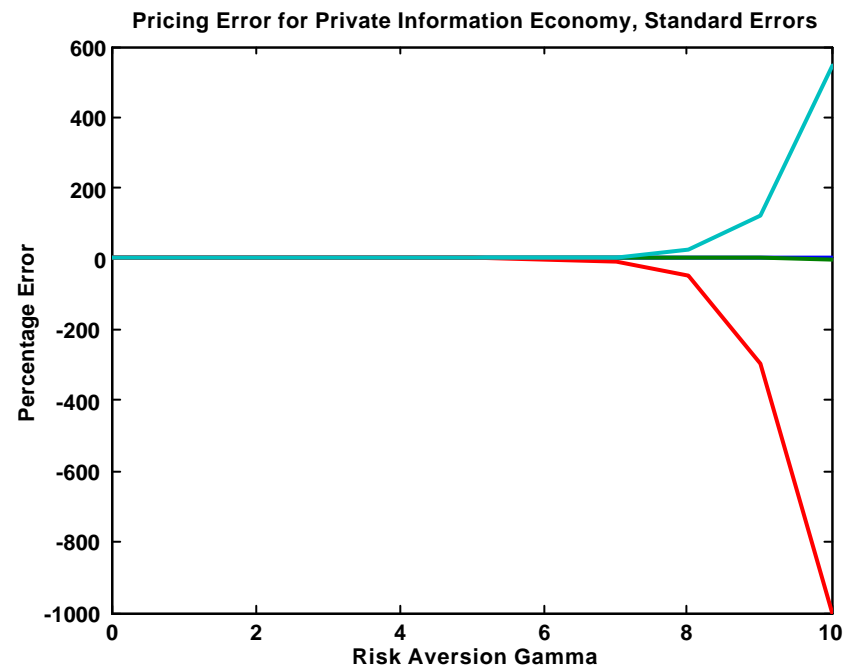
$$E \left\{ \left[\lambda(z^{t+1}) R_{t+1}^{bond}(z^{t+1}) - 1 \right] R_t^{bond}(z^t) \right\} = 0$$

Results: Equity Premium Pricing Error

$$\bar{e}_{mkt} = Mean \left\{ \hat{\lambda}(z^{t+1}) \left[R_{t+1}^{stock}(z^{t+1}) - R_{t+1}^{bond}(z^{t+1}) \right] \right\}$$







Evaluation of all Restrictions

$$\begin{aligned}\bar{e}_{mkt} &= \text{Mean} \left\{ \hat{\lambda}(z^{t+1}) \left[R_{t+1}^{stock}(z^{t+1}) - R_{t+1}^{bond}(z^{t+1}) \right] \right\} \\ \bar{e}_{b1} &= \text{Mean} \left\{ \hat{\lambda}(z^{t+1}) R_{t+1}^{bond}(z^{t+1}) - 1 \right\} \\ \bar{e}_{b2} &= \text{Mean} \left\{ \left[\hat{\lambda}(z^{t+1}) R_{t+1}^{bond}(z^{t+1}) - 1 \right] R_t^{bond}(z^t) \right\}\end{aligned}$$

Average Pricing Errors, Estimated (GMM) (γ, β)

Economy	Priv. Info.	Comp. Markets	Inc. Markets
\bar{e}_{mkt}	0.0247	0.0243	0.0242
\bar{e}_{b1}	≈ 0	0.001	≈ 0
\bar{e}_{b2}	≈ 0	≈ 0	≈ 0

Picky Points about the Empirical Analysis _____

- Why nondurable consumption? Small share of consumption. Shows no growth in sample period. Does not compare well with NIPA. Adding imputed service flows from durables helps (Krueger and Perri, 2005).
- Why not deflate consumption groups component by component? Substantial changes in relative prices.
- For CM, results and interpretation slightly different to Kocherlakota, 1996: For risk aversion greater than 8.5 results were not significant. Also doubtful that model that assumes risk aversion greater than 2.5 can be called a resolution of equity premium puzzle

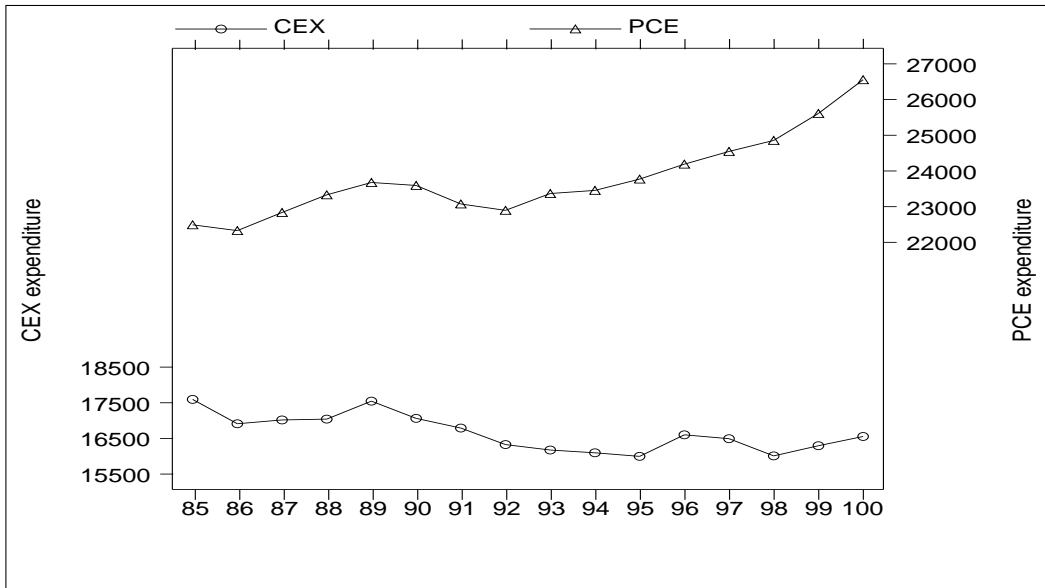


Figure 2: Non-durable expenditures in 2000 dollars - Consumer Expenditure Survey (CEX) and Personal Consumption Expenditures (PCE)

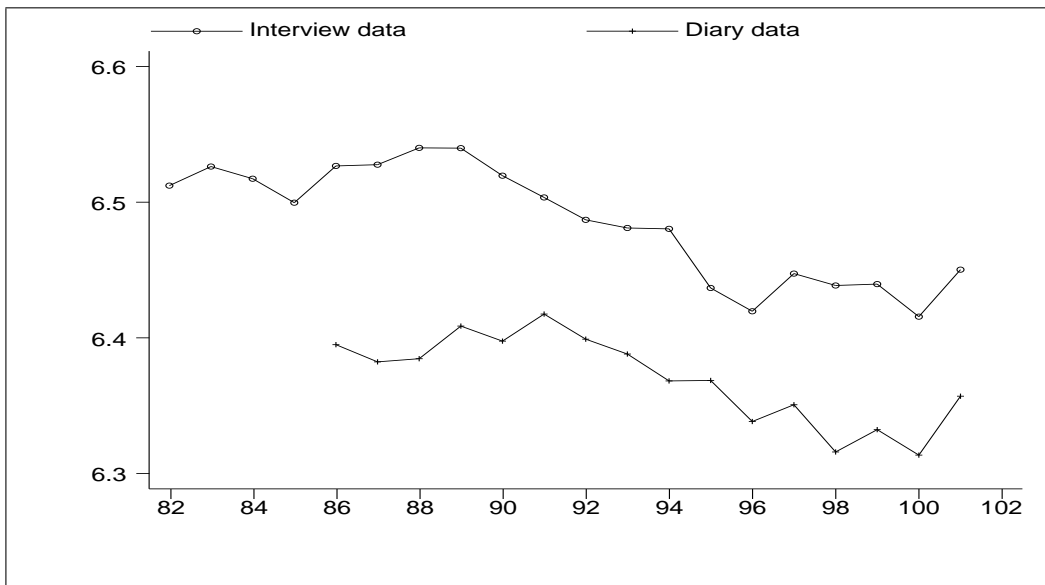


Figure 3: Mean of log monthly expenditure on non-durable goods (2001 dollars)

Direct Tests of Consumption Allocations Predicted by Both Models

- Before doing consumption-based asset pricing, may want to investigate whether models do good job describing actual consumption allocations.
- Ignore aggregate uncertainty and assume economy is in steady state and thus $\beta(1 - \delta + F_K) = 1$

- Private Information Economy

$$E_{\theta^{t+1}} \left\{ \left(\frac{c_{t+1}(\theta^{t+1})}{c_t(\theta^t)} \right)^\gamma - 1 \mid \theta^t \right\} = 0$$

- Standard Incomplete Markets Economy

$$E_{\theta^{t+1}} \left\{ \left(\frac{c_{t+1}(\theta^{t+1})}{c_t(\theta^t)} \right)^{-\gamma} - 1 \mid \theta^t \right\} = 0$$

- Estimate b in

$$E_{\theta^{t+1}} \left\{ \left(\frac{c_{t+1}(\theta^{t+1})}{c_t(\theta^t)} \right)^b - 1 \middle| \theta^t \right\} = 0$$

(by GMM, using orthogonality conditions and some instruments). If $\hat{b} > 0$, prefer private information model, if $\hat{b} < 0$, prefer standard incomplete markets model.

- Given their data and expertise, easy to do for the authors.
- Ligon (1997) implements such an exercise with consumption data for Indian villages and finds support for the private information model.
- Complete markets model? Independently rejected in CEX data many times (Mace, 1991, Nelson, 1994, Krueger and Perri, 2005).

Where to Go Next with Asset Pricing? _____

- PI model prices stocks well for high risk aversion ($\gamma \approx 9$).
- PI model prices bonds well for lower risk aversion ($\gamma \approx 3$) and thus higher $IES \approx \frac{1}{3}$ (which is within the range of values that people estimate from micro data, see Attanasio and Weber, 1993, 1995).
- Why not use preferences allowing to control attitudes towards consumption across states and across time separately? Recursive utility.
- But: hard to derive similar characterization because result in the paper relies crucially on consumption to be separable in the utility function.

Conclusions

- Elegant theory of incomplete consumption insurance.
- Very nice result that stochastic discount factor(s) is ratio of some moment of cross-sectional consumption distribution. Need only repeated cross sections of consumption to implement asset pricing tests.
- Can explain the equity premium, but not bond and stock market returns simultaneously.
- Final word on consumption-based asset pricing models?