Discussion of Rui Zhao "Repeated Two-Sided Moral Hazard"

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MAIN RESULTS

1. Under fairly general conditions Pareto optimal contracts under repeated two-sided moral hazard are recursive

2. Partial characterization of optimal contracts

3. Obvious next step: explore further properties of optimal contracts using numerical "examples"

MAIN ASSUMPTIONS

1. For all $\theta_i \in \Theta_i$, all $a_i \in A_i$

$$\pi(\theta_i | a_i) > 0$$

2. $u_i : (\underline{c}, \infty) \rightarrow \Re$ is strictly concave and

$$\lim_{c \rightarrow \underline{c}} (c) = -\infty$$

3. There exist $a_i, a_i' \in A_i$ such that $g_i(a_i) \neq g_i(a_i')$

4. (Infinite horizon)

RECURSIVE FORMULATION

- State variable

U : promise of expected discounted lifetime utility for agent 1

- Control Variables:

$$\begin{aligned} \alpha_i &: \text{ probability distribution over effort } a_i \in A_i \\ c_i(\theta) &: \text{ consumption, conditional on public signal} \\ \theta &\in \Theta_1 \times \Theta_2 \\ U(\theta) &: \text{ continuation utility of agent 1,} \\ & \quad \text{conditional on } \theta \end{aligned}$$

- Timing

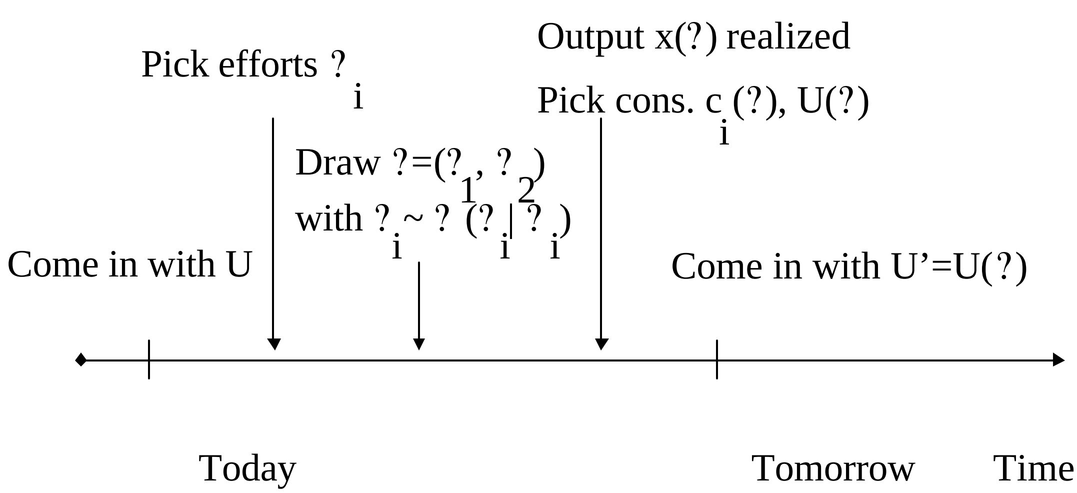

Figure 1:

- Bellman equation

$$
\begin{aligned}
& V(U) \\
= & \max_{\alpha_i, c_i(\theta), U(\theta)} \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|\alpha_2) * \\
& \{u_2(c_2(\theta)) + \delta V(U(\theta)) - g_2(\alpha_2)\}
\end{aligned}
$$

subject to

  - Promise Keeping

$$
\begin{aligned}
U = & \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|\alpha_2) * \\
& \{u_1(c_1(\theta)) + \delta U(\theta) - g_1(\alpha_1)\}
\end{aligned}
$$

– IC, agent 1: for all $a_1 \in A_1$

$$\begin{aligned} & \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|\alpha_2) * \\ & \{u_1(c_1(\theta)) + \delta U(\theta) - g_1(\alpha_1)\} \\ \geq & \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|a_1)\pi^2(\theta_2|\alpha_2) * \\ & \{u_1(c_1(\theta)) + \delta U(\theta) - g_1(a_1)\} \end{aligned}$$

– IC for agent 2: for all $a_2 \in A_2$

$$\begin{aligned} & \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|\alpha_2) * \\ & \{u_2(c_2(\theta)) + \delta V(U(\theta)) - g_2(\alpha_2)\} \\ \geq & \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|a_2) * \\ & \{u_2(c_2(\theta)) + \delta V(U(\theta))\} - g_2(a_2) \end{aligned}$$

– Resource Feasibility: for all $\theta \in \Theta_1 \times \Theta_2$

$$c_1(\theta) + c_2(\theta) = x(\theta)$$

MAIN CHARACTERIZATION

- Two-Sided Moral Hazard: Sub-martingale result

$$\begin{aligned}\frac{u_1'(c_1(\theta))}{u_2'(c_2(\theta))} &= \sum_{\theta_2'} \pi(\theta_2'|\alpha_2') \left[\sum_{\theta_1'} \pi(\theta_1'|\alpha_1') \frac{u_2'(c_2(\theta'))}{u_1'(c_1(\theta'))} \right]^{-1} \\ &\leq \sum_{\theta_1'} \sum_{\theta_2'} \pi(\theta_2'|\alpha_2') \pi(\theta_1'|\alpha_1') \frac{u_1'(c_1(\theta'))}{u_2'(c_2(\theta'))}\end{aligned}$$

where

$$\begin{aligned} c_i(\theta) &= c_i(U;\theta) \\ \alpha_i' &= \alpha_i'(U') = \alpha_i'(U(\theta)) \\ c_i(\theta') &= c_i(U';\theta') = c_i(U(\theta);\theta') \end{aligned}$$

- One-Sided Moral Hazard (Rogerson 1985)): Agent 1 is risk neutral principal; agent 2 has moral hazard problem; equal discount factors: Martingale result

$$
\begin{aligned}
\frac{1}{u_2'(c_2(\theta))} &= \sum_{\theta'} \pi(\theta'|\alpha_2') \left[\frac{u_2'(c_2(\theta'))}{1}\right]^{-1} \\
&= \sum_{\theta'} \pi(\theta'|\alpha_2') \frac{1}{u_2'(c_2(\theta'))} \\
\text{or } \frac{1}{u_2'(c_2(\theta))} &= E_{\theta'|\alpha_2'} \left\{\frac{1}{u_2'(c_2(\theta'))}\right\}
\end{aligned}
$$

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$$\frac{u_1'(c_1(\theta))}{u_2'(c_2(\theta))} = \sum_{\theta_2'} \pi(\theta_2'|\alpha_2') \left[\sum_{\theta_1'} \pi(\theta_1'|\alpha_1') \frac{u_2'(c_2(\theta'))}{u_1'(c_1(\theta'))} \right]^{-1}$$

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SMALL REMARKS

- What restrictions does the use of public actions impose?

- Implementation of optimal contracts?

- Connection with observables? Characterization of consumption (compensation) process? Note that the results about long-run behavior still involve endogenous choices.

NUMERICAL EXAMPLES

- Suppose $A_i = \{a_l, a_h\}$ and $\Theta_i = \{\theta_1, \theta_2\}$

- Only one (continuous) state variable U

- 14 control variables

- Potential problem: unbounded domain of U, $U(\theta)$

- Potential resolution: lower bounds on continuation utilities (because of limited commitment)

$$\begin{aligned} U(\theta) &\geq \underline{u}_1 \\ V(U(\theta)) &\geq \underline{u}_2 \end{aligned}$$

- But: this may change implications of the model drastically: see Atkeson and Lucas (1992) vs. (1995)