Pareto Improving Social Security Reform when Financial Markets are Incomplete!?*

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Abstract

This paper studies an overlapping generations model with stochastic production and incomplete markets to assess whether the introduction of an unfunded social security system leads to a Pareto improvement. When returns to capital and wages are imperfectly correlated a system that endows retired households with claims to labor income enhances the sharing of aggregate risk between generations. Our quantitative analysis shows that, abstracting from the capital crowding-out effect, the introduction of social security represents a Pareto improving reform, even when the economy is dynamically efficient. However, the severity of the crowding-out effect in general equilibrium tends to overturn these gains.

Keywords: Social Security Reform, Aggregate Fluctuations, Intergenerational Risk Sharing, Incomplete Markets

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1 Introduction

Should the government provide the elderly with a tax-financed pension or should each individual be left to save for her own retirement consumption in potentially risky privately traded assets? This question was one of the most controversial issues in the 2004 presidential campaign. Whereas the proponents of a partial privatization in the Bush camp primarily pointed to higher *average* returns that can be earned by privately investing in stocks, the opponents on the Kerry side stressed the *risk* of low returns to savings for entire generations due to large aggregate shocks.[1]

The current US pay-as-you go (PAYGO) social security system was introduced in 1935, partially as a response to the great depression, the biggest negative aggregate shock the US economy has experienced so far. In this paper we ask whether the introduction of an unfunded social security system which re-allocates the impact of aggregate shocks across generations and thus reduces the consumption risk in old age, provides a Pareto improving policy reform (that is, provides a welfare improvement for all generations then alive and for generations to be born into all future states of the world). We will show that the answer to this question depends on the quantitative importance of the positive *inter*generational risk sharing effect, relative to the negative effects from a declining aggregate capital stock. Choosing as a starting point of our thought experiment the economy without social security allows us to analyze the risk-return trade-off between social security and private assets, which is at the center of the current reform debate, without having to take a stand on how a potential transition from the current to a partially privatized system has to be financed.[2]

How can a social security system lead to enhanced intergenerational risk sharing? As Shiller (1999) and Bohn (1998, 1999) have argued, if returns

[1] To quote John Kerry "I do not support any of the current plans for privatization or partial privatization of Social Security because each would leave beneficiaries unacceptably vulnerable to volatility in the financial markets." The quote is from an interview, available at http://www.aflcio.org/issuespolitics/politics/candidates-i-retirement.cfm On the same issue, George Bush in the third Bush-Kerry debate, Oct 13, 2004: "Younger workers ought to be allowed to take some of their own money and put it in a personal savings account, because I understand that they need to get better rates of return than the rates of return being given in the current Social Security trust." The quote is available at http://www.issues2000.org/2004/George-W-Bush-Social-Security.htm. An academic discussion of this debate is contained in Aaron et al. (2001) or Burtless (2001).

[2] It is well known that a transition from an unfunded to a funded social security system generally cannot be Pareto-improving, independent of how outstanding benefits are honored and financed. See Feldstein and Liebman (2001) and the references they cite.

to capital and wages are imperfectly correlated and subject to aggregate shocks, the consumption variance of all generations can be reduced if government policies enable them to pool their labor and capital incomes. A social security system that endows retired households with a claim to labor income serves as such an effective tool to share aggregate risk between generations. It is absolutely crucial for this argument that financial markets are incomplete, for if there were private markets in which a full set of state-contingent claims is traded, social security can serve no further role as risk allocation device. The title of the paper is intended to reflect this argument.

The idea that missing asset markets provide a normative justification for a PAYGO social security system dates back at least to Diamond (1977). He points out that the absence of certain investment opportunities may lead to inefficient risk allocations. Merton (1983) analyzes the economic inefficiencies caused by the non-tradeability of human capital in an overlapping generations model with stochastic production and suggests that the present social security system can help to eliminate these. While incomplete financial markets can provide a rationale for social security, it is also well known that in a general equilibrium model a PAYGO social security system crowds out private savings and thus capital formation, and therefore leads to lower wages for future generations.[3] These two effects have opposite impacts on agents' welfare and only a careful *quantitative* analysis can reveal which of the two dominates. In this paper we undertake such an analysis.

Our economy is populated by overlapping generations that face stochastic, imperfectly correlated wages and returns to capital. Households have a preference for smooth consumption profiles and can transfer resources across time by purchasing claims to the risky aggregate capital stock and trade one period, risk-free bonds. Employing a recursive utility representation as in Kreps and Porteus (1978) and Epstein and Zin (1989) allows us to control risk aversion independently from the willingness to intertemporally substitute consumption, which is helpful in generating reasonable equity premia in general equilibrium. The government administers a pure PAYGO social security system by collecting a payroll tax and paying out (stochastic) benefits that balance the budget of the system. With the introduction of such a system the government in effect forces households to hold a third asset and thus to diversify capital income risk. This beneficial role has to be traded off against the crowding-out of physical capital that its introduction induces.

[3]See again Feldstein and Liebman (2001) for an elegant survey of the theoretical and empirical literature studying this crowding-out effect.

We insure that the equilibrium without social security is dynamically efficient in the sense of Samuelson (1958) by providing a sufficient theoretical condition that we check in our quantitative exercises. Therefore social security is *not* simply beneficial because it cures overaccumulation of capital or leads to better allocation of (average) resources across generations, as in Samuelson (1958) or Diamond (1965).

Our quantitative analysis exhibits three main findings. First, abstracting from the crowding-out effect of social security in general equilibrium, the introduction of social security does indeed represent a Pareto improving reform, even though the equilibrium without social security is dynamically efficient. This result is obtained even though the return differential between private returns to capital and implicit returns to the social security system amounts to 4.2 percentage points, indicating a strong positive effect of social security on the intergenerational allocation of risk. Second, the severity of the capital crowding-out effect in general equilibrium overturns these gains , at least if the economy is parameterized as is standard in the macroeconomic and public finance literature. However even in general equilibrium the introduction of social security is a Pareto-improving reform if households are highly risk averse and, in addition, have a fairly high intertemporal elasticity of substitution and physical capital is not too important in the production function.

In the next section we develop a simple, analytically tractable model that aims at formalizing the intuition for the intergenerational risk sharing effect and at providing a back-of-the-envelope calculation of the welfare consequences of social security reform. Section 3 describes the general equilibrium model and contains the sufficient condition for dynamic efficiency of equilibrium. Section 4 discusses the calibration of the model and Section 5 summarizes our main results, first for a partial equilibrium, then for a general equilibrium version of the model, including sensitivity analysis. Conclusions are contained in Section 6, with details about theoretical derivations and data used in the paper relegated to the appendix.

2 A Simple Model

We now present a simple, two period partial equilibrium model to formalize the intuition from the introduction. This model will abstract from dynamic consumption and portfolio choices, endogenous determination of asset returns and capital crowding-out. All of these elements will be endogenized in the full general equilibrium model below. With this simple model we seek to gain a first understanding which properties of the asset return processes are crucial for our argument to work, and to obtain a back-of-the envelope assessment whether the risk-sharing effect quantitatively significant.

Each agent lives for two periods, earns wage w in the first period on which she pays a payroll tax τ. The remainder of her wages is invested into a risky savings technology with stochastic gross return R. In the second period of her life she receives social security payments of τwG, where G is the stochastic gross return of the social security system. The agent values consumption in the second period of her live, with consumption given by

$$c = (1-\tau)wR + \tau wG \tag{1}$$

according to the differentiable utility function $v(c)$. Lifetime utility, as a function of the size of the social security system, is therefore given by

$$U(\tau) = Ev\left[(1-\tau)wR + \tau wG\right] \tag{2}$$

where $E(.)$ is the expectation with respect to uncertainty realized in the second period of the households' life.

We ask when a marginal introduction of a social security system is welfare-improving, that is, seek necessary and sufficient conditions under which $U'(\tau = 0) > 0$. Under the assumption that $v(c) = \ln(c)$ and that G and R are jointly lognormal this condition reduces to (see the appendix)

$$E\left\{\frac{G}{R}\right\} = \frac{E(G)}{E(R)} \cdot \frac{[cv(R)^2 + 1]}{[\rho_{G,R} \cdot cv(G) \cdot cv(R) + 1]} > 1 \tag{3}$$

where $\rho_{G,R} = Cov(G,R)/[Std(G)Std(R)]$ is the correlation coefficient between G and R and $cv(R) = Std(R)/E(R)$ is the coefficient of variation of the risky savings returns, with $cv(G)$ defined accordingly.

From (3) we see that the introduction of a marginal social security system is welfare improving if the implicit expected return to social security, $E(G)$, is bigger than the return on the risky saving technology, $E(R)$. But even if the latter is higher than the former, the introduction of social security may still be justified if the stochastic saving returns are very volatile ($cv(R)$ big) or the correlation between private saving returns and returns to social security is small. We will calibrate our general equilibrium model exactly to these statistics from the data which this simple model has pointed to as crucial in determining the welfare consequences of social security.

For a general CRRA utility function $v(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and without any distributional assumptions on (G, R) condition (3) can be generalized to

$$E\left(\frac{G-R}{R^{\sigma}}\right) = E\left(\frac{G}{R^{\sigma}}\right) - E\left(R^{1-\sigma}\right) > 0 \tag{4}$$

We use data on private returns to saving R and returns to the social security system G, as well as the condition (4) to provide a first quantitative assessment whether the introduction of a (small) social security system is justified on the grounds of a better risk allocation. This exercise also provides an estimate of the degree of risk aversion required for this argument to work. We construct the gross returns R in the model from a NYSE/AMEX value weighted portfolio, as reported in Campbell (2003), and the gross return to social security G by the gross growth rate of real total compensation of employees from NIPA, provided by the Bureau of Economic Analysis (BEA). Details are contained in the appendix.

A key question is what time interval in the data corresponds to a model period. The data is available in yearly frequency; since in our simple model agents live for two periods, a model period may more reasonably be interpreted as twenty years. We derived results for annual data, and for data of 17-year frequency (which exactly gives us four observations for our data). Computing the left hand side of (4) for varying degrees of risk aversion we find that condition (4) is satisfied for all $\sigma \geq 1.34$ if one uses annual data, and for all $\sigma \geq 1.54$ if one uses 17 year intervals. Thus, in our simple model the cutoff risk aversion above which social security provides a welfare improvement is at the lower end of values commonly used in macroeconomics and public finance. In the remaining part of the paper we assess whether the same result holds true once agents make dynamic consumption and asset allocation decisions, and asset returns as well as the capital stock are endogenously determined in general equilibrium

3 The General Model

Our model extends Diamond's (1965) economy to aggregate uncertainty. Time is discrete and runs from $t = 0, \ldots, \infty$. Aggregate uncertainty is represented by an event tree. The economy starts with some fixed event z_0, and each node of the tree is a history of exogenous shocks $z^t = (z_0, z_1 \ldots z_t)$. The notation $z^t \succ z^s$ means that z^t is a potential successor node of z^s, for $t > s$. The shocks are assumed to follow a Markov chain with finite support $\mathcal{Z}$ and strictly positive transition matrix π. Let $\pi(z^t|z_0)$ denote the probability that the node z^t occurs.

3.1 Demographics, Endowments and Preferences

The economy is populated by nine overlapping generations. This choice constitutes a compromise between realism and computational feasibility. The population grows at rate n. In each period t, $L_t = (1+n)L_{t-1}$ identical new households are born. $L_0 = 1$ denotes the number of newborns in period 0. A household is fully characterized by the node in which she is born (z^t). When there is no ambiguity we index them simply by their date of birth.

An agent born at node z^s has non-negative, deterministic labor endowment over her life-cycle, $(l^0, l^1, \ldots, l^8)$. The price of the consumption good at each date event is normalized to one and at each date event z^t the household supplies her labor endowment inelastically for a market wage $w(z^t)$.

Let $c^s(z^t)$ denote the consumption of an agent born at time s in period $t \geq s$ and let $U^s(c, z^t)$ be the expected continuation utility of an agent born in node z^s from node $z^t \succ z^s$ onwards. An agent born at node z^s therefore has expected lifetime utility from allocation c given by $U^s(c, z^s)$. Individuals have preferences over consumption streams representable by the recursive utility function (see Kreps and Porteus (1978) and Epstein and Zin (1989))

$$U^s(c, z^t) = \left\{ \left[c^s(z^t)\right]^\rho + \beta \left[\sum_{z_{t+1}} \pi(z_{t+1}|z_t) \left(U^s(c, z^{t+1}) \right)^{1-\sigma} \right]^{\frac{\rho}{1-\sigma}} \right\}^{\frac{1}{\rho}} \tag{5}$$

where $\frac{1}{1-\rho}$ is the intertemporal elasticity of substitution and σ measures the risk aversion of the consumer with respect to atemporal wealth gambles.[4]

Households have access to a capital storage technology which uses one unit of the consumption good today to produce one unit of the capital good tomorrow. We denote the investment of household s into this technology by $a^s(z^t)$. At time t the household sells capital goods accumulated from last period, $a^s(z^{t-1})$, to the firm for a market price $1 + r(z^t)$. In addition, households buy and sell one-period bonds $b^s(z^t)$ at price $q(z^t)$ today that pay one unit of consumption tomorrow. Agents are born with zero assets,

[4] We assume $\sigma > 0$ and $\rho < 1, \rho \neq 0$. Note that if $\rho = 1 - \sigma$, then households have standard constant relative risk aversion expected utility, with CRRA of σ, if the final continuation utility function is given by $U^s(c, z^{s+8}) = c^s(z^{s+8})$, which we assume.

and we allow households to borrow against future labor income. The budget constraint of household s in period $t \geq s$ reads as

$$c^s(z^t)+a^s(z^t)+q(z^t)b^s(z^t) = (1+r(z^t))a^s(z^{t-1})+b^s(z^{t-1})+(1-\tau(z^t))l^{t-s}(z_t)w(z^t)+I(s)ben(z^t) \tag{6}$$

where $\tau(z^t)$ is the social security payroll tax, $ben(z^t)$ are social security benefits received by a retired agent and $I(s)$ is an indicator function that equals 1 for retired agents and 0 otherwise. We assume that in period 0 there are $L_0/(1+n)^i$ households of ages $i = 0, \ldots, 8$ with given capital holdings $a^0_{-1}, \ldots, a^{-8}_{-1}$, where by assumption $a^0_{-1} = 0$.

3.2 Firms

There is a single representative firm which uses labor and capital to produce the consumption good according to a constant returns to scale production function $f_t(K, L; z_t)$. Since firms make decisions on how much capital to buy and how much labor to hire after the realization of the shock z_t they face no uncertainty and simply maximize current profits. In our quantitative work below we will always use the following parametric form for the production function

$$f_t(K, L; z_t) = \xi(z_t)K^\alpha \left[(1+g)^t L\right]^{1-\alpha} + K(1-\delta(z_t)) \tag{7}$$

where g is the rate of labour-augmenting technological progress, $\xi(.)$ is the stochastic shock to productivity and where $\delta(.)$ can be interpreted as the stochastic depreciation rate. Note that at the firm's optimum, the interest rate equals the marginal product,

$$r(z^t) = \frac{\partial f_t(K, L, z_t)}{\partial K} = \xi(z_t)\alpha \left[\frac{(1+g)^t L}{K}\right]^{1-\alpha} - \delta(z_t). \tag{8}$$

3.3 Government

The government levies payroll taxes to pay for social security benefits. We model social security as a PAYGO system that adheres to period by period budget balance. Taxes and benefits satisfy

$$\tau(z^t)w(z^t)L(z^t) = ben(z^t)L^{ret}_t \tag{9}$$

where $L(z^t)$ is total labor input at node z^t and L^{ret}_t is the total number of retired people in the economy.

3.4 Equilibrium and Pareto Efficiency

We assume that all markets, that is, spot markets for consumption, labor, capital and bonds, are perfectly competitive. We will present results for two versions of our model. The first is a general equilibrium version, in which all capital used in domestic production is owned by domestic agents and all asset markets clear. In the second version the productive capital stock is exogenously fixed at $\bar{K}_t$ which grows at rate $n+g$ per period, $\bar{K}_t = [(1+n)(1+g)]^t \bar{K}$. Prices for bonds q_t follow an exogenous stochastic process. We refer to this version as partial equilibrium model, because prices are exogenously determined.

For given initial conditions z_0, $(a^s_{-1}, b^s_{-1})^0_{s=-8}$ a competitive general equilibrium is a collection of choices for households $(c^s(z^t), a^s(z^t), b^s(z^t))^{s+8}_{t=s}$, for the representative firm $\{K(z^t), L(z^t)\}$, a policy $\{\tau(z^t), ben(z^t)\}$ as well as prices $\{r(z^t), q(z^t), w(z^t)\}$ such that households and the firm maximize, the government budget constraint (9) is satisfied, and markets clear: for all t, z^t

$$L(z^t) = (1+n)^t \sum_{s=0}^{8} \frac{l^s}{(1+n)^s} \quad (10)$$

$$K(z^t) = (1+n)^t \sum_{s=1}^{8} \frac{a^{t-s}(z^{t-1})}{(1+n)^s} \quad (11)$$

$$0 = (1+n)^t \sum_{s=1}^{8} \frac{d^{t-s}(z^{t-1})}{(1+n)^s} \quad (12)$$

$$(1+n)^t \sum_{s=0}^{8} \frac{c^{t-s}(z^t)}{(1+n)^s} + K(z^{t+1}) = f_t(K(z^t), L(z^t), z_t) \quad (13)$$

By Walras' law market clearing in the labor, bonds and capital market imply market clearing in the consumption goods market in general equilibrium. In the partial equilibrium version of the model the labor market clearing condition (10) remains the same and the capital market clearing condition now reads as[5] $K(z^t) = \bar{K}_t$.

An allocation (c, K) is (ex interim) Pareto efficient if it is feasible and there is no other feasible allocation $(\hat{c}, \hat{K})$ such that $U^s(\hat{c}, z^s) \geq U^s(c, z^s)$ for all z^s and $U^s(\hat{c}, z^s) > U^s(c, z^s)$ for at least one z^s.

In order to solve for the equilibrium numerically using recursive techniques we de-trend the economy by deterministic population growth and

[5] The bond markets and good markets no longer need to clear.

technological progress. Denoting growth-adjusted consumption by $\tilde{c}$, and other variables accordingly[6], the Euler equations from the individuals' optimization problem read as

$$\left[E_{z^t}\left(\tilde{U}^s_{t+1}\right)^{1-\sigma}\right]^{\frac{\sigma-1+\rho}{1-\sigma}} \tilde{\beta} E_{z^t}\left[\frac{\tilde{c}^s(z^{t+1})}{\tilde{c}^s(z^t)}\right]^{\rho-1}\left(\frac{1+r(z^{t+1})}{1+g}\right)\left(\tilde{U}^s_{t+1}\right)^{1-\sigma-\rho} = 1 \tag{14}$$

$$\left[E_{z^t}\left(\tilde{U}^s_{t+1}\right)^{1-\sigma}\right]^{\frac{\sigma-1+\rho}{1-\sigma}} \tilde{\beta} E_{z^t}\left[\frac{\tilde{c}^s(z^{t+1})}{\tilde{c}^s(z^t)}\right]^{\rho-1}\left(\frac{\left(\tilde{U}^s_{t+1}\right)^{1-\sigma-\rho}}{q(z^t)(1+g)}\right) = 1. \tag{15}$$

Since each agents' optimization problem is finite-dimensional and convex, these Euler equations are necessary and sufficient for optimal household choices. In order to compute equilibrium allocations numerically we formulate these Euler equations recursively and then compute Markov equilibria with the techniques developed by Krueger and Kubler (2004). These are equilibria with a compact state space, characterized by policy functions mapping from the state space $\mathcal{Z} \times \mathcal{S}$ to prices and actions. The set of exogenous shocks $\mathcal{Z}$ is finite and in our computations we assure that the space $\mathcal{S}$ of endogenous state variables (individual beginning-of-period wealth and aggregate capital) is compact.

3.5 The Thought Experiment

We consider the following thought experiment: In an equilibrium of the economy with a payroll tax rate $\tau \equiv 0$ at some event z^t there is an unanticipated increase of τ. What are the welfare effects for all individuals born or alive at z^t and born at all successor nodes? In order to determine whether such a reform improves welfare for all future generations, one needs, in principle, to compare welfare at infinitely many nodes. In our quantitative results below, however, the welfare consequences of the reform stabilize after at most 3 periods (ca. 20 years).

3.6 Dynamic Efficiency and Pareto Efficiency

Competitive equilibria in OLG models may not be Pareto efficient even when markets are sequentially complete (there exists a full set of Arrow

[6] More precisely, define $\tilde{c}^s(z^t) = \frac{c^s(z^t)}{[1+g]^t}$, $\tilde{\beta} = (1+g)^\rho \beta$ and $\tilde{U}^s_t = \frac{U^s(c,z^t)}{(1+g)^t}$.

securities), because of an inefficient allocation of average consumption across generations. It is well known since Samuelson (1958) that transfers from the young to the old (such as PAYGO social security) can help to cure this type of inefficiency, which we call *dynamic inefficiency*. Therefore we want to distinguish dynamic inefficiency from the inefficient risk allocation across generations that can occur if financial markets are incomplete.

For this we need a definition of dynamic efficiency when markets are incomplete that captures the notion that transfers between young and old within the existing *marketed subspan* cannot be Pareto improving. To make the concept of the marketed subspan precise, recall that an individual's investment at node z^t in the firm is denoted by $a(z^t)$ and his bond position by $d(z^t)$. Given bond prices and returns to capital $(q(z^t), r(z^t))$, define the marketed subspace $\mathcal{M}^{z^s}$ of the commodity space for an agent born at z^s by $(\eta(z^t)) \in \mathcal{M}^{z^s}$ if there exist a trading strategy $(d(z^t), a(z^t))$ such that

$$\eta(z^t) = d(z^{t-1}) + a(z^{t-1})(1 + r(z(s_t))) - d(z^t)q(z^t) - a(z^t) \text{ for all } z^t \succeq z^s. \tag{16}$$

Following Demange (2002) we now define dynamic efficiency with incomplete asset markets in the following way.

Definition: *A competitive equilibrium allocation c (given equilibrium bonds prices, capital returns and wages) is dynamically efficient, if there is no other feasible allocation* $\tilde{c}$ *in the marketed subspan (this is, with* $(\tilde{c}^s(z^t) - l^s(z^t)w(s^t)) \in \mathcal{M}^s$ *for all agents s) which Pareto-dominates c.*

3.6.1 Discussion

Below, we provide a sufficient condition for an allocation to be dynamically efficient which can be checked in our quantitative exercises. Before this we want to justify our focus on dynamically efficient equilibria and discuss what dynamic efficiency implies and what it does not imply.

There are three reasons that make us focus on dynamically efficient equilibria. First, if the equilibrium without social security is not dynamically efficient, incomplete markets and improved risk allocation are not needed to make a normative case for the introduction of PAYGO social security. Second, Abel et al. (1989) provide a sufficient condition for dynamic efficiency in a two period OLG model that can be tested empirically. When implementing the test they find strong support for the hypothesis that the US economy (as well the economies of other industrialized countries) is dynamically efficient. Third, while we abstract from land as an additional asset, it is well known (see e.g. Demange, 2002) that a competitive equilibrium in an economy with land necessarily is dynamically efficient. These arguments suggest that for the question addressed in this paper the main focus on economies that are dynamically efficient is appropriate.

What are the implications of dynamic efficiency the way we have defined it? First, if social security leads to a Pareto-improvement despite the fact that the old allocation was dynamically efficient, the improvement must be caused by the fact that the transfers did not lie in the span of the original assets. Second, our sufficient condition below will reveal that it is possible for the economy to be dynamically efficient even though the *average* return on the risk-free bond is lower than the average implicit gross return on PAYGO social security, $(1+g)(1+n)$. Of course if the bond return were *constant* over time and across states and smaller than $(1+g)(1+n)$, the economy is not dynamically efficient and social security is welfare improving because it helps to cure this dynamic efficiency. Since, as documented below, empirically average real gross bond returns are smaller than $(1+g)(1+n)$, it is crucial for dynamic efficiency that there exists states of the world in which the realized bond interest rate exceeds $(1+g)(1+n)$, which is also true empirically.

3.6.2 Sufficient Conditions for Dynamic Efficiency

In order to verify whether an allocation is dynamically efficient we need a sufficient condition that can be numerically verified for our economy. This condition is specifically formulated to be tractable for the economies we consider and may be far from being a necessary condition. To state the condition, recall that we compute Markov equilibria. Each state of current shock and current wealth for all generations, (z, Θ) in the compact state space can be viewed as a possible initial condition $\vartheta = (z, \Theta) \in \mathcal{Z} \times \mathcal{S}$ and induces (under our policy-functions) a unique competitive equilibrium with values for the state variables, returns to capital and bond prices at every date-event z^t, $(\Theta^{\vartheta}(z^t), r^{\vartheta}(z^t), q^{\vartheta}(z^t))$. We index these by the initial condition ϑ.

Define $\mathcal{Z}^H \times \mathcal{S}^H \subset \mathcal{Z} \times \mathcal{S}$, to be the set of states (z, Θ) in which the current equilibrium interest rate is above and bounded away from $(1+g)(1+n)$ and for which the endogenous state next period, given any shock $z \in \mathcal{Z}^H$ remains in $\mathcal{S}^H$. Note that in our partial equilibrium version of the model, prices only depend on the current exogenous state z_t and $\mathcal{Z}^H$ simply consists of all such states for which the risk-free rate is above $(1+g)(1+n)$.

For any equilibrium with prices $q(z^t), r(z^t)$, define a supporting[7] price system $(p(z^t))$ by $p(z_0) = 1$ and

$$\begin{aligned} E(p(z^t)(1 + r(z^t))|z^{t-1}) &= p(z^{t-1})(1 + n)(1 + g) && (17) \\ E(p(z^t)|z^{t-1}) &= p(z^{t-1})q(z^{t-1})(1 + n)(1 + g) && (18) \end{aligned}$$

For a given initial condition $\vartheta = (z, \Theta)$, collect all possible supporting price systems in $\mathcal{P}^\vartheta$. The following proposition now gives our sufficient condition for dynamic efficiency.

Proposition 1 *Suppose that*

1. *for all initial conditions with implied high interest rate,* $\vartheta = (z_0, \Theta_0) \in \mathcal{Z}^H \times \mathcal{S}^H$*, there exist two different shocks* z' *and* $\hat{z}'$ *such that in the induced equilibrium both* $q^\vartheta(z_0, z') < \frac{1}{(1+g)(1+n)}$ *and* $q^\vartheta(z_0, \hat{z}') < \frac{1}{(1+g)(1+n)}$ *and such that* $1 + r^\vartheta(z_0, z') > \frac{1}{q^\vartheta(z_0)}$ *and* $1 + r^\vartheta(z_0, \hat{z}') < \frac{1}{q^\vartheta(z_0)}$*.*
2. *for all possible initial conditions* $\vartheta = (z, \Theta) \in \mathcal{Z} \times \mathcal{S}$*, there exists a finite time horizon* $T < \infty$ *and supporting prices* $p \in \mathcal{P}^\vartheta$ *such that at time* T *supporting prices are less than one unless the economy is in a high-interest rate state, that is, if* $p(z^T) > 1$ *then* $(z^\vartheta_T, \Theta^\vartheta(z^T)) \in \mathcal{Z}^H \times \mathcal{S}^H$*.*

Then the economy is dynamically efficient.

Proof: See appendix.

If the only asset in our economy is the bond, condition 1 of the proposition says that there exists a set of states with high interest rates (above $(1+g)(1+n)$) and once the economy is in that set it stays there with positive probability. Condition 2 requires that the economy reaches the set $\mathcal{Z}^H \times \mathcal{S}^H$ of high interest rates in finite time, with positive probability. With capital, in addition condition 1 requires that across high bond interest rate states the return to capital varies sufficiently.

For the partial equilibrium version of the model, the two conditions in the previous proposition can be easily verified. First, since the Markov transition matrix for the exogenous shocks has strictly positive entries, for any state today it is always possible to reach a shock in $\mathcal{Z}^H$ in the next

[7] In fact, these are supporting prices, discounted by the population growth rate.

period. Then, it is sufficient that there exist two different shocks for which the interest rate is above $(1+g)(1+n)$, with returns to capital in one shock above the interest and returns to capital in the other shock below the interest rate. When stock and bond returns are stochastically independent this is possible if and only if for some state of the bond process the bond interest rate is above $(1+g)(1+n)$. For the partial equilibrium model it then suffices to have at least one state with risk-free rates bigger than $(1+n)(1+g)$ that is reached from all other states with positive probability. In our quantitative exercises this will be the case.[8]

4 Calibration

In order to quantify the welfare effects of introducing an unfunded social security system we first have to parameterize our model.

4.1 Aggregate Growth and Uncertainty

In our model economy agents live for 9 periods. Therefore we interpret one model period to last 6 years. As population growth rate we choose $n = 1.1\%$ per annum, and as average growth rate of wages we take $g = 1.8\%$, the long-run averages for the US. The labor share in the Cobb-Douglas production function is taken to be $\alpha = 0.3$, as in Hubbard and Judd (1987).

We assume that aggregate uncertainty is driven by a four-state Markov chain with support $\mathcal{Z} = \{z_1, z_2, z_3, z_4\}$ and transition matrix $\pi = (\pi_{ij})$. Since we want to model both shocks to total factor productivity and to depreciation, a particular state z_i maps into a combination of low or high TFP and low or high depreciation.

$$
\begin{aligned}
\xi(z) &= \begin{cases} 1.0 + \nu \text{ for } z \in \{z_1, z_2\} \\ 1.0 - \nu \text{ for } z \in \{z_3, z_4\} \end{cases} \\
\delta(z) &= \begin{cases} \bar{\delta} - \psi \text{ for } z \in \{z_1, z_3\} \\ \bar{\delta} + \psi \text{ for } z \in \{z_2, z_4\} \end{cases}
\end{aligned} \tag{19}
$$

We set $\bar{\delta}$, the average depreciation rate, to 0.31, or 6% per year.

[8] For the general equilibrium model, the second condition is harder to verify since there generally exist levels of aggregate capital for which the interest rate remains below $(1+g)(1+n)$, independently the exogenous shock next period. However, in our applications it turns out that after two periods the economy reaches $\mathcal{Z}^H \times \mathcal{S}^H$ with positive probability, and therefore the second condition can still be verified computationally, with $T = 2$.

The aggregate state z_1 is characterized by a good TFP-shock and a good depreciation shock (low depreciation), whereas z_4 features a bad TFP shock and a bad depreciation shock. To introduce persistence of the process over time we assume that the Markov process is a mixture between an *iid* process and the identity matrix I,

$$\pi = (1-w)\Pi + wI \tag{20}$$

where w is a parameter governing the persistence of the process and Π is composed of rows of the form $(\Pi_1, \Pi_2, \Pi_3, \Pi_4)$, and Π_j is the probability of state z_j in the stationary distribution of π. We assume symmetry in that $\Pi_1 = \Pi_4$ and $\Pi_2 = \Pi_3$. Given the restriction $\sum_j \Pi_j = 1$ the matrix π is then uniquely determined by two numbers (Π_1, w), which, together with (ν, ψ) and possibly $\bar{K}$ completely characterize the production technology.

In addition, in partial equilibrium we have to specify an exogenous stochastic bond price process (which we do below in subsection 5.1), whereas in general equilibrium this process is endogenously determined.

4.2 Endowments and Preferences

Labor endowments follow the life cycle pattern documented in Hansen (1993). This profile is given as $(l^0, l^1, \ldots, l^8) = (1, 1.35, 1.54, 1.65, 1.67, 1.66, 0, 0)$. It implies that, absent aggregate shocks, individual labor earnings have a hump-shaped profile, with peak around the age of 48; at that age individuals earn 67% more than at their entry into the labor force in their early 20's. Households of age 63 retire and possibly receive social security benefits.

Our recursive preferences are characterized by the intertemporal elasticity of substitution $\frac{1}{1-\rho}$, the time discount factor β and the risk aversion parameter σ. Since our results depend sensitively on these parameters we report outcomes for different combinations of these parameters, choosing as benchmark an intertemporal elasticity of substitution of 0.5 (that is, $\rho = -1$) and document our welfare numbers for several degrees of risk aversion. An IES of 0.5 lies in the middle of the empirical estimates from the micro consumption literature (see, e.g. Attanasio and Weber, 1993, 1995), and is commonly used in the macro and public finance literature (it implies a coefficient of relative risk aversion of 2 with standard CRRA preferences).

4.3 Social Security

The benchmark size of the social security system is $\tau = 0$ (no social security) and our experiment consists of the "marginal" introduction of social security of size $\tau = 2\%$ (its size when it was first introduced in the U.S.).

4.4 Calibration Targets

The technology parameters $(\Pi_1, w, \nu, \psi, \bar{K})$ are chosen jointly so that the benchmark model competitive equilibrium delivers the following statistics from aggregate data on wages and returns to capital, which we empirically interpret as stock market returns.[9] These data, and thus the equilibrium of our model, exhibit exactly the return-risk trade-off on which the current political debate about social security reform centers. The empirical statistics we target with our model are: a) an average (log of the gross) real return on risky capital of 42% (7% per annum), b) a coefficient of variation for the return of capital of 1.15, c) a coefficient of variation of wages of 0.11 d) a correlation coefficient between wages and returns to risky capital of -0.38, and e) an autocorrelation of wages of 0.78.

Note that in partial equilibrium, model-generated statistics for wages and returns are independent of the preference parameters and thus need not be re-calibrated as we perform sensitivity analysis with respect to these parameters. In the general equilibrium version of our model capital accumulation is endogenous, and therefore the parameter $\bar{K}$ is absent. Consequently we choose one of the preference parameters, namely the time discount factor β, so that the general equilibrium with $(\Pi_1, w, \nu, \psi, \beta)$ delivers equilibrium observations consistent with the facts above. In anticipation of this we choose as time discount factor for the partial equilibrium version of the model $\beta = 0.92$, or a time discount rate of 1.4% per year.

The parameters required in partial equilibrium for model-generated statistics to coincide with the five empirical observations stated above are summarized in Table 1, together with the other parameters of the model.

Table 1: Parameterization

Par.	$n(pa)$	$g(pa)$	α	Π_1	ν	ψ	$\bar{\delta}$	w	β	τ
Val.	1.1%	1.8%	0.3	0.12	0.11	0.82	0.31	0.78	0.92	0

Note that a probability $\Pi_1 = \Pi_4 = 0.12 < 0.25$ is required to match the negative correlation of returns to labor and capital for *six year time*

[9] Our model period lasts for 6 years, and thus the statistics reported below refer to wage and return data over six year periods. Loosely speaking, the parameter $\bar{K}$ determines the average return on capital, the shock to TFP, ν, determines the variability of wages, conditional on ν the shock to depreciation ψ determines the variability of returns to capital, the probability Π_1 determines how correlated returns to capital and labor are and finally w controls the autocorrelation of wages.

periods in the data. For the model to reproduce this observation it has to be sufficiently likely that TFP-shocks and depreciation shocks of opposite direction occur simultaneously. The relative magnitude of TFP-shocks and depreciation shocks is explained by the fact that returns to capital are much more volatile in the data than are wages. Since TFP-shocks affect both returns as well as wages directly, the size of these shocks have to be moderate for wages not to be too volatile. Given this, depreciation shocks have to be of large magnitude to generate returns to capital that are sufficiently volatile in the model. In general equilibrium these parameters have to be re-calibrated to generate the same statistics as in partial equilibrium; the required changes in parameters are relatively modest, however.

5 Results

5.1 Partial Equilibrium

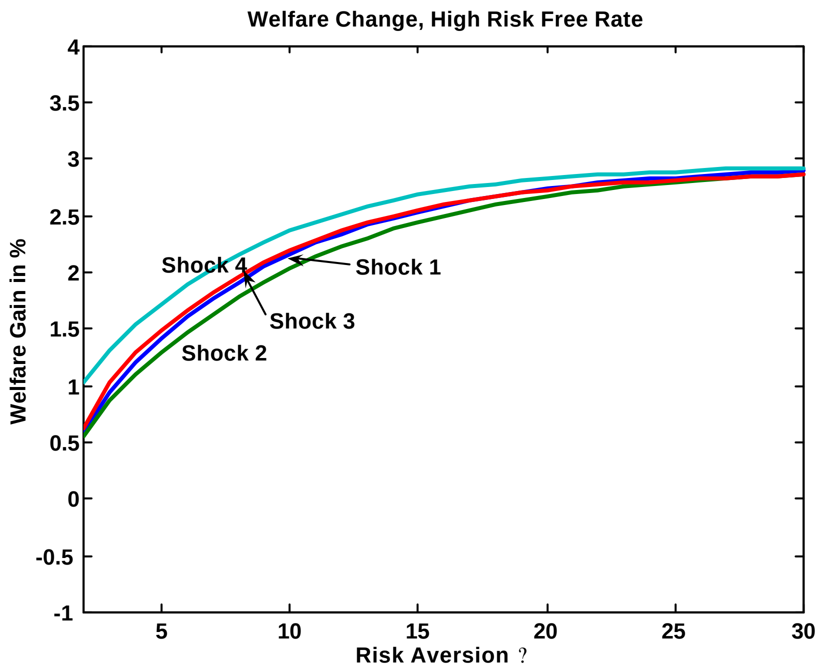
We first investigate whether the basic results from our simple model in section 2 carry over to a model with nontrivial intertemporal and portfolio choices. In addition to the parameters specified above we assume in this section that the exogenous price of the risk free bond is driven by an *iid* process that is independent of the stochastic process driving the technology shocks. The bond price q takes one of two values, $q \in \{q_l, q_h\}$ with equal probability. The values for $\{q_l, q_h\}$ are chosen such that the implied risk-free interest rate $r_b = \frac{1}{q} - 1$ has mean and variability as in the data. The empirical targets from the Campbell (2003) data are 3.31% for the mean of the six-year (logarithm of the gross) real interest rate (i.e. 0.55% per annum) and a standard deviation of roughly 20%, again for six year intervals.[10]

5.1.1 Benchmark Results

Figure 1 displays the welfare impact *for a newborn agent* of introducing a marginal unfunded social security system $\tau = 2\%$, as a function of the coefficient of relative risk aversion. Since the aggregate capital stock is fixed and wages and returns to capital therefore only vary with the exogenous shock z, the welfare consequences from such a reform for any newborn agent depend only on the current shock and the current bond interest rate $\frac{1}{q} -$

[10] This yields bond prices $q_l = 0.79$ and $q_h = 1.19$. Note that while bond prices are risky, our bond is still riskless in that it pays one unit of consumption for sure tomorrow. But what is not risk-free is a trading strategy that re-invests $1 year after year in the bond.

1. The figure displays the effects for $q = q_l$, that is, for currently high bond interest rates. We measure welfare changes in consumption equivalent variation (or "consumption", for short): we ask what percentage of extra consumption, in each state, an agent would require in the old equilibrium to be as well off as with the introduction of social security. Positive numbers thus indicate welfare gains from an introduction of social security for a newborn agent.[11]



The figure shows several features worth commenting on. First, introducing social security is welfare improving: it increases welfare by between 0.5% and 2.8%, in terms of consumption equivalent variation. Second, the welfare gains are monotonically increasing in the households' risk aversion. Third, they are fairly uniform in the aggregate shock z, with slightly higher welfare gains, if today's shock is $z = z_4$ (low wages, low returns) and slightly lower

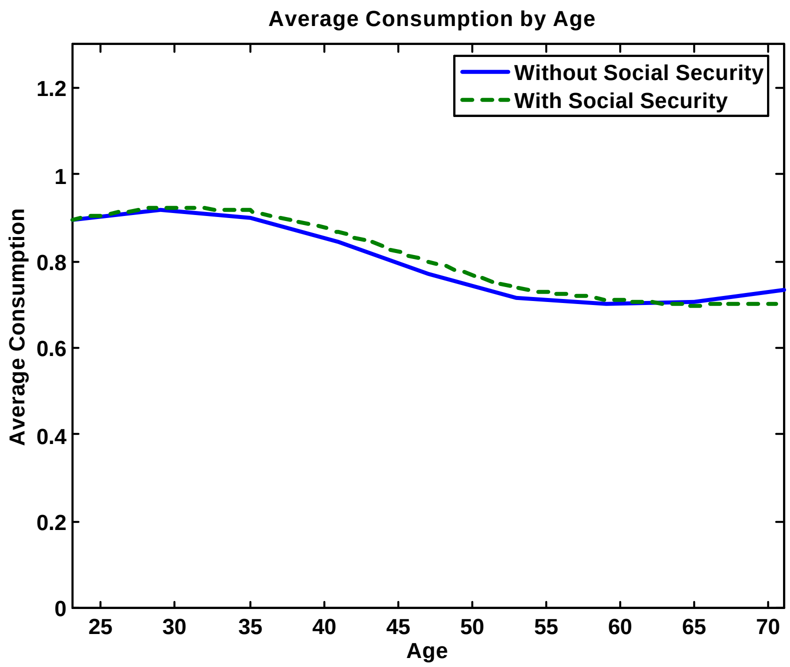
[11] Newborn agents are crucial in the determining whether the introduction of social security is Pareto improving. If newborns gain, then older agents that will receive full benefits but only pay social security taxes for part of their working lives benefit from the introduction of social security as well.

for $z = z_2$ (high wages, low returns). Note that plotting the same figure for low current bond interest rates (not shown) yields the same qualitative results, but with welfare gains that are slightly higher (between 1% and 3%).

What drives the results in figure 1? There are two potential effects that social security can have on households consumption and asset allocation decisions, and thus welfare. First, social security gives old households a source of income that is imperfectly correlated with returns to capital, the principal source of financing of old-age consumption. As such, social security can serve to reduce the variance of consumption of older households; henceforth we will refer to this effect as the *consumption insurance effect*. The size of the welfare benefit from this effect depends crucially on the households' attitude towards risk, as measured by its risk aversion.

Second, social security can have an *effect on mean consumption.* On one hand it reduces disposable labor income when young because of the payroll tax. While households are free to borrow in our model, they are already optimally leveraged without social security. Simply borrowing the payroll tax would lead, in states of the world with bad returns to capital tomorrow, to suboptimally low consumption. Consequently households with high risk aversion decide to partially borrow more, but partially adjust their demand for risky capital (and, to a smaller part, current consumption) to the reduction in current disposable labor income. But this strategy leads to lower average consumption in the future, due to lower receipts from savings in capital. On the other hand, even though the economy is dynamically efficient, the average return on social security is higher than on the risk free bond. Therefore it is feasible for households to obtain higher average consumption with social security than without, if they are willing fully offset the payroll tax by borrowing more, and thus to tolerate significantly more consumption risk.

In our quantitative exercise, agents with lower risk aversion indeed opt to increase average consumption over the life cycle and consumption risk at all dates, when faced with social security. When agents are highly risk averse (to a degree that is required to obtain a reasonable equity premium in general equilibrium) social security has a small impact on average consumption over the life cycle, and it does not uniformly increase it. To the contrary, during the retirement periods average consumption without social security is higher than with social security, as Figure 2 shows (which is derived for households with $\sigma = 20$). This suggests that the mean consumption effect is of secondary order in explaining the welfare gains from social security. On the other hand note that social security makes consumption in old age significantly less risky (the *consumption insurance effect*); the standard deviation of consumption in the last period of life declines by 8% and in the second to last period by 2%.



In an economy without risk-free bonds agents with low risk aversion cannot increase average consumption by borrowing around social security. In fact, the reduction in disposable income requires a one-for-one reduction in the demand for capital or consumption. Thus in that economy the introduction of social security constitutes a Pareto improvement only if households are sufficiently risk-averse (for a σ bigger than 5). The main difference to the model with the bond is that now the negative mean consumption effect is significantly more powerful for households with low risk aversion, turning the welfare gains in the model with the bond into welfare losses.

5.1.2 Fixed Benefits vs. Fixed Tax Rates

We now assess how our results change when modeling social security as providing safe benefits system rather than be defined by constant payroll taxes. This is important because historically congress has reacted to surpluses of the social security trust fund with benefit increases, rather than tax reductions. On the other hand, revenue shortfalls have tended to give rise to tax increases, rather than benefit cuts. It is therefore hard to decide on empirical grounds whether modeling the system as defined benefit- or defined contribution system is more reasonable, even if one imposes budget balance on the system, as we do in this paper.

We now model the social security system as paying out a fixed benefit b in retirement and as levying a stochastic payroll tax that adjusts to insure budget balance. Evidently this requires higher tax rates when wages are currently low. In our calibration we choose the size of the introduced benefit b such that the associated average (over states) tax rate equals 2%, making our results quantitatively comparable to the previous sections.

Qualitatively, our results are similar to our benchmark system. But now the welfare gains from social security are higher (approximately 0.5 percentage points), due to the fact that a defined benefits system provides safe, rather than risky benefits, yielding less volatile retirement consumption.

In summary, our partial equilibrium analysis has exhibited the following findings: allowing households to intertemporally trade a risky and riskless asset leads to welfare consequences from social security that are uniformly positive in sign, despite the fact that the economy without social security is dynamically efficient. The size of the welfare gains is an increasing function of the households' risk aversion, testifying to the quantitative importance of the consumption insurance role played by social security. Finally, comparing a system of fixed benefits to a system with constant payroll taxes shows that both qualitatively and quantitatively the exact modelling choice of the balanced budget social security system is of secondary importance.

The remaining question, addressed in the next section, is whether and to what extent the well-known crowding-out effect of capital from a PAYGO social security system in general equilibrium can overturn our positive findings from the partial equilibrium model.

5.2 The Crowding-Out Effect in General Equilibrium

In order to address this question we now endogenize the capital stock and bond and capital returns, by requiring that the market clearing conditions, equations (11)-(13) hold. We keep the intertemporal elasticity of substitution as above, and choose technology and preference parameters such that the same empirical statistics as in partial equilibrium are matched. Since now average bond returns are endogenous, as is the capital stock (which therefore cannot be fixed anymore to determine the average return on capital), we pick the preference parameters (σ, β) to ensure that the average bond and stock returns in general equilibrium equal the empirical targets of 0.55% and 7% per annum, respectively. This requires $\sigma = 18, \beta = 0.92$.[12]

In partial equilibrium this parameterization yields solid welfare gains from introducing social security for newborns of 2-2.5%, depending on the state it is being introduced. Now the same agents suffer welfare *losses* from the same reform of between 1.6% and 1.8%. The welfare losses increase for future newborns to roughly 2% for the generation three periods (18 years) after the reform and then settle down.[13] Note that all generations already alive at the introduction of the reform experience welfare gains, apart from the two youngest generations.

What explains these welfare losses in general equilibrium? Table 2, column 1, summarizes average bond and stock returns in the equilibrium without and with social security, the percentage change in the capital stock and output, the percentage change in the coefficient of variation of consumption in the last period of life, and the range of welfare gains/losses. Here r_0 stands for the average stock return in the economy without social security, and r_∞ for the average stock returns with social security in the long run, that is, once the transition period induced by the introduction of social security has been completed. The percentage change in the capital stock, output and the coefficient of variation are measured in the same way, and the range of welfare numbers refers to newborns along the transition and in the new stochastic steady state

Table 2: General Equilibrium Results

Calibration	Benchmark	Cal. with SocSec	Lower Cap. Share
r_0 p. a.	7.0%	5.9%	6.7%
r_∞ p. a.	7.7%	6.3%	6.9%
r_{b0} p. a.	0.6%	−1.0%	0.5%
$r_{b\infty}$ p. a.	1.3%	−0.4%	0.8%
ΔK in %	−6.1%	−5.9%	−2.2%
Δ output in %	−2.0%	−1.2%	−0.37%
Δ cv cons. last period %	−2.3%	−1.9%	−3.4%
ΔWelf. in %	[−2.2%, −1.6%]	[−1.4%, 0.8%]	[0.02%, 0.8%]

[12] Strictly speaking, in general equilibrium all free parameters determine all equilibrium statistics jointly and thus the technology parameters have to be adjusted from their partial equilibrium values. However, to match the average returns the preference parameters (σ, β) are quantitatively most important.

[13] The welfare consequences of the reform in general equilibrium depend on the state of the economy in which social security is introduced, but not in a quantitatively crucial way.

The crucial statistic is the size of the capital crowding out induced by the reform. The capital stock declines, in the long run, by more than 6%, which leads to a decline in available output by about 2%. Since it takes time for the capital stock to fall to its new (stochastic) long run level, the welfare losses for newborn generations from the introduction of social security are rising over time. This detrimental crowding-out effect offsets the positive insurance effect of the policy reform (old age consumption becomes less risky, as the decline in the coefficient of variation of consumption in old age shows). Also note from table 2 that our economy without social security displays an empirically plausible endogenous return on capital and risk-free rate, and thus a realistic equity premium of about 6.5%. This is achieved with the help of large technology shocks, a fairly high intertemporal elasticity of substitution and a large risk aversion.[14] Note that despite the low average bond return the equilibrium without social security is not dynamically inefficient, as a numerical verification of our sufficient condition in Proposition 1 reveals. The reason for this is (as in partial equilibrium) substantial volatility in bond interest rates, so that states with gross bond interest rates strictly above $(1+g)(1+n)$ are sufficiently likely.

The large crowding-out effect in general equilibrium is explained as follows. Without social security households have to save privately for old age. They do so mostly with physical capital, which carries high return risk and thus a significant amount has to be acquired to guarantee a certain amount of retirement consumption. Social security provides some retirement income, and provides it in a fairly safe way. Thus upon the introduction of this program households partially offset lower income due to payroll taxes by borrowing in bonds, plus they reduce their retirement savings through capital substantially. Both these effects lead to a strong decline in aggregate savings and thus to a strong reduction of the physical capital stock in general equilibrium.[15] We conclude that even for high risk aversion the crowding-out effect of social security dominates the intergenerational risk sharing effect, and therefore the reform does not provide a Pareto improvement.

[14] A value of $\sigma = 18$ lies outside the range of values commonly deemed reasonable by macroeconomists, but is not uncommon in the finance literature (see, e.g., Kandel and Stambaugh (1991) and Cecchetti et al., 1993) and has some empirical support from experiments (see Gollier (2001) for a summary of this evidence). Because of the high σ and the large technology shocks required in our analysis we do not claim to jointly have resolved the equity premium and the risk free rate puzzle.

[15] The first effect is absent in an economy without a bond, and thus the crowding-out effect is less severe in such an economy

5.2.1 Social Security and Stock Market Returns

The data on returns of the stock market we use in our calibration section stem from the years 1926-1998. A pay-as-you go social security system was in place in the US since the late 1930's. It is therefore possible that high stock market returns in the sample period are partially due to the presence of social security. This possibility is important for our calibration exercise. The main reason why social security has such adverse welfare consequences in general equilibrium is the return differential between risky capital and an unfunded social security system, before its introduction, of roughly 4.2%. Above, in the partial equilibrium section, we argued why households don't find it optimal to completely offset the social security tax by borrowing at the bond interest rate, if they are fairly risk-averse.

Suppose we calibrate our economy in such a way that *with* an unfunded social security system our model economy reproduces the empirical targets set forth in the calibration section. Qualitatively, since returns on the risky capital stock in the *absence* of social security now are closer to the potential implicit returns of an unfunded social security system, we expect the welfare consequences of a social security reform to be more favorable. We now ask whether under such a calibration the economy without social security is dynamically inefficient, and if not, if a (marginal) reform now provides a Pareto improvement. We calibrate to the same observations as above, but now use as social security tax rate $\tau = 6.2\%$, half in between the current payroll tax rate and the situation in 1926 (no social security tax). Column two of table 2 shows that the welfare losses from the reform are somewhat smaller than under the benchmark calibration; in fact, for some states even newborn generations enjoy welfare gains. The crowding-out effect is equally severe as before and still dominates risk allocation considerations, insofar as the reform is still no Pareto improvement. Also note that bond interest rates are now so low that our sufficient condition for dynamic efficiency is not satisfied anymore; thus if the reform were a Pareto improvement it could be because it cured dynamic inefficiency in addition to leading to better risk allocation across generations.

5.2.2 Pareto-Improving Social Security Reform in General Equilibrium?

Is there a defendable calibration that renders social security a Pareto improving reform without the initial equilibrium being dynamically inefficient?

Our previous results indicate that for this to happen the capital crowding-out effects is required to be relatively small, or the impact of the reduction of capital on output and thus welfare needs to be reduced. The third column of table 3 provides such an example. The calibration on which it is based is similar to the benchmark (i.e. roughly matches the same empirical statistics), but now uses an intertemporal elasticity of substitution of one and capital share of $\alpha = 0.2$ in the production function, lower than commonly used in the literature.[16] Now the crowding out effect is substantially smaller, and given that the importance of the capital stock for production is diminished because of the smaller α, output merely falls by about 0.5% with the introduction of social security. A better risk allocation dominates the small negative effect from the crowding-out, leading to welfare gains for all current and future generations from the introduction of social security. Note that assuming a very high intertemporal elasticity of substitution also curbs the capital crowding-out effect and results in a Pareto improving social security reform even for $\alpha = 0.3$, but it renders the economy dynamically inefficient in the absence of social security, because it reduces the mean and variability of interest rates.

Summarizing our results from the general equilibrium version of the model we conclude that it is possible to calibrate the economy in a reasonable way such that the introduction of a small unfunded social security system constitutes a Pareto improvement even though the initial equilibrium allocation without social security is dynamically efficient. For this it is crucial that the capital crowding out effect induced by reduced private savings with social security is not too large, requiring either a very high intertemporal elasticity of substitution (which makes the economy dynamically inefficient) or a capital share that is not too big. For standard selection of parameters commonly used in the quantitative macro and public finance literature, however, the crowding out effect in general equilibrium quantitatively dominates the positive impact of social security on the risk allocation across generations, and the introduction of social security does not constitute a Pareto improvement.

[16] Auerbach and Kotlikoff (1987) use a value of $\alpha = 0.25$. Poterba (1997), table 4, computes a labor share $1 - \alpha$ of slightly more than 0.8, if labor income is measured as the sum of employee compensation and proprietors' income.

6 Conclusion

Our general equilibrium results suggest that the current political debate about the return-risk trade-off may be settled in favor of the return-dominance argument. However, because of the transition cost implied by a reform that reverses the introduction of a PAYGO social security system in 1935 no clear-cut policy recommendation about the desirability of a (partial) privatization of social security should be derived from our work.

Future research may extend our work along several important dimensions. First, we abstract from several beneficial roles of an unfunded, redistributive social security system. In the presence of incomplete financial markets social security provides a partial substitute for missing insurance markets against *idiosyncratic* labor income and lifetime uncertainty. On the other hand the distortive effects of payroll taxes on the labor supply decision remain unmodeled. We abstract from these features to more clearly isolate the potential magnitude of the beneficial intergenerational risk sharing role of social security.

Second, in this paper we are setting a very demanding bar that social security has to pass in order to be judged as welfare improving. Employing the Pareto criterion our normative analysis is silent about the political conflict surrounding the historical adoption or current reform of social security. Extensions of the work of Cooley and Soares (1997) and Boldrin and Rustichini (2000) to our environment with aggregate uncertainty are needed to address the questions why, though not mutually beneficial, the US social security system was introduced when it was introduced and who one would expect the major supporters of this reform (and of its reversal) to be.

Finally, we took the market structure to be incomplete and invariant to government policy. It is conceivable that in the absence of social security private markets would have developed after 1935 that could play the same role of providing intergenerational risk sharing as social security did in our paper. Ignoring this endogeneity may bias our results in favor of social security. While we view endogenizing the deeper reasons for market incompleteness and their interaction with public policy as important future research, it is unlikely to overturn the main message of this paper, given that it turns out to be somewhat unfavorable for social security already.

A Theoretical Appendix

A.1 Derivation of Equation (3)

With $v(c) = \ln(c)$ we have

$$U'(\tau = 0) = E\left\{\frac{G - R}{(1-0)R + 0 * G}\right\} > 0 \text{ if and only if } E\left\{\frac{G}{R}\right\} > 1 \quad (21)$$

We note that

$$E\left(\frac{G}{R}\right) = E\left(e^{\ln(G)-\ln(R)}\right) = E\left(e^{\ln(Z)}\right) \quad (22)$$

where $\ln(Z) := \ln(G) - \ln(R)$, so that $Z = \frac{G}{R}$. Since $(\ln(G), \ln(R))$ are jointly normal, both $\ln(G)$ and $\ln(R)$ are normal random variables, and thus $\ln(Z)$ is normal with mean $\mu_{\ln Z} = \mu_{\ln G} - \mu_{\ln R}$ and variance $\sigma^2_{\ln Z} = \sigma^2_{\ln G} + \sigma^2_{\ln R} - 2\sigma_{\ln G, \ln R}$. Since Z is lognormal we have

$$E\left(\frac{G}{R}\right) = E(Z) = e^{\mu_{\ln G} + \frac{1}{2}\sigma^2_{\ln G}} \cdot e^{-(\mu_{\ln R} + \frac{1}{2}\sigma^2_{\ln R})} \cdot e^{\sigma^2_{\ln R}} \cdot e^{-\sigma_{\ln G, \ln R}} \quad (23)$$

Since G and R are log-normal we have

$$\begin{aligned} E(G) &= e^{\mu_{\ln G} + \frac{1}{2}\sigma^2_{\ln G}} \text{ and } E(R) = e^{\mu_{\ln R} + \frac{1}{2}\sigma^2_{\ln R}} \\ Var(R) &= e^{2\mu_{\ln R} + \sigma^2_{\ln R}} \cdot \left(e^{\sigma^2_{\ln R}} - 1\right) = E(R)^2 \cdot \left(e^{\sigma^2_{\ln R}} - 1\right) \quad (24) \end{aligned}$$

We thus obtain

$$e^{\mu_{\ln G} + \frac{1}{2}\sigma^2_{\ln G}} = E(G) \quad (25)$$

$$e^{-(\mu_{\ln R} + \frac{1}{2}\sigma^2_{\ln R})} = \frac{1}{E(R)} \quad (26)$$

$$e^{\sigma^2_{\ln R}} = \frac{Var(R) + E(R)^2}{E(R)^2} \quad (27)$$

Finally we want to obtain an expression for $e^{-\sigma_{\ln G, \ln R}}$. But

$$\begin{aligned} Cov(G, R) &= E(GR) - E(G)E(R) = E(e^{\ln(G)+\ln(R)}) - E(G)E(R) \\ &= e^{\mu_{\ln G} + \mu_{\ln R} + \frac{1}{2}\sigma^2_{\ln G} + \frac{1}{2}\sigma^2_{\ln R} + \sigma_{\ln G, \ln R}} - E(G)E(R) \\ &= E(G)E(R)\left(e^{\sigma_{\ln G, \ln R}} - 1\right) \quad (28) \end{aligned}$$

and thus

$$e^{-\sigma_{\ln G, \ln R}} = \frac{E(G)E(R)}{Cov(G, R) + E(G)E(R)} \quad (29)$$

Plugging in (25) − (29) into (23) yields

$$E\left(\frac{G}{R}\right) = \frac{E(G)}{E(R)} \cdot \frac{\frac{Var(R)+E(R)^2}{E(R)^2}}{\frac{Cov(G,R)+E(G)E(R)}{E(G)E(R)}} = \frac{E(G)}{E(R)} \cdot \frac{\left[cv(R)^2+1\right]}{\left[\rho_{G,R} \cdot cv(G) \cdot cv(R) + 1\right]} \tag{30}$$

as in the main text.

A.2 Proof of Sufficient Condition for Dynamic Efficiency

Our proof strategy is to show that the sufficient condition stated in the main text implies the following condition by Demange (2002) that she shows to be sufficient for dynamic efficiency (see Theorem 1 in Demange (2002) as well as the proof). Recall that $\mathcal{P}$ is the set of supporting prices.

Proposition 2 *An equilibrium allocation is dynamically efficient if*

$$\lim_{t\to\infty} \inf_{p\in\mathcal{P}} E_0 \left(\sum_{s=t}^{t+8} p(z^s)\right) = 0 \tag{31}$$

The proposition states that it is sufficient for dynamic efficiency that the *infimum* over all supporting prices tends to zero as time goes to infinity. Therefore we can assure that an allocation is dynamic efficiency if we find *some* supporting price system that satisfies condition (31).

At a first glance it seems that (31) is not easy to verify since it involves prices at infinity. However, the fact that we compute Markov equilibria simplifies the analysis crucially. The following lemma will be the key to tractable sufficient conditions for (31).

Lemma 3 *Suppose that there exists an $\epsilon > 0$ such that for all initial conditions $\vartheta = (z_0, \Theta_0) \in \mathcal{Z} \times \mathcal{S}$, there is an integer T for the resulting equilibrium such that there exist supporting prices $(p(z^t)) \in \mathcal{P}^\vartheta$, such that*

$$\sum_{z^T \succ z_0} \pi(z^T|z_0) p(z^T) < (1-\epsilon). \tag{32}$$

Then condition (31) is satisfied.

Proof: Since our computed equilibria are Markov equilibria, for each z^T, the values of the state will be in the state space, $(z_T, \Theta^\vartheta(z^T)) \in \mathcal{Z} \times \mathcal{S}$, and can be viewed as initial conditions themselves. Thus for any integer $i = 1, ...,$ there are prices $p(z^t)$ such that

$$\sum_{z^{iT} \succ z_0} \pi(z^{iT}|z_0)p(z^{iT}) < (1-\epsilon)^i \tag{33}$$

from which condition (31) follows by letting $i \to \infty$. **QED**

We are now ready to prove the result from the main text.

Proof of Proposition 1: It suffices to show that condition (32) holds. From Lemma 3 it then follows that the allocation is dynamically efficient.

To show condition (32) we first consider initial conditions $\vartheta = (z_0, \Theta_0) \in \mathcal{Z}^H \times \mathcal{S}^H$. For each $\delta > 0$ there exists a T and $p \in \mathcal{P}^\vartheta$ such that

$$\sum_{z^T} \pi(z^T|z_0)p(z^t) < \delta \tag{34}$$

This is true because by condition 1. of the proposition, for all $t = 1, ..., T$, there exist $p(z^t)$ such that

$$\begin{aligned} q^\vartheta(z^t)(1+n)(1+g) - \sum_{z' \notin \mathcal{Z}^H} \pi(z'|z_t)\epsilon &= \sum_{z' \in \mathcal{Z}^H} p(z^t, z')\pi(z'|z_t) \\ 1 - \sum_{z' \notin \mathcal{Z}^H} \pi(z'|z_t)\frac{(1+r^\vartheta(z^t, z'))\epsilon}{(1+g)(1+n)} &= \sum_{z' \in \mathcal{Z}^H} p(z^t, z')\pi(z'|z_t)(1+r^\vartheta(z^t, z')) \end{aligned} \tag{35}$$

By construction, for sufficiently large T, equation 34 holds.

For any other initial condition $\vartheta \in \mathcal{Z} \times \mathcal{S}$, condition 2. of the proposition implies that there is a T such that

$$\sum_{z^T : (z_T, \Theta(z^T)) \notin \mathcal{Z}^H \times \mathcal{S}^H} p(z^T)\pi(z^T|z_0) < 1 \tag{36}$$

By the above argument, taking $\delta = \min_{z^T} \frac{1}{p(z^T)}$, there exists a $\hat{T}$ such that

$$\sum_{z^{T+\hat{T}}} p(z^{T+\hat{T}})\pi(z^{T+\hat{T}}|z_0) < 1. \tag{37}$$

Together with Lemma 3 this proves the theorem. **QED**

B Data Appendix

We use data for 1926-1998, since reliable wage and asset return data are available only for this period. The financial data (stock returns, interest rate data) and the price index numbers from the CPI stem from Campbell (2003), and are available publicly at http://kuznets.fas.harvard.edu/˜campbell/data.html. The wage and employment data come from the Bureau of Economic Analysis and are available at http://www.bea.doc.gov/bea/dn/nipaweb/Index.asp. Nominal stock returns are computed from prices and dividends of a NYSE/AMEX value weighted portfolio. Nominal returns are adjusted for inflation by the CPI reported in the Campbell data set. Nominal interest rates are derived from 30 day T-bill nominal rates in the CRSP. The interest rate data are in quarterly frequency, and adjusted for inflation by the inflation rate, again computed with the CPI. Our measure of wages is real per worker total compensation. We remove a constant growth rate of 1.8 per annum from the wage data; the statistics referring to the wage data pertain to the so detrended data. Where applicable, we aggregate yearly data into 12 six-year intervals to obtain data of frequency comparable to that of our models.[17]

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