Population Aging and the Market for Higher Education: Implications for Education Finance Reform*

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Abstract

We construct a general equilibrium life cycle model of the college market with heterogeneous colleges, student college quality and major choice, and subsequent labor market outcomes of workers in different occupations. The cross-sectional distribution of college quality and college specialization is an equilibrium outcome and is shaped by the demographic structure of the economy, by public education spending and college loan policies, as well as by the endogenous relative wages that college graduates with different majors command in the labor market. We use the model to evaluate the aggregate and distributional consequences of the "demographic cliff" that will reduce the number of high-school graduates in the next decades, and against the backdrop of technological change (e.g., the emergence of AI) in the labor market. We then discuss the ability of recently proposed public education finance reforms by the recent administrations to shape these consequences.

JEL Classification: D15, D31, E24, I24

Keywords: Equilibrium Model of College Market, Education Subsidies, Demographic Cliff, Technological Change, College Major Choice.

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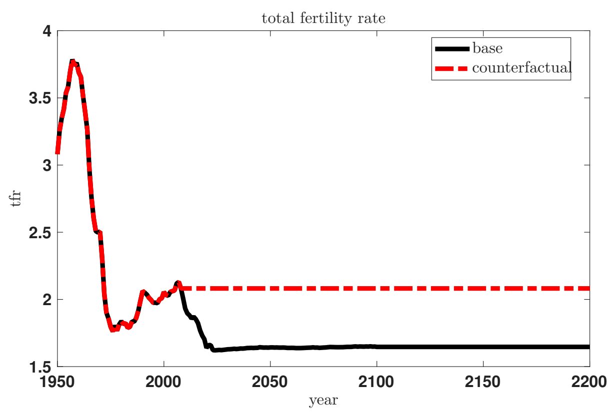
1 Introduction

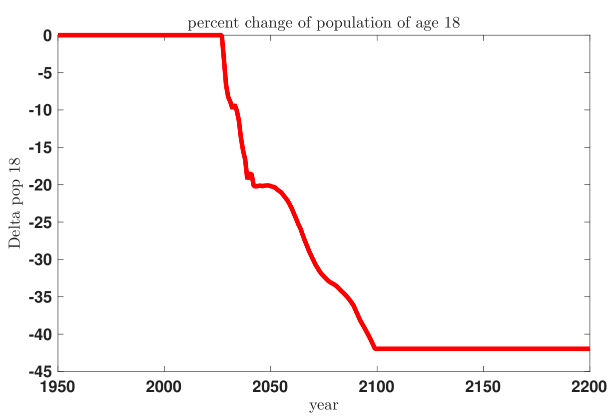
The U.S. is undergoing a demographic cliff that is expected to have a profound impact on the higher education as well as the labor market in the coming decades in the U.S. Just between 2025 and 2032 the number of domestic high school graduates is expected to fall by 5% (NCES Digest of Education Statistics, 2023). Figure 1 shows the actual and predicted fertility rate in the US and illustrates the number of potential college students lost due to the decline of the fertility rate since 2008.

Figure 1: Fertility Rate and Population Size

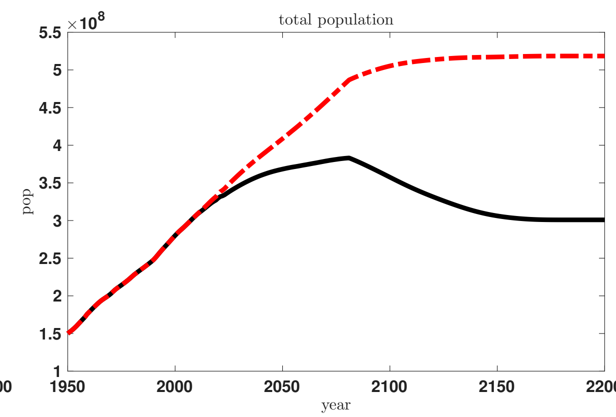


(a) Total Fertility Rate

(b) Population at Age 18



(c) Percent Change of Population Aged 18



(d) Total Population

Notes: Projected fertility rate, population at age 18, percent of population aged 18 and total population. Own calculations based on UN and HMD.

The college supply side is already responding to this demographic shift. About 100 institutions closed between AY 2022/23 and AY 2023/24, according to 2024 NCES statistics, and 2020 enrollments were ca 10% lower than in 2010, according to the 2023 NCES Digest of Education Statistics. At the same time, not only will lower enrollments eventually translate into smaller graduating classes, but there are concerns that shortages in specific areas such as STEM fields, will be especially prominent. BLS (2025) estimates that STEM-based occupations are expected to grow faster than non-STEM occupations. At the same time, STEM fields (both in college and in the labor market) are populated heavily by immigrants, see e.g., (NSF Board, 2024) and changes in immigration policy will (and perhaps already have) acerbate domestic concerns about skill shortages in key labor market areas. Furthermore, there is evidence (see, e.g., Deming and Noray, 2020) that STEM careers are getting shorter and are subject to increased risk of technological obsolescence.

The broad question this paper is concerned with is: how should private education choices and public education policy respond to these demographic and technological trends, and how do these trends alter the trade-offs involved in designing public education and redistributive policies? More specifically, we construct a general equilibrium life cycle model of the college market with heterogeneous colleges, student college quality and major choice, and subsequent labor market outcomes of workers in different occupations. The cross-sectional distribution of college quality and college specialization is an equilibrium outcome and is shaped by the demographic structure of the economy, by public education spending and college loan policies, as well as by the endogenous relative wages that college graduates with different majors command in the labor market. We use the model to evaluate the aggregate and distributional consequences of the "demographic cliff" that will reduce the number of high-school graduates in the next decades, and against the backdrop of technological change (e.g., the emergence of AI) in the labor market that raises the relative demand for STEM-intensive occupations and college majors. In the context of the model we then quantify the aggregate and distributional effects of recent federal funding cuts in higher education, focusing on their impact on enrollment, the composition of college quality, tuition levels, wages, and public finances.

To conduct our analysis we embed a life cycle model with college quality and college major choice into the competitive equilibrium model of the college market in the spirit of Cai and Heathcote (2022). Households that differ in their initial resources (due to, e.g., inter-vivos transfers by their parents), their innate ability to study and their residence status (determining whether they qualify for in-state tuition at public colleges) first decide whether and what quality colleges they want to attend, given the equilibrium tuition schedule that depends on these characteristics. After enrolling in college, they are subject to college-major specific preference shocks, and given these shocks as well as their expected labor earnings processes from a given college major they decide which major to pursue. After college, household life follows a standard life cycle with consumption-savings decisions, eventual exogenous retirement and stochastic death. On the college supply side, competitive colleges can freely enter the market at each level of college quality, where quality is determined by a combination of average student quality as well as per-student discretionary spending. The college-quality-, student-ability-, and residence-status-specific tuition schedule is determined in competitive equilibrium by household demand and the supply of competitive colleges.

After calibrating the model to U.S. macroeconomic, demographic and education data we first use it to characterize how, according to the model, the demographic cliff, modeled as a permanent decline in the U.S. fertility rate starting in 2008 impacts the U.S. college market, the labor market and public finances in the long run. In order to quantify the importance of these demographic shifts we then consider a counterfactual (from the perspective of today) demographic scenario in which the fertility rate is at replacement level and therefore the population is constant in the long run. Finally, for both demographic scenarios we evaluate the aggregate, distributional, college market, public finance and welfare consequences of abolishing public subsidies for higher education.

The declining population due to the demographic cliff, perhaps not surprisingly, has large negative effects on macroeconomic aggregates in the long run, simply because the population shrinks by 16%, relative to the counterfactual scenario without changes in demographics. However, per-capita outcomes look much more favorable. First, the demographic cliff has modest but favorable effects on the college market. Concretely, the long-run college enrollment rate increases, the distribution of students attending college shifts toward higher-quality colleges, and the share of STEM majors rises at all quality levels. Nevertheless, on account of dwindling absolute numbers of high-school graduates, colleges experience a decline in absolute student numbers throughout the quality distribution. The share of high-ability students remains almost unchanged at top-quality colleges but falls somewhat at low- and medium-quality colleges due to the inflow of less-prepared marginal students, which in turn increases the tuition-ability discount in the lower and middle segments of the quality distribution.

Given our overall focus on the interaction between public education policies and long-run demographic and technological trends (ro shocks), we next evaluate a public college funding cut akin to the one recently implemented by the Trump administration, both under a baseline scenario with the initial steady-state population growth rate and under the demographic cliff scenario described above. We find that the long-run aggregate losses induced by a public funding cut are much smaller than those generated by the demographic shock. However, the subsidy cut qualitatively undoes the positive effects of the demographic shock on college enrollment, the quality distribution of students, and college tuition. When the subsidy cut takes place against the backdrop of the demographic cliff, perhaps the currently most relevant *factual* scenario, the contractionary aggregate and fiscal effects of the subsidy cut are somewhat mitigated by the college market adjustments triggered by the demographic shock. As a result, the combined effect of the two shocks is smaller than the sum of their individual effects in isolation.

The next steps in our analysis will focus on demographic shocks, more broadly construed, to also include immigration-related policy changes, such as changes in the number of H-1B visas or exempting STEM immigrants from green card caps, and on technological shocks that asymmetrically affect STEM and non-STEM occupations.

1.1 Related Literature

Our paper contributes to the broad literature on (long-run) demographic change and its implications for the college landscape and college closures, see e.g., NCES Digest of Education Statistics (2023), Kelchen, Ritter, and Webber (2024) and Conesa, Kehoe, Nygaard, and Raveendranathan (2020). The second long-run trend (and associated literature) crucial to our analysis studies skill-biased technological change, STEM shortages and the implied wage changes in the labor market, with Deming and Noray (2020), Braxton and Taska (2023), Hendricks and Schoellman (2023) as well as Kogan, Papanikolaou, Schmidt, and Seegmiller (2023) being especially pertinent. Additionally, we contribute to the literature on the long-run trends in the US college market and college attendance patterns - see e.g. Hoxby (2009) and Hendricks et al. (2021). Also relevant for us in the literature on college applications, dropout risk and college major choice, see e.g., Vardishvili (2024).

More specifically, we construct a quantitative macro model with a college market equilibrium, adapting the competitive market framework of Cai and Heathcote (2022). Other recent examples of macroeconomic studies incorporating a college market equilibrium include Alon, Capelle, and Matsuda (2025b) and Alon, Capelle, and Matsuda (2025a). The industrial organization literature on equilibrium models of the college market is likewise highly relevant, see, e.g., Arcidiacono (2005), Epple, Romano, and Sieg (2006), Fu (2014), and more recently Marto and Wittmann (2024). Our model explicitly incorporates quality differences across colleges and household choice among these differentiated education products, an approach shared by Blair and Smetters (2021), Hendricks, Herrington, and Schoellman (2021), Hendricks, Koreshkova, and Leukhina (2021), and Hendricks, Koreshkova, and Leukhina (2024).

To inform our calibration and model validation we turn to the empirical literature on the costs and returns college majors, and major choice, and specifically, to Altonji, Arcidiacono, and Maurel (2016) as well as Altonji and Zimmerman (2018). Our specification and calibration of aggregate production relies on the quantitative literature on capital-embodied technical change, starting from Krusell, Ohanian, Ríos-Rull, and Violante (2000) and more recently Caunedo, Jaume, and Keller (2023). Finally, our thought experiment of reducing public subsidies to higher education builds on the large literature in macroeconomics and public finance on education finance reform, see, e.g. Krueger, Ludwig, and Popova (2025) and the references therein.

2 Population Dynamics in the Quantitative Model

Before we describe the economic model we first discuss the demographic model underlying our thought experiments. The dynamics of the population as a whole follows:

$$N_t = (1 + n_t)N_{t-1} \tag{1}$$

where n_t denotes the aggregate population growth rate. In the following, denote the fertility rate by f_t and the age-specific conditional survival rates by $\psi_{t,j}$. The latter is defined as the weighted average of gender-, skill-, and origin-specific survival rates $\varsigma_{t,j,i,s,g}$ from the demographic model introduced in Appendix A. The uniform fertility rate f_t is chosen to match the long-run population growth rate implied by the demographic model for the period 2000–2008.

Cohort laws of motion are:

$$N_{t+1,0} = f_t N_{t,j_f} \tag{2}$$

$$N_{t+1,j+1} = \psi_{t,j} N_{t,j}. \tag{3}$$

In steady state, fertility is constant f and each cohort grows at the same rate n. Equation (3) then implies:

$$(1 + n)N_{j+1} = \psi_j N_j, \tag{4}$$

or equivalently,

$$N_{j+1} = \frac{\psi_j}{1 + n} N_j. \tag{5}$$

Iterating this recursion up to age j_f starting from age 0, and assuming $\psi_j = 1$ for $j < j_f$ (certain survival to fertility age), yields:

$$N_{j_f} = \left(\frac{1}{1 + n}\right)^{j_f} N_0. \tag{6}$$

Using equation (2), this becomes:

$$(1 + n)^{j_f+1} = f, \tag{7}$$

so the long-run steady state population growth rate is

$$n = f^{\frac{1}{j_f+1}} - 1. \tag{8}$$

For completeness, observe that in the general case with $\psi_j \neq 1$ for $j \leq j_f$ the above equation is:

$$n = \left(f \cdot \prod_{j=0}^{j_f-1} \psi_j \right)^{\frac{1}{j_f+1}} - 1. \tag{9}$$

Observe that this equation implies that the steady-state population growth rate is pinned down by the fertility of the childbearing cohorts and the survival up to fertility age. Survival after fertility only affects the age distribution, but not the growth rate.

Thus, in steady-state demographic counterfactuals there are three key objects that can be varied: (i) the fertility rate, which determines the long-run population growth rate; (ii) survival rates, which shape the age distribution; and (iii) the total population stock, which scales the overall size of the economy. Finally, during the transition the aggregate population growth rate follows directly from equation (1):

$$n_t = \frac{N_t}{N_{t-1}} - 1, \tag{10}$$

where $N_t = \sum_j N_{t,j}$ is total population. The cross-sectional age distribution in each period t is jointly determined by the history of fertility and survival rates in preceding periods. In steady state, by contrast, the growth rate is constant, $n_t = n$, and the age distribution stabilizes.

3 The Model

Time is indexed by t and in each period a new cohort of size $N_{0,t}$ is born. The size of newborn cohorts $N_{0,t}$ is exogenous but may vary over time. There is a unit mass of households in each cohort. Households live from age $j = 0$ (which represents the college phase or the first labor market period of life) to at most age $j = J$, so that life after college starts at age $j = 1$. The retirement age j_r is exogenous and after this age households face age-specific survival risk: the probability of surviving from age $j-1$ to age j is given by ψ_{j-1}, with $\psi_j = 1$ for $j \leq j_r - 1$ and $\psi_J = 0$. Therefore, the number of individuals of age j alive in period t is defined recursively as

$$N_{t+1,j+1} = \psi_j N_{t,j} \tag{11}$$

We will describe the model in a stationary recursive setting, therefore suppressing time subscripts t until we introduce unexpected MIT transitions. The stationary population structure is then denoted by the vector $(N_0, ...N_J)$ and determined by (11) as well as the normalization $N_0 = 1$.

3.1 Household Endowments, Preferences, Budget Sets ad Decision Problems

3.1.1 Endowments and Budget Constraints

Households are endowed with innate ability e which falls into a discrete set $e \in \mathcal{E} = \{e_1, e_2, \ldots, e_E\}$, and we denote by $\mu_e(.)$ the measure of households of a given e. Following Cai and Heathcote (2022), we assume that in addition to ability households also differ by residence status $r \in \mathcal{R} = \{r_1, r_2\}$ which determines the generosity of government subsidies received by universities when these households become students (and hence the tuition these students pay).[1] We denote by $\mu_r(.)$ the measure of households in residence r. In addition, households start with initial resources $b \in \mathcal{B}$, interpreted as initial wealth transfers from parents, which are drawn from an exogenous child ability-specific probability measure $F_e(.)$. Thus, initial household heterogeneity is summarized by the vector $(e, r, b) \in \mathcal{E} \times \mathcal{R} \times \mathcal{B} \equiv \mathcal{S}$. Probability measures over cohort age j populations will be denoted generically by Φ_j and are endogenous for ages $j \geq 1$, but the measure Φ_0 is exogenous and determined by the initial distributions (μ_e, μ_r, F_e) for the economically newborn (aged 18 in the real wold) cohort.[2]

Households can choose to attend a college of quality $q \in \mathcal{Q} = \{0, [q_{\min}, q_{\max}]\}$, where $q = 0$ is non-college and $q_{\min}$ is the lowest and $q_{\max}$ is the maximum college quality available on the market. Given the choice of college, households also choose which degree program $p \in \mathcal{P} = \{p_1, \ldots, p_P\}$ to study. These choices are described in more detail below.

College Age In the first period of life, in addition to the wealth transfer by their parents b, households earn an exogenous income y_0, standing in for working full-time at the non-college wage. If they attend college, which we denote by indicator $\mathbb{1}_{q>0}$, households lose a fraction ω of that income, which represents the opportunity cost of time required to study (and which is the purpose of introducing y_0 in the first place). Gross income is therefore $(1 - \mathbb{1}_{q>0}\omega)\, y_0$ on which households pay taxes according to a possibly non-linear tax function $T(\cdot)$. Net labor income is $y_0^n(q) = y_0 (1 - \mathbb{1}_{q>0}\omega) - T(y_0 (1 - \mathbb{1}_{q>0}\omega))$. Besides labor income taxation, households are also subject to proportional capital income and consumption taxes, with rates τ_k and τ_c, respectively.

If a household attends college ($\mathbb{1}_{q>0}$), she receives a potentially means tested transfer that is a fraction $\varsigma(b)$ of the ability e, residence r and quality q dependent college tuition $t(e, r, b; q)$ which

[1]This feature of the model allows us to generate the substantial heterogeneity in tuition that in-state and out-of state students pay at public universities.

[2]Concretely, denote by $\mathbf{B}(\mathcal{E})$, $\mathbf{B}(\mathcal{R})$, and $\mathbf{B}(\mathcal{B})$ the (Borel) σ-algebras of $(\mathcal{E}, \mathcal{R}, \mathcal{B})$ and by $\mathbf{B}(\mathcal{S})$ the σ-algebra of $\mathcal{S}$. Then for any $E \subset \mathbf{B}(\mathcal{E})$, $R \subset \mathbf{B}(\mathcal{R})$ and $B \subset \mathbf{B}(\mathcal{B})$ we have $\Phi_0(E, R, B) = \mu_e(E)\mu_r(R) \sum_{e \in E} F_e(B)$.

is determined in equilibrium. Denoting by c_0 consumption and by a_1 asset choices of households in the first period of life, the budget set for the first period of life is defined by

$$(1+\tau_c)c_0 + a_1 = b + y_0\left(1 - \mathbb{1}_{q>0}\omega\right) - T\left(y_0\left(1 - \mathbb{1}_{q>0}\omega\right)\right) + \mathbb{1}_{q>0}(1-\varsigma(b))t(e,r,b;q) \tag{12}$$

$$a \geq -\underline{\mathsf{a}}_1(q) \tag{13}$$

where $\underline{\mathsf{a}}_1(q)$ denotes the education-specific borrowing limit: only households that choose $q > 0$ are allowed to borrow for their college education (at a level we specify in the calibration section), whereas for those with $q = 0$ we impose a tight borrowing constraint at zero on the asset choice a_1. The direct utility of going to college is given by

$$\mathbb{1}_{q>0}\left(\nu_0 + \nu_1 \ln(q) + \mathbb{1}_{p=2}\nu_2(e)\right) \tag{14}$$

where ν_0 captures the utility flow from attending any college, $\nu_1 \ln(q)$ encodes the direct preference for high-quality college and $\nu_2(e)$ measures the (dis-)utility for studying STEM, which is assumed to be student ability-dependent.

Working Life Ages After college, for all ages $j \in \{1, ...j_r\}$ households participate in the labor market; if they have chosen $q = 0$, their labor market career commences at age $j = 0$ already. Their labor income is (potentially) stochastic and depends on the quality of college attended as well as the program studied. The deterministic age-productivity component depends only on whether a worker attended college. Specifically, let $\gamma_j(0)$ denote the life-cycle productivity profile of non-college workers and $\gamma_j^{q>0}$ the common profile for all college graduates, independent of q. We assume that it has the following structure:

$$\begin{aligned}
&\gamma_j(0,0) = \gamma_j(0) \quad \text{(No-College)}\\
&\gamma_j(q,1) = \gamma_j(q>0)\left(1 + a\cdot(q)^c\right) \quad \text{(No-Stem)}\\
&\gamma_j(q,2) = \gamma_j(q>0)\left(1 + a\cdot(b\cdot q)^c\right) \quad \text{(STEM)}
\end{aligned} \tag{15}$$

where $a, b, c > 0$ are constants, the coefficient a controls the average college skill premium, c determines how strongly productivity and therefore wages increase with college quality q, and the parameter $b > 0$ captures the STEM ($p = 2$) premium, conditional on college quality q.

At the end of every working age $j \geq 1$ there is a chance that an individual with state $p = 2$ turns into an individual with state $p = 1$. Consequently labor productivity is deterministic, conditional on the type of a worker (e, q, p), but is rendered stochastic by the random transitions of p (in every period of working life).[3]

Labor markets are segmented by $p \in \{0, 1, 2\}$, and the wage for labor with training p is given by $w(p)$. Realized gross labor income is given by

$$y_j(q, p) = \gamma_j(q, p)w(p)$$

net labor income of a type (q, p) worker then is $y_j^n(q, p) = y_j(q, p) - T(y_j(q, p))$, and a typical working age budget constraint is given by

$$(1 + \tau_c)c_j + a_{j+1} = y_j^n(q, p) + (1 + r(1 - \tau_k))a \tag{16}$$

Retirement For retired people $j > j_r$, the budget constraint is similar to (16), but labor income $y_j(q, p)$ is replaced by retirement benefits $pen_j(q)$. Households that die prematurely leave accidental bequests that are confiscated by the government and form part of tax revenue.

3.1.2 Preferences

In each period, households derive per-period utility from a period utility function $u(c)$. We further assume that during college $j = 0$ households derive direct utility benefits from attending college given by equation (14). Finally, after making the college quality decision but before choosing program p, households draw a p-specific preference shock from a a type 1 extreme value distribution; we specify the details of these preference shocks below when we discuss the p-choice.

3.2 Timing of Events

At every age $j \geq 1$ households simultaneously make consumption and savings choice. At age $j = 0$, the timing of events is as follows. A household of type (e, r, b) first decides on whether to go to college, and conditional on going, what quality q of college to attend. Then the program-specific preference shocks realize, and based on these realizations the household chooses a field of study p. Finally the household makes consumption-savings choices; at the end of the period the relevant state vector of the household is (e, q, p, a) with associated cross-sectional measure Φ_1.

3.2.1 Recursive Formulation of the Household Problem

For all ages $j \geq 1$, households solve a standard consumption-savings problem with labor income being determined by by the productivity process $\gamma_j(.)$ and the risk of skill obsolescence if $p = 2$.

[3] In the current version of the paper, labor market productivity does not depend explicitly on innate ability e, and thus e is not currently a state variable for ages $j \geq 1$. Also, currently p-transitions during working life are not yet implemented.

Denote the value function at the beginning of model age $j = 1$ as $V_1(e, q, p, a)$. It depends on the education states (q, p), initial ability e (all of which determine labor productivity) as well as asset holdings a (which can be negative if the household borrowed to pay the tuition in college). We spell out the recursive problem(s) giving rise to $V_1(e, q, p, a)$ in Appendix B; this includes determining all age-specific value functions $\{V_j(e, q, p, a)\}$ and associated policy functions $\{(c_j(e, q, p, a)), a_{j+1}(e, q, p, a)\}$, for $j \geq 1$.

We now describe the sequence of dynamic programming problems the household solves sequentially during the college age $j = 0$, working backward from the consumption-saving choice to the college major choice and finally the college quality choice; we spell them out in the main paper as these are the more novel aspects of our model.

The Consumption-Saving Choice After having made higher education choices (q, p) the relevant state vector of the household is (e, q, p, a) and the household solves the problem:

$$W(e, r, b, q; p) = \max_{c_0, a_1} \{u(c_0) + \beta V_1(e, q, p, a_1)\} \qquad s.t. \tag{17}$$

$$(1 + \tau_c)c_0 + a_1 = b + y_0^n + \mathbb{1}_{q>0}(1 - \varsigma(b))t(e, r, b; q) \tag{18}$$

$$a_1 \geq -\underline{\mathsf{a}}_1(q), c_0 \geq 0, q \in \mathcal{Q} \tag{19}$$

where $y_0^n = y_0 (1 - \mathbb{1}_{q>0}\omega) - T(y_0 (1 - \mathbb{1}_{q>0}\omega))$ is net labor income during the college phase (which depends on whether the household chooses to attend college or not). The resulting policy functions are denoted by $(c_0(e, r, b, q, p), a_1(e, r, b, q, p))$. It is understood that for $q = 0$ (the household not attending college) the p-dimension of the value function is irrelevant.

The College Major Decision Working backwards, a household with initial state (e, r, b) that has chosen to attend a college of quality $q > 0$ and with updated state (e, r, b, q) now chooses what field p to study, subject to preference shocks $\tilde{\eta}_p$ (a vector of size P) that follow a type 1 extreme value distribution with scale parameter σ_{η_p}. Define the expected lifetime utility from a given major p choice as $\bar{W}(e, r, b, q; p)$. It is given by

$$\bar{W}(e, r, b, q; p) = \mathbb{1}_{q>0} (\nu_0 + \nu_1 \ln(q) + \mathbb{1}_{p=2}\nu_2(e)) + W(e, r, b, q, p) \tag{20}$$

where $W(e, r, b, q, p)$ was defined in the previous section. Furthermore, define the auxiliary value function $U(e, r, b, q)$ as expected lifetime utility prior to the major choice. It is given by

$$U(e, r, b, q) = \mathbf{E}_{\tilde{\eta}_p} \left[\max_{p \in \mathcal{P}} \{\bar{W}(e, r, b, q; p) + \sigma_{\eta_p}\tilde{\eta}_p(p)\} \right]$$

where the expectation is taken with respect to the random variables $\tilde{\eta}_p$. The associated policy function is given by $p(e, r, b, q; \eta_p)$ and depends on the realizations for the preference shocks for all p. Given the assumed distribution for the preference shocks, choice probabilities and the associated auxiliary value function can be given in closed form as:

$$\pi(p|e,r,b,q) = \frac{\exp\left(\bar{W}(e,r,b,q;p)\right)}{\sum_{i\in\mathcal{P}}\exp\left(\bar{W}(e,r,b,q;i)\right)} \tag{21}$$

$$U(e,r,b,q) = \sigma_{\eta_p}\log\left(\sum_{p\in\mathcal{P}}\frac{\exp(\bar{W}(e,r,b,q;p))}{\sigma_{\eta_p}}\right) \tag{22}$$

For a person that chooses not to attend college, $U(e,r,b,0) = W(e,r,b,0)$.

The College Attendance and Quality Decision In period 0, prior to the college program decision, households decide which college (if any) to attend, rationally forecasting their college major choice probabilities. Households choose q to maximize the following program:

$$V_0(e,r,b) = \max_{q\in\mathcal{Q}}\{U(e,r,b,q)\} \tag{23}$$

The optimal college choice q is a function of the initial state variables (e,r,b) of the household, and the household takes the tuition and subsidy functions $t(e,r,b;q), \varsigma(b)$ as well as the tax function $T(y)$ and the borrowing limit $\underline{a}_1(q)$ as given in the consumption-savings problem that in turn defines $U(e,r,b,q)$.

3.3 Cross-Sectional Distributions Implied by Household Choices

The decision problem at age $j=0$ gives rise to college decision rules $q(e,r,b)$, choice probabilities $\pi(p|e,r,b,q) = \pi(p|e,r,b,q(e,r,b))$ and consumption savings policies (c_0,a_1). Given these function and the initial exogenous probability measure Φ_0 across characteristics (e,r,b) we can now define the cross-sectional distributions at each decision stage for households of age $j=0$.

In order to prepare the definition of a college market equilibrium we first specify the cross-sectional college quality distribution implied by the household desired college attendance decisions as well as the distribution of initial household characteristics. First, the mass of individuals seeking to attend a college of quality $q\in Q$ is given by:

$$\chi(Q) = \sum_e\sum_r\int_b \mathbb{1}_{\{q(b,e,r)\in Q\}}d\Phi_0(e,r,b) \tag{24}$$

Note that the same equation applies to $Q=\{0\}$ which determines the set of households not attending college. The mass of households of ability e wanting to attend a college in Q is given by

$$\Phi_e(Q) = \sum_r \int_b \mathbb{1}_{\{q(e,r,b)\in Q\}} d\Phi_0(\{e\}, r, b) \tag{25}$$

and thus for all Q with $\chi(Q) > 0$ the conditional distribution of student ability e (i.e., the share of high-ability students) in quality set Q is given by

$$\Phi(e|Q) = \frac{\Phi_e(Q)}{\chi(Q)}. \tag{26}$$

Of course, the set Q can be a singleton, as long as that singleton has positive mass (as will be the case once we discretize the college quality set $\mathcal{Q}$). A similar set of calculations defines as the mass of people attending schools in Q and choosing program p. It is given by

$$\Phi_p(Q) = \sum_e \sum_r \int_b \pi(p|e, r, b, q(e, r, b)) \mathbb{1}_{\{q(e,r,b)\in Q\}} d\Phi_0(e, r, b) \tag{27}$$

and the cross-section share of students graduating with major p from college of quality in Q is given by

$$\Phi(p|Q) = \frac{\Phi_p(Q)}{\chi(Q)}. \tag{28}$$

At the end of the college age, the remaining relevant state variables are given by (e, q, p, a) and the cross sectional probability measure, for each $e \in \mathcal{E}$, and for all sets $(Q, P, A) \in (\mathbf{B}(\mathcal{Q}), \mathbf{B}(\mathcal{P}), \mathbf{B}(\mathcal{B}))$ with $\{0\} \neq Q$ it given by

$$\Phi_1(\{e\}, Q, P, A) = \sum_r \int_b \sum_{p\in P} \pi(p|e, r, b, q(e, r, b)) \mathbb{1}_{\{q(e,r,b)\in Q, a_1(b,e,r,q(e,r,b),p)\in A\}} d\Phi_0(\{e\}, r, b) \tag{29}$$

For $q = 0$ (households not going to college) it is understood that the college program dimension is irrelevant and

$$\Phi_1(\{e\}, \{0\}, P, A) = \sum_e \sum_r \int_b \mathbb{1}_{\{q(e,r,b)=0, a_1(b,e,r,q(e,r,b),p)\in A\}} d\Phi_0(\{e\}, r, b) \tag{30}$$

Finally in order to appropriately aggregate the consumption and asset choice at the end of period 0, we need the cross-sectional population measure $\bar{\Phi}_0$ across characteristics (e, r, b, q, p)

(since this is what the consumption-savings policies depend upon). This distribution is given by

$$\bar{\Phi}_0(\{e\},\{r\},B,Q,P) = \int_{b\in B} \sum_{p\in P} \pi(p|e,r,b,q(e,r,b)) \mathbb{1}_{\{q(e,r,b)\in Q\}} d\Phi_0(\{e\},\{r\},b). \tag{31}$$

where again it is understood that for $q = 0$ the college program dimension is irrelevant.

3.4 College Supply

Having characterized the demand of households for college of different qualities, we now build upon Cai and Heathcote (2022)'s equilibrium model of college supply. To do so, we assume that colleges operate in a competitive environment with free entry into any quality segment $q \in \mathcal{Q} = [0, [q_{\min}, q_{\max}]]$ of the college market. Suppose a college has a student ability distribution given by $\{\eta_e\}$ where η_e is the share of students of ability e; recall that $e \in \mathcal{E}$ falls in a finite set. Denote by

$$\bar{e} = \sum_e \eta_e e \tag{32}$$

the average student ability in college q. Given $\bar{e}$ colleges decide on instructional spending per student i and produce quality q according to CRS production function

$$q = \bar{e}^{\theta} i^{1-\theta}. \tag{33}$$

In addition to tuition revenue, colleges also receive per student public subsidies $s(e, r; q)$ that potentially depend on the quality they deliver and on the characteristics of students they admit. Conditional on ability e, profit-maximizing colleges strictly prefer to admit only students who generate the most revenue. Let $v(q, e) = \max_{b,r} t(e, r, b; q) + s(e, r; q)$ denote revenue from admitting the highest-revenue students (by resources b and residence status r) of ability level e. Producing programs p also comes at a program specific fixed cost $\kappa(p)$. For a given q, colleges maximize profits:

$$\pi(q) = \max_{i,\{\eta_e\}} \left\{ \sum_{e\in E} \eta_e v(q, e) - i - \sum_p \Phi(p|q)\kappa(p) \right\} \tag{34}$$

subject to (32) and (33).

We assume that students make choices with respect to majors upon being assigned to a college of a given q. Thus, colleges cannot condition tuition or spending decisions on chosen majors. But the fixed cost $\kappa(p)$ exogenously depends on the program. Colleges observe students' preferences over programs with $\Phi(p|q)$ denoting the distribution of students across majors at each q based on their expected application behavior[4].

For the two-level ability case with e^l and e^h denoting low and high ability, respectively, we can rewrite the above problem as follows:

$$\pi(q) = \max_{i,\eta(q,e^h)} \left\{ \eta(q,e^h)v(q,e^h) + (1-\eta(q,e^h))v(q,e^l) - i - \sum_p \Phi(p|q)\kappa(p) \right\} \tag{35}$$

subject to

$$q = \bar{e}^{\theta} i^{1-\theta} \tag{36}$$

$$\bar{e} = \eta(q,e^h)e^h + (1-\eta(q,e^h))e^l. \tag{37}$$

Thus, as in the general case, colleges choose per student expenditures and average student ability with the latter being equivalent to choosing the share of high-ability students $\eta(q,e^h)$ in the setting with two ability levels.

Rewrite the college optimization problem as follows:

$$\pi(q) = \max_{i,\eta(q,e^h)} \left\{ \eta(e^h)[v(q,e^h) - v(q,e^l)] + v(q,e^l) - i - \sum_p \Phi(p|q)\kappa(p) \right\} \tag{38}$$

subject to

$$q = \bar{e}^{\theta} i^{1-\theta} \tag{39}$$

$$\bar{e} = \eta(q,e^h)e^h + (1-\eta(q,e^h))e^l. \tag{40}$$

The first-order conditions to the problem result in the following optimality condition:

$$v(q,e^h) - v(q,e^l) = -\frac{\theta}{1-\theta} \frac{(e^h - e^l)}{\eta(q,e^h)e^h + (1-\eta(q,e^h))e^l} i := d(q) \tag{41}$$

[4]I.e. major choice probabilities as implied by the student optimization problem.

Use this optimality in the zero-profit condition to obtain ability-specific tuition schedules:

$$v(q,e^l) = i\left(1+\eta(q,e^h)\frac{\theta}{1-\theta}\frac{(e^h-e^l)}{\eta(q,e^h)e^h+(1-\eta(q,e^h))e^l}\right)+\sum_p \Phi(p|q)\kappa(p) \tag{42}$$

$$v(q,e^h) = i\left(1-(1-\eta(q,e^h))\frac{\theta}{1-\theta}\frac{(e^h-e^l)}{\eta(q,e^h)e^h+(1-\eta(q,e^h))e^l}\right)+\sum_p \Phi(p|q)\kappa(p) \tag{43}$$

Thus, equilibrium tuition for low- and high-ability students, respectively, can be written as:

$$v(q,e^l) = i-\eta(q,e^h)d(q)+\sum_p \Phi(p|q)\kappa(p) \tag{44}$$

$$v(q,e^h) = i+(1-\eta(q,e^h))d(q)+\sum_p \Phi(p|q)\kappa(p) \tag{45}$$

Using the quality constraint, the above expressions can be rewritten as:

$$v(q,e^l) = q^{\frac{1}{1-\theta}}(\eta(q,e^h)(e^h-e^l)+e^l)^{\frac{\theta}{\theta-1}}-\eta(q,e^h)d(q)+\sum_p \Phi_t(p|q)\kappa(p) \tag{46}$$

$$v(q,e^h) = q^{\frac{1}{1-\theta}}(\eta(q,e^h)(e^h-e^l)+e^l)^{\frac{\theta}{\theta-1}}+(1-\eta(q,e^h))d(q)+\sum_p \Phi(p|q)\kappa(p) \tag{47}$$

The high-ability share $\eta(q,e^h)$ at each q can also be pinned down from the optimality condition derived above and the quality constraint:

$$\begin{aligned}
d(q) &= \frac{\theta}{\theta-1}\frac{(e^h-e^l)}{\eta(q,e^h)e^h+(1-\eta(q,e^h))e^l}i \\
&= \frac{\theta}{\theta-1}(e^h-e^l)q^{\frac{1}{1-\theta}}\bar{e}^{\frac{1}{\theta-1}} \\
\Leftrightarrow \bar{e} &= q\left(d(q)\frac{\theta-1}{\theta}\frac{1}{e^h-e^l}\right)^{\theta-1} \\
\Leftrightarrow \eta(q,e^h) &= \frac{q\left(d(q)\frac{\theta-1}{\theta}\frac{1}{e^h-e^l}\right)^{\theta-1}-e^l}{e^h-e^l} && (48) \\
&= A-\frac{e^l}{e^h-e^l} && (49)
\end{aligned}$$

where $A := \frac{q\left(d(q)\frac{\theta-1}{\theta}\frac{1}{e^h-e^l}\right)^{\theta-1}}{e^h-e^l}$.

Thus, equilibrium tuition for low- and high-ability students can be written as:

$$v(q,e^l) = q^{\frac{1}{1-\theta}}[A(e^h-e^l)]^{\frac{\theta}{\theta-1}} + \frac{e^l}{e^h-e^l}d(q) - Ad(q) + \sum_p \Phi(p|q)\kappa(p) \tag{50}$$

$$v(q,e^h) = q^{\frac{1}{1-\theta}}[A(e^h-e^l)]^{\frac{\theta}{\theta-1}} + \frac{e^h}{e^h-e^l}d(q) - Ad(q) + \sum_p \Phi(p|q)\kappa(p) \tag{51}$$

3.5 Production and the Firm's Problem

Denote by $L(p)$ the aggregate labor efficiency units trained in a given program p (e.g., non-college, college non-stem, college STEM), aggregated across all households across initial wealth b, ability e, residence status r and that attended colleges of quality q

$$L(p) = \sum_r \sum_e \int_b \int_Q \pi(p|e,q(e,r,b),a(e,r,b)) \mathbb{1}_{\{q(e,r,b)\in Q\}} \sum_{j=0}^{j_r-1} \gamma_j(q,p) d\chi(Q) d\Phi_0(e,r,b) N_j. \tag{52}$$

Note that this formulation assumes that labor inputs of college graduates from different quality q colleges are perfect substitutes, but that graduates from higher quality colleges are more productive. Likewise, denote aggregate non-college $p=0$ labor by

$$L(0) = \sum_r \sum_e \int_b \mathbb{1}_{\{q(e,r,b)=0\}} \sum_{j=0}^{j_r-1} \gamma_j(q=0) d\Phi_0(e,r,b) N_j. \tag{53}$$

Denote by $K(p)$ the capital stock used by labor of type p and $\mathbf{K}$ the vector $(K(0),...K(P))$. Total output in the economy is then produced according to the aggregate constant returns to scale production function

$$Y = F(\mathbf{K},\mathbf{L}) = \left[\sum_{p\in\{0,\mathcal{P}\}} \lambda(p) y(\Psi K(p), \Gamma(p)L(p))^{\nu}\right]^{\frac{1}{\nu}} \tag{54}$$

$$= \sum_{p\in\{0,\mathcal{P}\}} \left[\lambda(p)\left(\alpha K(p)^{\rho(p)} + (1-\alpha)\left(\Gamma L(p)\right)^{\rho(p)}\right)^{\frac{\nu}{\rho(p)}}\right]^{\frac{1}{\nu}}. \tag{55}$$

where $y(\Psi K(p), \Gamma(p)L(p))$ is output produced in a "sector" that uses labor from program p, the term $\Gamma(p)$ captures labor-augmenting technological progress (which may differ across p) and the factor Ψ captures capital-augmenting technological change (assumed to be uniform across all capital stocks $K(p)$). The weights $\lambda(p)$ with $\sum_p \lambda(p) = 1$ determine the relative importance (or productivity) of the p-specific sectors, and the elasticity of substitution $\frac{1}{1-\nu}$ between output produced with different labors is determined by the parameter ν. Within each p, production is a CES between capital and labor, with weights $\alpha(p)$ on capital and p-specific substitution elasticity between labor and capital $\sigma(p) = \frac{1}{1-\rho(p)}$. We will calibrate the model such that $\sigma(0) > \sigma(1) > \ldots > \sigma(P)$, so that the highest ranked college program P (STEM-trained college graduates) is most complementary to capital, and non-college labor is most substitutable with capital. These substitution elasticities will determine the direction and the size of relative and absolute wage effects of changes in the supply of labor induced by changes in demographics and/or education policy or by changes in labor and/or capital productivity designed to mimic potentially skill-biased technological change.

The production technology (55) is operated by a representative firm that operates in perfectly competitive factor and output markets, hires labor efficiency units of the different types p for type-specific wages $w(p)$, rents capital K_t at a rental rate r_t and also decides how to allocate the capital stock across different sectors so that $\sum_p K_t(p) = K_t$. The capital stock depreciates at rate δ. Note that since the firm can freely allocate capital across all uses p, the marginal products across all $K_t(p)$ are equalized and equal to the rental rate plus depreciation, $r_t + \delta$. Since labor is differentiated by p and outputs across "industries" are not perfect substitutes, there are $P + 1$ separate labor markets and associated wages for p-specific labor efficiency units. The firm optimality conditions (as well as the algorithm to compute equilibrium factor prices r_t and $w_t(p)$ from the capital and labor market equilibrium conditions) are provided in equations (70) and (71) in the computational appendix, Section E.2.

3.6 Government

The government provides subsidies $s(e, r; q)$ to colleges, proportional to sticker tuition $t(e, r; q)$.[5] In addition, the government finances need-based financial aid $\varsigma(b)$, which captures federal and state grants and is specified in detail in Section 4.

The government balances in each period both the general tax-and-transfer system budget and the pension system budget. Expenditures within the general tax-and-transfer system consist of an exogenous stream of non-education-related spending and an endogenous stream of education-related spending, as summarized above. Revenues are generated from taxes on consumption, capital income, and labor income.

[5] As in Cai and Heathcote (2022), we assume that public subsidies are provided only to in-state students.

We assume that the government has to balance the following per-period budget:

$$\sum_r \sum_e \int_b \int_Q \left(s(e,r;q) + \varsigma(b)t(e,r;q)\right) \mathbb{1}_{\{q(e,r,b)\in Q\}} d\chi(Q) d\Phi_0(e,r,b) + G = T^c + T^k + T \tag{56}$$

where the left-hand side captures education-related expenditures (public subsidies $s(e,r;q)$ and financial aid $\varsigma(b)$) plus exogenous non-education spending G. On the revenue side, T^c denotes consumption tax revenue, T^k denotes capital income tax revenue (including revenue from taxing accidental bequests of deceased households), and T denotes labor income tax revenue:

$$\begin{aligned}
T^c &= \tau_c \sum_r \sum_e \int_b c_0(e,r,b,q,p) d\bar{\Phi}_0(e,r,b,q,p) N_0 \\
&\quad + \tau_c \sum_{j=1}^{J} \sum_e \int \int_{q_0}^{q^{max}} \sum_p c_j(e,q,p,a) \Phi_j(e,q,p,a) N_j \\
T^k &= \tau_k r \sum_{j=1}^{J} \sum_e \int \int_0^{q^{max}} \sum_p a_j(e,q,p,a) \Phi_j(e,q,p,a) N_j \\
&\quad + \sum_{j_r}^{J} \sum_e \int \int_0^{q^{max}} \sum_p (1-\psi(j)) a_j(e,q,p,a) \Phi_j(e,q,p,a) N_j \\
T &= \sum_r \sum_e \int_b \int_Q \pi(p|e,r,b,q(e,r,b)) \mathbb{1}_{\{q(e,r,b)\in Q\}} T(y_0(b,e,r,q,p)) d\chi(Q) d\Phi_0(e,r,b) N_0 \\
&\quad + \sum_{j=1}^{J} \sum_e \int \int_0^{q^{max}} \sum_p T(y_j(e,q,p,a)) \Phi_j(e,q,p,a) N_j
\end{aligned}$$

The pension system budget constraint is also balanced period by period, with benefits paid to current retirees equaling contributions collected from current workers.

3.7 Equilibrium Definition: College Market and Government

An equilibrium is a measure of colleges $\chi(Q)$, college tuition $t(b,e,r;q)$, average ability of admitted students $\bar{e}(q)$, per student college expenditures $i(q)$ and profit $\pi(q)$ functions of colleges, household choices $c_0(b,e,r)$ and $c_j(e,q,a;p)$, $a_0'(b,e,r)$ and $a_j'(e,q,a;p)$, and $q(b,e,r)$ and $p(e,q,a;\eta_p)$, and government policies that satisfy the following conditions:

1. Given college tuition $t(e,r,b;q)$ and government policies (subsidies to colleges, student financial aid and taxes), the household consumption $c_0(e,r,b,q,p)$ and $c_j(e,q,p,a)$, savings $a_1(e,r,b,q,p)$ and $a_{j+1}(e,q,p,a)$ and college quality $q(e,r,b)$ and college major $p(e,r,b,q;\eta_p)$ choices solve the household optimization problem for all (e,r,b).

2. For all $q > 0$ in the feasible set $\Omega = [0, q_{max}]$, the maximal revenue function is determined as $v(q,e) = \max_{b,r} t(e,r,b;q) + s(e,r;q)$, the college input choices $\bar{e}(q)$ and $i(q)$ solve the college's profit maximization problem, and $\pi(q)$ is the associated profit per student.

3. Zero profits: For all college quality subsets $Q \subset \Omega \setminus \{0\}$, $\int_Q \pi(q) d\chi(q) = 0$, and $\pi(q) \leq 0$ for all $q \in Q$.

4. Goods market clearing:

$$C + G + I^{college} + FC^{college} + E = Y - \delta K \tag{57}$$

where $I^{college}$ denotes aggregate expenditure of colleges on students, $FC^{college}$ is the total fixed cost paid by colleges, and E denotes the aggregate financial aid provided to students by the government.[6]

5. College market clearing: for all $e \in \mathcal{E}$ and all $Q \subseteq \Omega \setminus \{0\}$

$$\sum_r \int \mathbf{1}_{q(e,r,b) \in Q} d\Phi_0(e,r,b) = \int_Q \eta(q,e) d\chi(Q) \tag{58}$$

where

$$q(b,e,r) = q^* \;\Rightarrow\; (b,r) \in \arg\max\{t(b,e,r;q^*) + s(e,r;q^*)\} \tag{59}$$

6. The per-period government budget constraint in equation (56) holds in every period.

3.8 Sources of Inefficiency of the Equilibrium and the Scope for Higher Education Subsidies

In our framework, several sources of inefficiency could potentially call for government intervention, and higher education subsidies specifically. First, the presence of borrowing constraints prevent households from smoothing consumption and investment optimally across periods. Although we permit households to borrow to attend college, the potential risk of dropping out of college as well as the risk of post-college low productivity might push households into the borrowing constraint later in life, in turn inducing suboptimal (relative to a complete financial markets benchmark)

[6] Written out explicitly, aggregate consumption is given by

$$C = \sum_r \sum_e \int_b c_0(e,r,b,q,p) d\bar{\Phi}_0(e,r,b,q,p) N_0 + \sum_{j=1}^{J} \sum_e \int_b \int_0^{q^{max}} \sum_p c_j(e,q,p,a) \Phi_j(e,q,p,a) N_j$$

higher education choice (both at the extensive margin – whether to go to college – and along the intensive margin – what quality college to go to). This friction is especially relevant for low-resource households.

Second, households face uninsurable idiosyncratic risks - most notably, the risk of failing to complete college and the risk of skill obsolescence later in life (e.g., in STEM occupations). In the absence of private insurance markets for such shocks, households must bear them individually, potentially leading to precautionary behavior and inefficient underinvestment in education. Reducing the private costs of attending college through public subsidies mitigates this risk (but of course also induces a moral hazard issue: some individuals might attend college even though private returns are low since the government pays for part of the cost).

Third, government fiscal policies interact with these frictions. In our incmplete markets model, attending a (high-quality) collge and/or studying a high potential wage major turns an individual into a high tax payer, a fiscal externality that the individual does not take into account when making higher education decisions. More broadly, subsidies to colleges, financial aid for students, and the progressive labor income tax code might positively impact the distribution of lifetime utilities across individuals. At the same time, these interventions may also distort individual incentives, for instance by affecting equilibrium tuition in the college market or altering the returns to education through the tax-and-transfer system.

4 Calibration

Each period in the model corresponds to four years. Households enter economic life at age $j_a = 18$. College education lasts one model period, so all households enter the labor market full time at biological age 22. The statutory retirement age is 66, and the maximum lifespan is 101 years. We calibrate the initial steady state of the model to broadly approximate the U.S. economy in the 2010s.

4.1 Households

Demographics. We take the survival probabilities from the Human Mortality Database (HMD). The baseline long-run population growth rate is set to 1% which is the average US population growth rate between 2000 and 2008. Following Cai and Heathcote (2022), the share of in-state students in the population is set to 0.529.

Preferences. Recall that the psychological cost of attending college depends on child ability and is specified as $\nu_0 + \nu_1 \ln(q) + \mathbb{1}_{p=2}\nu_2(e)$. The parameter ν_0 is calibrated to match the average college enrollment rate, while $\nu_2(e)$ governs the ability-specific shares of students in STEM majors. Finally, ν_1 controls the taste for college quality and is currently set to 1. Qualitatively, this parameter captures an additional, non-pecuniary motive - a "warm-glow" - to invest in college

Table 1: Calibration Parameters: First Stage Parameters

Parameter		Value	Target/Interpretation
j_a	Age at beginning of econ life (age 18)	1	
j_c	Age at finishing college (age 22)	2	
j_r	Retirement Age (age 66)	13	
J	Max. Lifetime (age bin 98-101)	21	
$\{\phi_j\}$	Survival Probabilities	see main text	Life Tables SSA
$E = 2$	Number of ability types	2	[low, high]
$P = 2$	Number of programs	2	[No-STEM, STEM]
$\eta(e^h)$	Aggregate share of high ability students	0.5	Cai and Heathcote (2022)
e^h	High ability level	1.0	Normalization
e^l	Low ability level	0.375	Cai and Heathcote (2022)
	College quality production		
θ	Average ability elasticity	0.5	Normalization
$\bar{\kappa}$	Average college fixed cost	$\approx \$9,000$	Altonji and Zimmerman (2018)
$\frac{\kappa(STEM)}{\kappa(no-STEM)}$	Ratio of STEM to no-STEM college fixed costs	1.71	Altonji and Zimmerman (2018)
	Household preferences and endowments/productivity		
σ	Relative risk aversion parameter	1	-
ψ	Frisch elasticity	0.6	
$\gamma_q(q = 0)$	Non-college labor productivity	1	Normalization
ℓ	Average hours worked	1/3 of time endowment	-
ℓ^{stud}	Student average hours worked	0.25 ℓ	NCES
$\bar{b}$	Mean initial (family) resources	\$61,200	Average IVT transfer per child, 2013 Rosters and Transfers Module of PSID
ς	Ratio of average initial resources by ability $\frac{\bar{b}(e^{high})}{\bar{b}(e^{low})}$	1.35	Cai and Heathcote (2022)
σ^b	Variance of initial resource distribution	0.55	Cai and Heathcote (2022)
$\{\epsilon(q = 0, j)\}$	Age Profile, non-college	see main text	PSID 1968-1997
$\{\epsilon(q > 0, j)\}$	Age Profile, college	see main text	PSID 1968-1997
$\underline{a}(j \in [j_a], q > 0)$	College borrowing limit	45,590\$	Krueger and Ludwig (2016)
	Aggregate production		
α	Capital share	33.3%	
δ	Depreciation rate	5%	
$\rho(p)$	Subst. Elas. $1/(1 - \rho(p))$ by p	[-0.11,-0.43]	Caunedo et al. (2023)
$\rho(q = 0)$	Subst. Elas. $1/(1 - \rho(q = 0))$ non-coll.	0.33	Caunedo et al. (2023)
	Government		
ϑ	Public subsidy of college education	38.8%	Krueger and Ludwig (2016)
ϑ^{pr}	Private subsidy of college education	16.6%	Krueger and Ludwig (2016)
τ_c	Consumption Tax Rate	5.0%	legislation
τ_k	Capital Income Tax Rate	36%	Trabandt and Uhlig (2011)
ξ	Labor Income Tax Progressivity	0.18	Heathcote et al. (2017)
τ^p	Soc Sec Payroll Tax	12.4%	legislation
G/Y	Government consumption to GDP	13.8%	current value

Notes:. This table summarizes the exogenously (first stage) calibrated parameters used for the quantitative analysis.

quality beyond labor market returns. Households discount utility at rate β, which is calibrated such that the implied steady-state annual interest rate in general equilibrium equals 3.5%.

Table 2: Calibration Parameters: Second Stage Parameters

Parameter		Value	Target/Interpretation
	Household preferences and endowments		
β	Discount factor	0.9882	Interest rate
ν_0	Utility (psychological) cost of attending college	2.64	College enrollment rate
ν_1	Weight on direct utility from q (strength of warm glow q-investment motive)	1.0	Net tuition spending to GDP ratio
$\nu^{STEM}(e)$	Ability-specific utility (psychological) cost of studying STEM	$[1.65, 1.20]$	STEM shares by ability
a, b, c	Parameters of $\gamma(q, p)$	0.35, 3.80, 0.55	Average college wage premium, wage difference between top and bottom quartile of q distribution, average STEM wage premium
	Aggregate production		
Γ	Aggregate wage normalization parameter	1.34	Normalization $w(q = 0) = 1$
	Government		
τ	Labor income taxation level parameter	0.226	Marginal labor income tax rate evaluated at average earnings

Notes:. This table summarizes the endogenously (second stage) calibrated parameters used for the quantitative analysis.

Table 3: Targeted Moments: Empirical Values and Data Sources

Moment	Empirical Value	Data Source
Interest rate (annual)	3.5%	
College enrollment rate	51%	PSID 2011-2017
Net tuition spending to GDP	1.57%	OECD 2020
Average STEM share	24%	NSF 2024 report
STEM share for high ability students	40%	currently a bit arbitrary, but will use NSLY97
Average college wage premium	77%	PSID 2011-2017
Earnings difference between top and bottom quartile of q distribution	98%	Leukhina (2023)
Average STEM wage premium	57%	PIACC 2011 & 2012
Marginal labor income tax rate evaluated at average earnings	29%	

Notes:. This table summarizes the targeted moments, their corresponding empirical values, and the data sources used.

Endowments, Constraints and Costs. Households enter the model at college age, endowed with heterogeneous initial human capital $h \in \mathcal{H} = h_1, \ldots, h_H$. We measure h_i as well as its CDF $\Phi(h)$, with associated PDF $\phi(h)$ by SAT test scores. Specifically, we build the probability distribution function $\phi(h)$ by assigning the mid points of the SAT composite score bins to the corresponding percentile scores, see https://blog.prepscholar.com/sat-percentiles-and-score-rankings. Thus, by merging percentile scores -1 and 1 the lowest 1 percent of students gets assigned SAT composite score of $h_1 = 480$ (the mid point of 400 to 660), the next 1 percent of students gets an SAT score of $h_2 = 675$, and so forth. Accordingly, the highest scoring 1 percent of students obtains an SAT score of $h_H = 1585$.

Equipped with this fine distribution of test scores, we map those into an initial ability distribution on grid $e \in \mathcal{E} = \{e_1, \ldots, e_E\}$, where $E < H$, with corresponding CDF $\Phi(e)$ and PDF $\phi(e)$ as follows. First, we normalize the bounds of the support to be the same, thus $e_1 = h_1/h_H = 480/1585 = 0.3$ and $e_E = 1$. Second, the probability mass on the bounds of the support $\phi(h_1), \phi(h_H)$ is mapped on the mass $\phi(e_1), \phi(e_E)$, respectively. Third, for each $h_i \in \{2, \ldots, h_H\}$ we distribute the mass of $\phi(h_j)$ by linear interpolation, that is we locate for each h_j the neighboring points on the e-grid, denote those by e_{i-1} and e_i, and the additional mass on point e_{i-1} is then $\Delta\phi(e_{i-1}) = \left(1 - \frac{h_j - e_{i-1}}{e_i - e_{i-1}}\right) \cdot \phi(h_j)$ and the remaining mass $1 - \Delta\phi(e_{i-1})$ is distributed to gridpoint e_i. In our current application, we assume that children can be of two ability types, high and low, thus $E = 2$ and $e \in \{e_l, e_h\}$.

Following Cai and Heathcote (2022), the probability of college graduation γ_e depends on child ability, with $\gamma_{e^l} = 0.52$ and $\gamma_{e^h} = 0.78$. While in college, students cannot work full time and, in addition to incurring the psychological cost described above, must also forgo earnings. Consistent with Cai and Heathcote (2022), we assume that students forgo 20 weeks of median weekly earnings of full-time workers aged 16 to 24 while enrolled.

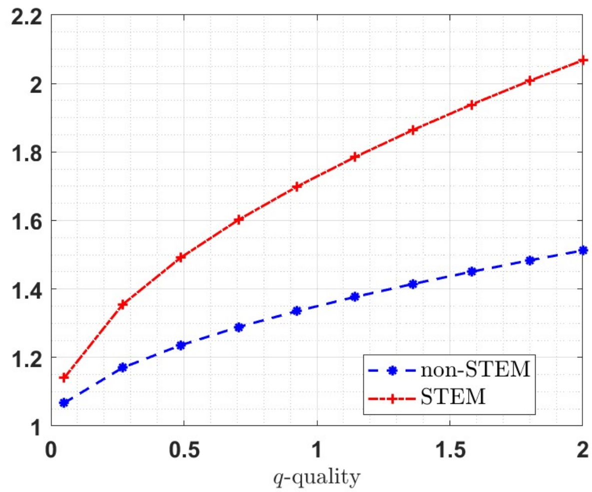
Earnings differences by college quality and major/program of study. Following Hendricks et al. (2021), we classify colleges into four quality types. The lowest quality type consists of two-year community colleges offering transferable associate degrees. The other three types are defined based on the average SAT scores of their freshmen. Type 2 includes four-year institutions with freshmen's average SAT scores in the first tercile of the SAT distribution (least selective public and private colleges). Type 3 includes four-year institutions with scores in the second tercile (many flagship universities and directional schools). Finally, Type 4 includes the most selective four-year institutions with scores in the third tercile (Ivy League, selective private schools, most flagship universities, and many other selective public universities).

As described in Leukhina (2023), after controlling for family income and AFQT scores, graduates from Type 2, Type 3, and Type 4 colleges earn on average 65%, 75%, and 98% more, respectively, than graduates from Type 1 colleges.

Regarding earnings differences by college major, in calibrating $\gamma_j(q, p)$ we target that average earnings of STEM workers are about 57% higher than those of non-STEM workers, based on the Programme for the International Assessment of Adult Competencies (PIAAC, 2011–2012).

The productivity function $\gamma_j(q, p)$ is specified in equation 15. The three parameters a, b, and c are endogenously calibrated to match the average college wage premium, the average STEM wage premium, and the wage gap between colleges in the bottom and top quartiles of the q distribution, respectively. We obtain $a = 0.35$, $b = 3.8$, and $c = 0.55$. Figure 2 illustrates the resulting $\gamma(q, p)$ productivity schedule.

Figure 2: Labor Market Productivity



(a) labor productivity by quality q, conditional on major p

Notes: tbc

4.2 College Supply

Following Cai and Heathcote (2022), we normalize the share parameter θ in the quality production function to 0.5. The fixed cost that colleges must bear independent of their quality, net of public per-student subsidy, is set to \$4,610, as in Cai and Heathcote (2022). Finally, following Altonji and Zimmerman (2018), we assume that the cost of producing graduates in the highest-cost majors is about 1.71 times higher than in the lowest-cost majors.

4.3 Aggregate Production

We parameterize the aggregate production function specified in equation (55) as follows. The outer substitution elasticity parameter ν is set to 0.8 to match the lower bound of the substitution elasticity estimates between college and non-college labor from Bils et al. (2024). The parameters $\rho(0)$, $\rho(p=1)$ and $\rho(p=2)$ governing the skill- and major-specific substitution elasticities between capital and labor are based on Caunedo et al. (2023) estimates and are set to 0.33, -0.11 and -0.43, respectively.

Non-college wages are normalized to $w(0) = 1$. We achieve this by scaling Γ using (72) for $p = 0$ to get

$$\Gamma = \frac{\alpha(0)}{1-\alpha(0)} \cdot \frac{k_t(0)^{\rho(0)-1}}{r_t + \delta}$$

4.4 Government

College subsidies, student financial aid and student loans. As in Cai and Heathcote (2022), colleges receive an average per-student subsidy for all in-state students at a rate of 49%. We calibrate the student financial aid function $\varsigma(b)$ to capture the two main components of U.S. aid programs: (i) federal need-based grants, primarily Pell Grants, which do not scale with sticker tuition, and (ii) state-level aid programs, which generally do depend on tuition. To this end, we assume that financial aid is provided as a fixed fraction of sticker tuition, but subject to a cap that depends on financial need:

$$\varsigma(b) = \min\left(\bar{\varsigma}(b), \varsigma \cdot t(e, r; q)\right), \tag{60}$$

where $t(e, r; q)$ denotes the sticker tuition faced by a student of ability e, residence status r, and college quality q. Currently, we assume a single resource threshold $\underline{\mathrm{b}}_1$. All households with family resources below this threshold qualify for financial aid, up to a cap of \$6,870, which corresponds to the average Pell Grant and state-level financial aid reported in Cai and Heathcote (2022). Households with resources above $\underline{\mathrm{b}}_1$ do not qualify for aid. The threshold $\underline{\mathrm{b}}_1$ is endogenously calibrated such that, in equilibrium, 32% of enrolled students receive financial aid.

Following Krueger and Ludwig (2016), the maximum amount of publicly provided students loans per year is given by \$11,397, which is the borrowing limit for college students in the model.

Taxes and (non-education) transfers. The consumption tax rate is set at 5% (see Mendoza et al. (1994)), and the capital income tax rate is fixed at 36% following Trabandt and Uhlig (2011). The progressive labor income tax code is approximated using a two-parameter tax function as in Heathcote et al. (2017),

$$T(y) = y - \tau_\ell y^{1-\xi},$$

where the progressivity parameter ξ is set to 0.18 - an average estimate across demographic groups in Heathcote et al. (2017) - and the level parameter τ_ℓ is endogenously calibrated so that the implied average tax rate for the median household in the model matches its empirical counterpart.

5 Results

In this section we present our basic quantitative results. Section 5.1 displays properties of the steady state equilibrium under the baseline demographic scenario in which the population grows at a constant rate of 1%, the long-run U.S. population growth rate (which factors in immigration) prior to 2008. Section 5.2 then subjects the baseline economy to an unexpected (MIT shock) reduction in public subsidies for higher education; we model a (possibly too) stark reform in which all public subsidies are reduced to zero. In Section 5.3 we turn to the impact of the demographic cliff on the macro economy and the college market, and Section 5.4 discusses the interaction between demographic shifts and public education funding reform. In all scenarios the change in demographics and/or public education subsidies is completely unexpected, and we (currently) compare the long-run (steady state) consequences of these changes.[7]

5.1 Baseline Demographics

In Figure 3 we display the equilibrium in the college market under our benchmark demographic scenario with a population growth rate of 1% (and age-specific mortality rates determined directly from the data). Panel (a) displays the histogram of college students by school quality, including the ca. 50% of high school students that choose not to go to college (and thus select $q = 0$, the spike at the left end of panel (a)). Panel (b) of Figure 3 displays the equilibrium tuition schedule. It shows two prominent (but fully expected) features: first, tuition is monotonic in school quality since per-student discretionary spending is one of the two determinants of school quality and colleges have to break even. Second, high ability students receive a tuition discount that compensates them for raising the average quality of the student body, and thus the quality of the college itself, given that average student quality is the second determinant of college quality.[8]

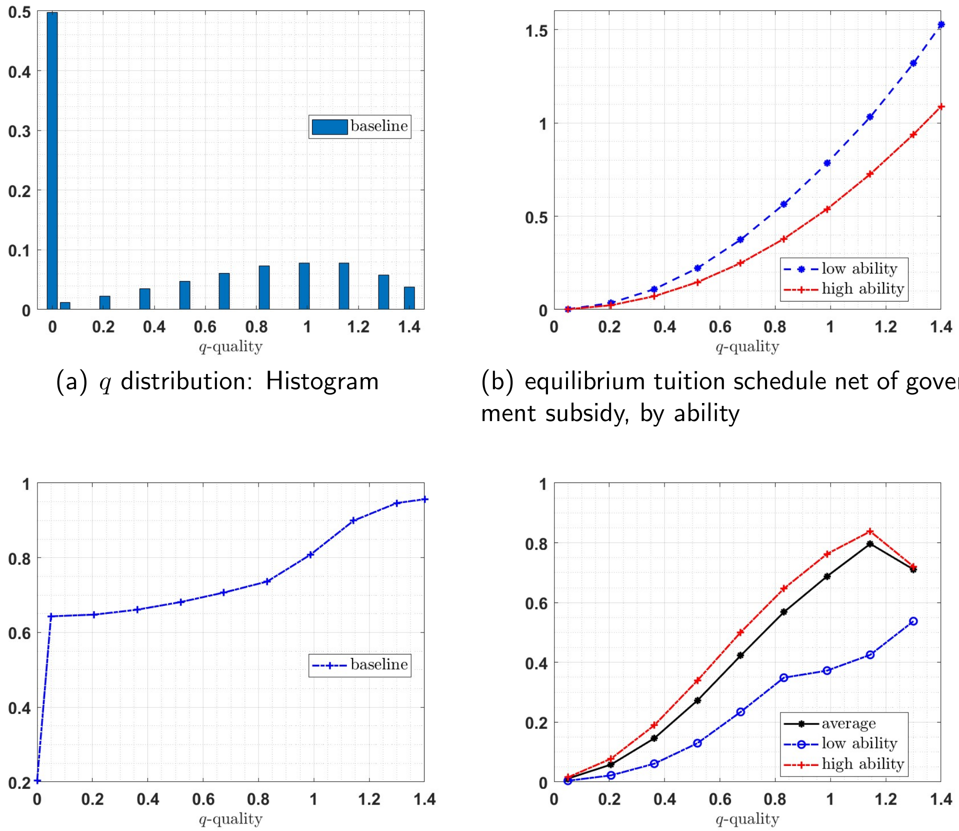
Panel (c) displays the share of high ability $(e = e_h)$ students by college quality. It shows that about 2/3 of high-school students not attending college come from the low-ability group; the outcome that one third of individuals come from the high-ability group stems from the fact that these individuals start their life with a low b draw (which is a low-probability event but can happen) and are subject to idiosyncratic taste shocks. Given the tuition schedule, the foregone wages while in college and the utility cost of studying, they find it optimal to not attend college despite their high innate ability. The higher the quality of the college, the larger is the ability tuition discount and the share of high-ability students attending colleges of that quality. Given the college quality production function in which per-capita expenditures and average quality enter multiplicatively, the value of a high-ability student is larger at a high per-student expenditure school, a property that is reflected in the equilibrium tuition schedule.

Panel (d) focuses on the college program (p) or major choice. Recall that studying a STEM field (which in many universities now includes economics) incurs a larger disutility, the more so the less innately able a student $(e = e_l)$ is, but it carries a larger wage premium in the labor market (and this effect is the stronger the better the attended college, given the log-linear (and

[7]Future versions of this paper will characterize the entire transition paths induced by demographic change and education policy reform. This will be especially crucial for the normative assessment of these changes.

[8]The resulting tuition-ability discount in our model can be interpreted as capturing merit-based institutional financial aid in the U.S.

Figure 3: Baseline: Initial Steady State



(a) q distribution: Histogram

(b) equilibrium tuition schedule net of government subsidy, by ability

(c) share of high ability students

(d) share of students in STEM fields, by student ability

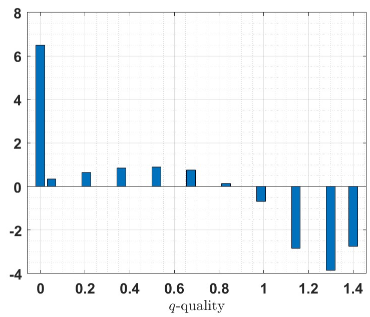
Notes: College market equilibrium under the benchmark demographic scenario in which the population grows at a constant rate of 1%.

thus multiplicative in levels) structure of labor productivity and thus wages in equation (15)). Consequently, the share of STEM students is higher at high-quality colleges, and larger among high ability students (which in turn sort more strongly into high-quality colleges). Additionally, both college quality and field-of-study decisions are influenced by wealth effects, since our model features savings. For households that are not borrowing constrained, higher initial resources b make financially and psychologically costly majors such as $p = 2$ (STEM) relatively less attractive. Thus, given that students with higher resources sort on average into higher-q colleges, the wealth effect can reduce the share of STEM majors in top segments of the quality distribution. This mechanism explains the non-monotonicity of the STEM share among high-ability students at top-quality colleges.

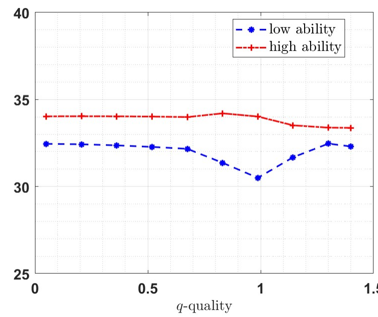
5.2 Education Subsidy Cuts in General Equilibrium

In this section we quantify how a reduction in public higher education subsidies impacts the college market equilibrium. This exercise is meant to prepare the discussion for the same thought experiment when concurrently the economy is subject to a demographic cliff in Section 5.4. The direct effect of lowering $s(e, r; q)$ uniformly to zero is to increase net tuition for all households seeking to go to college. Of course, in equilibrium the pre-subsidy tuition schedule adjusts, and with it the equilibrium allocation of students across colleges.

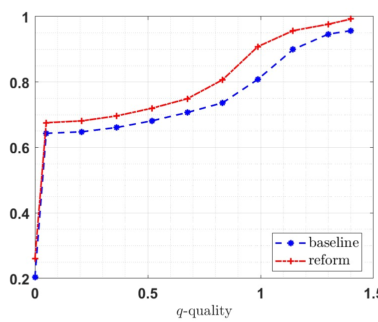
Figure 4: College Funding Cut: Eliminating the 25% Subsidy



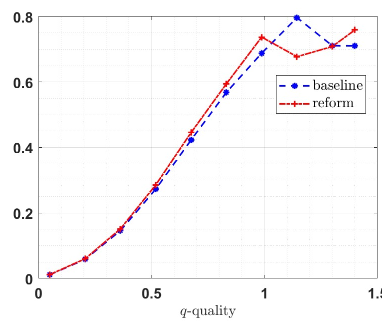
(a) q distribution: absolute change (%p)



(b) equilibrium tuition schedule net of subsidy: %-change relative to baseline



(c) share of high ability students: baseline vs reform scenario



(d) share of students in STEM: baseline vs reform scenario

Notes: Change in the college market equilibrium in response to a cut in public higher education subsidies from 25% to zero.

Panel (a) of Figure 4 shows the change in the distribution of college students, and panel (b) plots the percentage change in tuition net of the government subsidy (that is, the tuition students actually pay in equilibrium), both as a function of college quality. To interpret panel (b), note that *if* gross tuition charged by colleges had not changed, net tuition would have increased by $1/(1-0.25)-1 = 33.3\%$. The actual change is less than that for low-ability students and slightly more than that for high-ability students. The associated change in the college quality distribution in panel (a) shows first, and not surprisingly, a substantial increase (close to seven percentage points) in the share of households not going to college, and a shift away from the highest quality and most expensive colleges. The college population shifts towards higher-ability students (see panel (c)) since at the higher price tag college is not a plausible choice for low-ability students that find it harder to study (especially STEM) and for whom the labor market payoff is not as large. The improvement in the quality distribution of students also explains why the share of students pursuing a STEM degree increases mildly for just about all college qualities, as panel (d) displays. However, since the STEM share is increasing in college quality and students shift away from the highest quality colleges (due to their higher cost), the overall share, and certainly the absolute number of STEM students falls significantly in response to the public funding cut. Therefore, to the extent that STEM shortages due to biased technological change are expected in the future, reducing funding for higher education would acerbate these shortages, from the perspective of our model.

5.3 The Demographic Cliff and the College Market

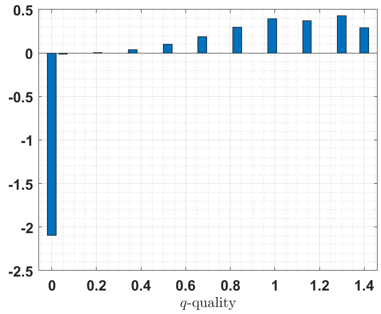
We now consider a shift from a baseline population growth rate of 1% in the initial steady state to 0% in the final steady state. In addition, using our demographic projections (see Appendix A), we assume that the level of the total population stock shrinks by approximately 16% in the final steady state, relative to the benchmark. The latter information is important for the comparison of not only per capita variables, but aggregate levels of economic activity, student body, and fiscal variables across the two demographic scenarios.[9]

Figure 5 illustrates the long-run general equilibrium effects of the demographic shock on the college market and households' higher-education choices. Panel (a) reports percentage-point changes in the equilibrium shares of students choosing a given q (with $q = 0$ denoting non-attendance). The aging shock increases the long-run capital-labor ratio, which lowers the return on physical capital, see Table 4 below. Since capital is more complementary to college-educated than to non-college labor, equilibrium college enrollment rises (the non-enrollment share falls), by about 2 percentage points. In addition, the relative shares of students attending top- and middle-quality colleges also increase.

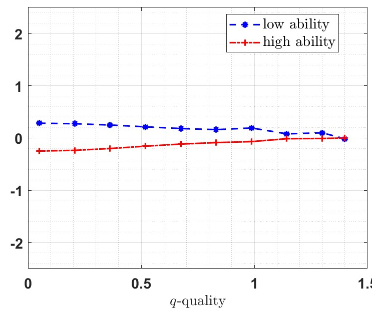
Panel (c) shows that the share of high-ability students declines at low-q colleges, although the effects is relatively small. This is because the marginal students who switch from non-attendance in the initial steady state to attendance in the final steady state have, on average, lower ability than infra-marginal students, and some high-ability inframarginal students move from low-q to higher-quality colleges. At top- and middle-q colleges, the change in the high-ability share is

[9] If we were only interested in per capita variables the computation, using the demographic model, how the absolute number of bodies in the economy will change between the old and the new steady state would not be required.

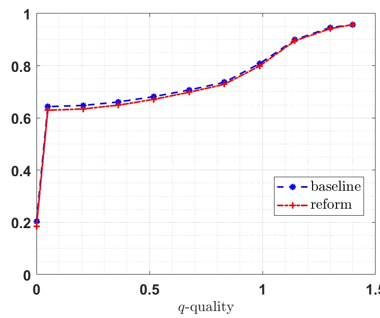
Figure 5: Exogenous Aging Shock



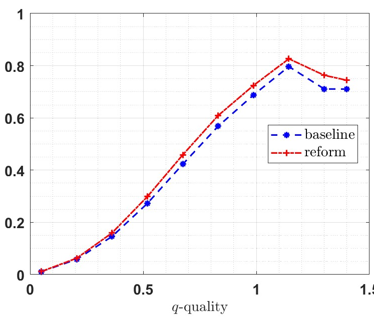
(a) q distribution: absolute change (%p)



(b) equilibrium tuition schedule NET OF SUBSIDY, by ability: %-change relative to the baseline



(c) share of high ability students: baseline vs reform scenario



(d) share of students in STEM: baseline vs reform scenario

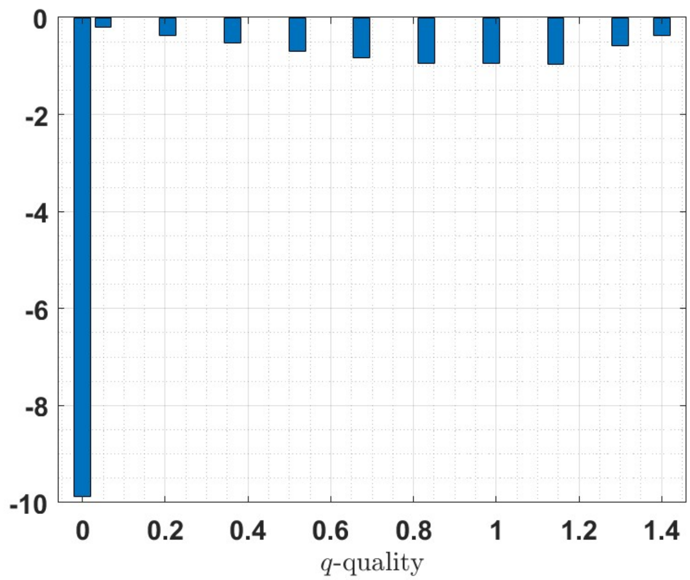
Notes: Change in the college market equilibrium in response to a demographic cliff in which the population growth rates falls from 1% to 0%.

negligible, as the downward pressure from the inflow of less-qualified marginal students is almost fully offset by an inflow of high-ability infra-marginal students moving up from lower-q colleges.

As stated, the aggregate production technology assumes that workers in high-skilled and more technologically intensive jobs are more complementary with capital, and thus demand for those workers increases most. In addition, the labor productivity (wage) earnings function features complementarity between college quality and more technologically intensive fields of study. In addition, the decline in the interest rate reduces the incentive of the household to invest in financial assets rather in human capital (going to a higher q college and/or obtaining a STEM degree).This further contributes to the increase in the STEM share. Together, these model features explain the increase in the share of STEM majors at middle- and high-quality colleges following the aging shock, as shown in panel (d).

Finally, note that because the shifts in the share of high-ability students across the q-distribution are relatively small in magnitude, the resulting adjustments in equilibrium gross tuition are also fairly modest (see panel (b)). Qualitatively, two mechanisms are at work: (a) the inflow of less-prepared marginal students into low- and middle-quality colleges and (b) the outflow of high-ability students from those colleges. Together, these effects increase the tuition-ability discount at the bottom and middle of the q-distribution.

Figure 6: Exogenous Aging Shock: Absolute change (Normalizing Initial Population to 1)



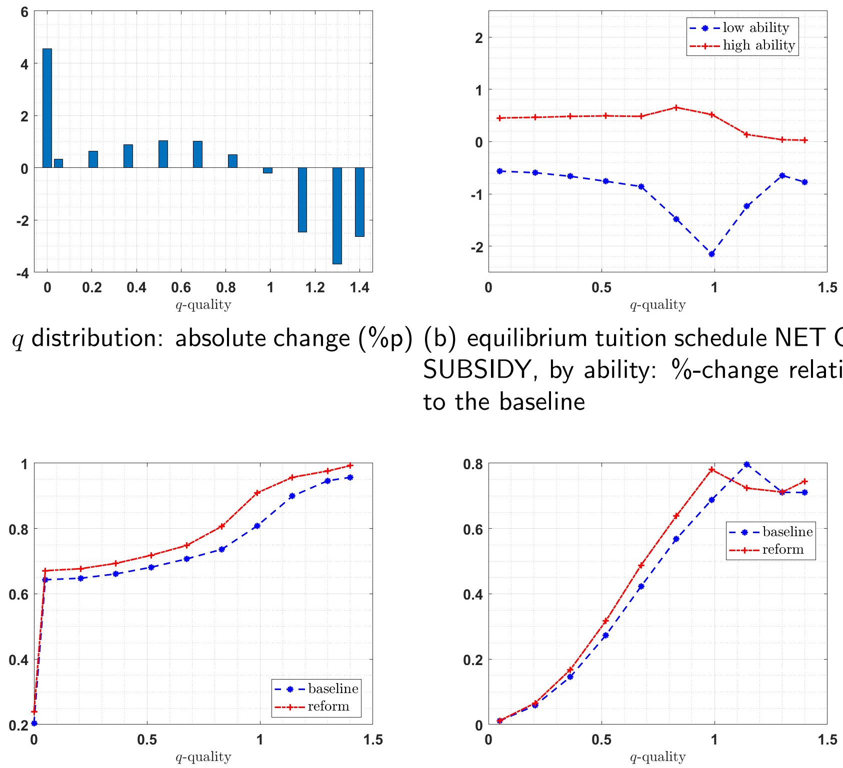
Notes: Change in the absolute number of students at different points in the college quality distribution.

The results thus far seem to suggest that the impact of the demographic cliff on the college market (and by implication, the labor market and the macro economy at large) is rather benign, even positive since the scarcity of labor and capital-skilled labor complementarity draws a larger share of high-school students into college and makes them study more labor-market lucrative STEM majors. However, the demographic change also implies a substantial reduction in the absolute *number* of bodies, 16% according to our demographic model. Figure 6 shows the absolute changes, relative to the initial (high-population) benchmark. It shows a massive decline in the number of people not going to college, but also the loss in the absolute number of students across the entire quality distribution. Given our model with constant returns to scale, it is silent about whether the fall in the number of students will lead to smaller universities and/or a reduction in their number, Figure 6 clearly shows that the largest reductions in enrollment are expected in the upper middle quality class of colleges (i.e. the big state schools).

5.4 Public Subsidy Cuts when Population is Aging

Figure 7 shows the college market response when, in addition to the demographic shock just discussed, public funding of colleges is fully withdrawn at the same time, i.e. the subsidy rate is reduced from its baseline level to zero precisely when the demographic cliff hits. Although reality is of course more complex than that, but one could interpret this scenario as the environment the U.S. is currently best described by (or, given the steady state assumption, will be described by in a couple of decades).

Figure 7: Exogenous Aging Shock plus Subsidy Cut



(a) q distribution: absolute change (%p)

(b) equilibrium tuition schedule NET OF SUBSIDY, by ability: %-change relative to the baseline

(c) share of high ability students: baseline vs reform scenario

(d) share of students in STEM: baseline vs reform scenario

Notes: Change in the college market equilibrium in response to a cut in public higher education subsidies from 25% to zero in conjunction with the emergence of a demographic cliff

Qualitatively, the responses closely resemble those induced by the pure subsidy cut discussed in Section 5.2. In other words, the favorable effects of the aging shock on the college market in per capita terms (higher enrollment and a right shift in the college quality distribution) are largely undone by the withdrawal of public funding. The magnitudes of the responses are somewhat smaller than in Figure 4, suggesting that the demographic shock making labor scarcer and dearer partially offsets the disincentive effects of the subsidy cut.

Anticipating the discussion of aggregate and fiscal effects in Section 5.5, it is worth noting that although the aging shock has favorable effects on the college market in per capita terms, as discussed above it induces a decline in the absolute number of high-skilled households (which are the main tax payers in the economy) and generates a substantial decline in macroeconomic aggregates and a fiscal contraction. Importantly, the magnitude of these negative responses is considerably larger than those induced by a pure college funding cut. However, the joint effect of both shocks is smaller than the sum of the individual responses, suggesting that the demographic shock mitigates some of the adverse aggregate and fiscal effects of the subsidy cut.

5.5 Aggregate and Fiscal Effects of Demographic Shocks and Subsidy Cuts

In this section we summarize the aggregate and fiscal policy implications of the thought experiments discussed thus far. Table 4 contains the impact on interest rates and relative wages, whereas Table 5 contains the reaction of macroeconomic aggregates.

Table 4: General Equilibrium: Wages, Interest Rate and Tax Adjustments (% and %p Changes)

Reform	Interest Rate	Non-Coll. Wage	Coll. Non-STEM Wage	Coll. STEM Wage	Lab. Inc. Tax
Subsidy cut	0.07%p	-4.11%	-0.64%	5.82%	0.75%p
Demogr. shock	-0.58%p	5.07%	4.24%	0.09%	7.32%p
Dem. shock & subs. cut	-0.52%p	0.69%	3.51%	5.31%	7.87%p

Notes:. This table summarizes % changes of aggregate wages and the rate of return in general equilibrium.

From Table 4 we observe that the demographic shock has a strong impact on the equilibrium interest rate and the tax rate required to balance the government budget. It also increases the wage in the economy, but the wage of which group rises depends on the government's involvement in the college market. If college is subsidized, then it is mostly the wage for non-college labor that rises (since college, and within college, STEM attendance responds strongly to the relative scarcity of labor). If, on the other hand, college subsidies fall as well, then the reduction of college attendance makes college labor especially scarce, and college wages, and especially STEM-college wages rises dramatically, and so does the labor income tax rate required to compensate for the smaller number of college graduates paying high taxes.[10]

Table 5: General Equilibrium: Aggregate Variables (% Changes)

Reform	Output	Capital	Assets	Aggregate Labor
Subsidy cut	-5.35%	-5.74%	-5.73%	-3.12%
Demogr. shock	-27.12%	-21.40%	-21.40%	-30.38%
Dem. shock & subs. cut	-31.01%	-26.22%	-26.22%	-32.71%

Notes:. This table summarizes % changes of several aggregate variables in general equilibrium.

[10] Table A3 in Appendix D shows that even though the reduction of the subsidies leads to a substantial reduction in government expenditures (close to one 1% of benchmark GDP), the collapse in tax revenue is quantitatively dominant so that the labor income tax rate has to increase even in the subsidy cut experiment to balance of government budget. In this sense, reducing public higher education subsidies does not in fact save the government resources, at least on in the long run.

Table 5 shows that the demographic cliff also leads to a large collapse in aggregate labor, capital and output, simply on account of the reduction of the population implied by the new demographic reality, and further reinforced by contractionary fiscal adjustments needed to balance the budget. The decline in public subsidies and associated reduction in college attendance and the shift in the quality distribution to the left magnifies these demographic forces, as the comparison between the second and third row shows.[11]

6 Conclusion

In this paper we have constructed a general equilibrium life cycle model of the college market with heterogeneous colleges, student college quality and major choice, and subsequent labor market outcomes of workers in different occupations. We have applied the model to evaluate the aggregate and distributional consequences of the "demographic cliff" that reduce the number of high-school graduates in the next decades. We have then discussed the consequences of a funding cut towards public higher education akin to the recently imposed measures.

Our demographic cliff experiments so far were driven by a change in the domestic fertility and therefore domestic population growth rate of the economy. However, significant changes in immigration policy also will have a substantial impact on the age- and the skill distribution of the U.S. population. In a next step we will expand our demographic projections to include this force, with specific attention being placed on the reduction of high-skill migrations and against the backdrop of the quantitative importance of immigrants in STEM fields and innovation.

Second, when conducting our counterfactual experiments thus far, and even though different types of labor have a differential degree of substitutability with capital, we held technology constant (beyond a constant growth trend). Especially when it comes to the importance of STEM education, considering capital-skill-biased technological change (whether exogenously or, eventually, endogenously generated) is important for a full assessment of the changes in the higher education market, and the impact of public funding policies for that market. Whether college should be subsidized, and which programs should be publicly funded, will likely depend on the direction technological progress will take, and we plan to use our environment to shed light on this question. Finally, our analysis thus far compared long-run steady states under different demographic and higher education policy scenarios. Especially a normative assessment of policy reform will require the explicit consideration of transitions, given that demographics and the cross-sectional distribution of higher education attainment are slow-moving variables that will take time to converge to their long run values. We defer this to future versions of this paper.

[11] For the demographic shock (the second and third row of the table), of course part of the decline in macroeconomic aggregates is driven by the 16% decline in the population. Table A2 in Appendix D displays the corresponding declines in per-capita terms, which is simply mechanically accounting for the reduction by 16% of the number of adult individuals in the economy.

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Appendix

A Detailed Population Dynamics and Projections

Households are born at age $j = 0$ and live at most until age $J > 0$. We distinguish between natives ($i = 1$) and immigrants ($i = 2$). Within each population group we allow for heterogeneity by skill s and gender g.

We denote by $N_{t,j,i,s,g}$ the stock of the population at time t, age j, in population group i, skill group s, and gender g. Let $\mu_{t,j,i,s,g}$ denote the net migration rate of group (j, i, s, g) at time t. Assuming end-of-period survival at rate $\varsigma_{t,j,i,s,g}$, population dynamics are given by

$$N_{t+1,j+1,i,s,g} = (1 + \mu_{t,j,i,s,g})\, \varsigma_{t,j,i,s,g} N_{t,j,i,s,g}.$$

We denote by $f_{t,j,i,s,g}$ the time t, age j, group i, skill s, gender g specific fertility rate. The rates $f_{t,j,i,s,g}$ are calibrated from data and are nonzero only for the fertile age range, which we take to be biological ages 15 to 50. The number of newborns in each period is then given by

$$N_{t+1,0,i,s,g} = \sum_{j=0}^{J} f_{t,j,i,s,g} N_{t,j,i,s,g}.$$

B Dynamic Programming Problems for $j \geq 1$

To be Completed

C Quantitative Model Calibration Appendix

C.1 Mapping Tasks to Occupations

Assume that each occupation j can be described as a bundle of tasks that changes over time. For example, Acemoglu et al. (2024) consider the following level of disaggregation: 96 job types, which is 16 industries, and 6 broad occupation types.

Tasks are grouped in four broad categories: Abstract, Routine, Manual and Contact. We use replication files from Deming (2017) to get the occupational task measures for over 300 harmonized 3-digit occupation codes from the census IPUMS data.

So far for illustration, Figure A1 below shows the table from Hurst et al. (2024) constructed using the same replication files of Deming (2017) that displays task content of a selected set of occupations. Task measures are converted into z-score space by taking unweighted differences across occupations. As a result, task measures are expressed in terms of standard deviation differences in the task content of a given occupation relative to all other occupations. For

Figure A1: Table R1 from Hurst et al. (2024)

Table R1: Task Content of Selected Occupations

Occupation	*Abstract*	*Contact*	*Routine*	*Manual*
Automobile mechanics	-0.39	-0.38	1.21	0.73
Carpenters	-0.27	-0.87	1.26	2.23
Chief executives and public admin	1.16	1.25	-1.18	-0.52
Civil engineers	2.30	0.09	1.22	0.59
Clergy and religious workers	0.05	0.96	-1.47	-0.90
Computer scientists	1.07	0.14	-0.76	0.03
Financial managers	1.99	0.50	-1.10	-0.89
Gardeners and groundskeepers	0.42	-0.50	-0.82	0.86
Janitors	-0.82	-0.52	-0.33	0.70
Lawyers	1.11	1.01	-1.67	-0.89
Machine operators, n.e.c.	-0.82	-1.22	0.47	0.04
Mail carriers for postal service	-0.80	0.01	-1.48	-0.72
Nursing aides, orderlies, and attendants	-0.37	0.95	-0.48	0.15
Physicians	2.17	1.15	0.05	0.29
Police, detectives, and private investigation	-0.55	0.86	-1.47	1.62
Primary school teachers	-0.14	0.76	-1.44	0.65
Retail sales clerks	-0.63	1.71	-0.84	-0.69
Secretaries	-0.39	0.80	1.76	-0.90
Social workers	1.66	1.53	-1.41	-0.85
Truck, delivery, and tractor drivers	-0.87	0.58	-1.37	1.98
Waiter/waitress	-0.78	1.51	-1.43	0.66

Notes: Table shows the task content (in z-score units) of various occupations.

example, an Abstract task measure of 2.0 in a given occupation means that occupation has an Abstract task requirement that is two standard deviations higher than the average occupation.

C.2 Mapping Tasks to College Majors/Programs

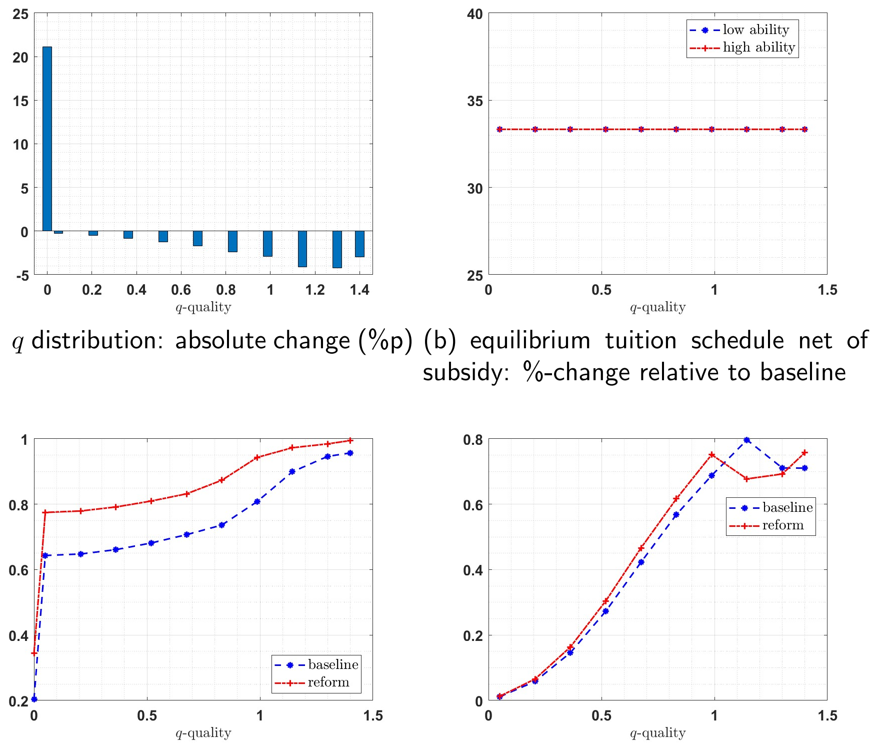
Each occupation can be described by a distribution of college majors that respective workers hold degrees in. On the one hand, the education system (college supply side) determines for which occupations graduates in a given major are qualified (i.e. have occupation-specific labor market productivity above certain threshold) upon graduation. On the other hand, changing task content of occupations over time also affects the set of occupations for which college graduates from different fields of study are qualified.

Empirically, comparing task portfolios of majors and task requirements of occupations in DOT/O-NET can define how strongly certain occupations are related to different majors (that is for which occupations graduates are broadly qualified). This has been done in the literature - for example Freeman and Hirsch (2008). Also Altonji et al. (2014) do that - and we could even rely on their replication files. So, constructing a mapping from majors to tasks, and from occupations to tasks, and based on the two determining closely and less closely related occupations - has been done in the literature and we can do it with O-NET and DOT.

D Additional Results

D.1 Subsidy Cut: Partial Equilibrium

Figure A2: College Funding Cut: Eliminating the 25% Subsidy



(a) q distribution: absolute change (%p)

(b) equilibrium tuition schedule net of subsidy: %-change relative to baseline

(c) share of high ability students: baseline vs reform scenario

(d) share of students in STEM: baseline vs reform scenario

Notes: Change in the college market equilibrium in response to a cut in public higher education subsidies from 25% to zero.

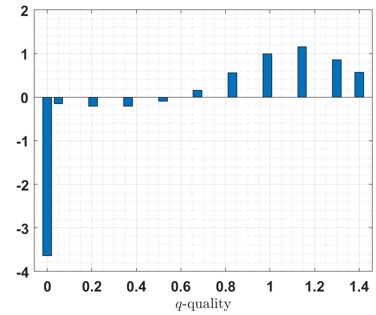
Table A1: Partial Equilibrium: Aggregate Variables(% Changes)

Reform	Output	Capital	Aggregate Labor
Subsidy cut	-12.39%	-8.45%	-4.65%

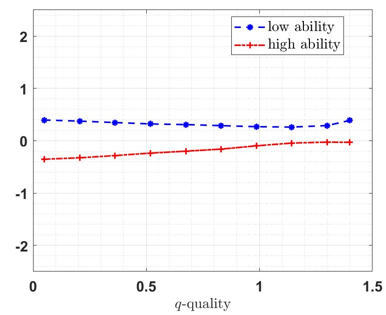
Notes:. This table summarizes % changes of several aggregate variables in partial equilibrium.

D.2 Demographic Shock with G/N constant

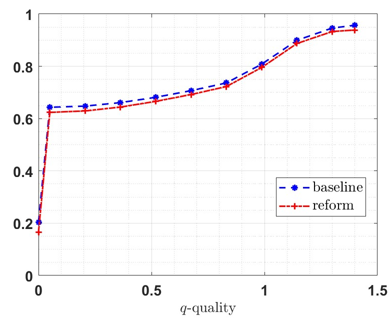
Figure A3: Exogenous Aging Shock



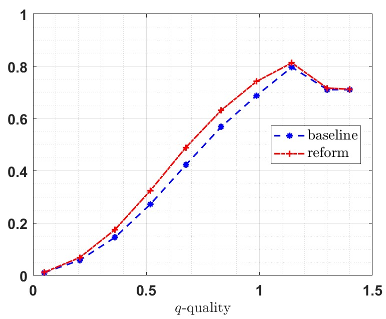
(a) q distribution: absolute change (%p)



(b) equilibrium tuition schedule net of subsidy: %-change relative to baseline



(c) share of high ability students: baseline vs reform scenario



(d) share of students in STEM: baseline vs reform scenario

Notes: Change in the college market equilibrium in response to a cut in public higher education subsidies from 25% to zero.

D.3 Subsidy Cut and Population Aging: Per Capita Variables

Table A2: General Equilibrium: Aggregate Variables Per Capita (% Changes)

Reform	Output	Capital	Aggregate Labor
Subsidy cut	-5.35%	-5.74%	-3.12%
Demogr. shock	-12.93%	-6.05%	-16.82%
Demogr. shock, G/N constant	-10.15%	0.85%	-16.26%
Dem. shock & subs. cut	-17.57%	-11.65%	-19.61%

Notes:. This table summarizes % changes of several aggregate variables in per capita terms in general equilibrium.

Table A3: Fiscal Variables as a Fraction of Baseline GDP (%p changes)

Reform	Edu. Spending	Total Revenues (PE)	Lab. Tax Rate Change (GE)
Subsidy cut	0.88%p	-2.33%p	0.75%p
Demogr. shock	0.00%p	-1.52%p	7.32%p
Demogr. shock, G/N constant	0.00%p	-1.52%p	1.41%p
Dem. shock & subs. cut	0.88%p	-3.48%p	7.87%p

Notes:. This table reports percentage point changes in public education expenditures and government tax revenues as fractions of baseline GDP, in partial equilibrium (i.e., under baseline tax parameters, factor prices and college tuition). The last column shows the required increase in the average labor income tax rate (evaluated at baseline average income) in general equilibrium.

E Computational Appendix

Below we describe the computational algorithms used to compute (a) equilibrium college tuition (Section E.1) and (b) equilibrium aggregate wages and the interest rate (Section E.2). In the full implementation, these objects are solved jointly, together with government policies that ensure budget balance. For expository clarity, however, we present the two algorithms separately.

E.1 Solution Algorithm: College Market Equilibrium

Computing a college market equilibrium implies solving an $Q \times E$ dimensional system of non-linear equations in the unknowns $v(q, e)$, i.e. ability-specific *gross* tuition for each college quality q that exists in equilibrium.

The pass-through from subsidies to colleges - which also depend on student residence status - to net tuition (i.e., the out-of-pocket cost of college paid by students) is 100% by construction. Accordingly, we treat $v(q, e)$ as the equilibrium object.

Steps:

1. Solve the household problem by backward induction[12] from the final period J to age $j = j_a + 1$, i.e., up to the initial period in which all education-related decisions are made.[13]

2. For the initial period $j = j_a$:

    (a) Use the endogenous grid method to solve for optimal consumption as a function of savings (end-of-period resources).

    (b) Using the budget constraint, recover the beginning-of-period total resources net of educational expenses (i.e. after college tuition payment and receipt of college-related

[12] We use an adaptation of the endogenous grid method.

[13] From period $j = j_a + 1$ onwards, the household problem is independent of college tuition prices or any other variables related to the college market equilibrium, unless these directly affect labor market earnings.

financial aid):

$$b - \mathbb{1}_{q>0}\left(t(b,e,r;q)(1-\zeta(b))\right) \tag{61}$$
$$= (1+\tau_c)c_0 + a_1 - (y_0(1-\mathbb{1}_{q>0}\omega) - T(y_0(1-\mathbb{1}_{q>0}\omega))) \tag{62}$$
$$\Leftrightarrow b^{totnet} = c_0 + s_1 - (y_0(1-\mathbb{1}_{q>0}\omega) - T(y_0(1-\mathbb{1}_{q>0}\omega)))$$

This gives consumption policy function $c_0(e,q,a;p)$ evaluated on the endogenous grid of total resources $b^{totnet}(e,q,a;p) = b - \mathbb{1}_{q>0}\left(t(b,e,r;q)(1-\zeta(b))\right)$.

3. Take as given the following objects pre-computed in **Step 2**:
period j_a value function $c_{j_a}(b^{totnet},e,q,a;p)$ (computed on an endogenous grid of initial resources after educational expenses and receipt of college financial aid $\mathcal{G}^{b^{totnet}}$),
   (a) For each q, initialize a guess for the ability-specific revenue schedule $v(q,a)$.
   (b) Recall that the distribution of initial resources before financial aid, $\Phi(b|e)$, is exogenous and given. We span an exogenous grid of pre-aid-and-tuition resources, b, $\mathcal{G}^b = \{b^1, b^2, ..., b^n\}$, i.e. the resources the distribution of which is given by $\Phi(b|e)$.
   (c) Taking $v(q,a)$ as given, for each realization of the household state vector (b,e,r), compute the optimal q^*-choice, $q^*(b,e,r)$, using simple grid search:
      i. Obtain residence-status-specific *net* tuition schedule:

$$t(e,r,q) = v(q,a) - s(e,r,q) \tag{63}$$

      ii. Given $t(e,r,q)$ and $zeta(b)$, compute q-specific ex-ante value functions $W(b,e,r;q)$:
         A. For given q, compute the total financial cost of education which equals $t(e,r,q)$ for $q>0$, and is set to zero for $q=0$.
         B. Given exogenous initial resources b (grid $\mathcal{G}^b$), evaluate the amount of financial aid $\zeta(b)$ received[14], and compute initial resources after financial aid and net of college financial cost:

$$b^{totnet} = b - \mathbb{1}_{q>0}\left(t(e,r,q)(1-\zeta(b))\right) \tag{64}$$

         C. For each major p, obtain optimal major-specific beginning-of-period value function interpolating the stored value function $v_{j_a}(b^{totnet},e,r;q,p)$.

[14] Which is readily available in closed form because both college quality q and initial resources b are taken as given, i.e. at this stage we compute q-specific value and policy functions for given resources b.

D. Compute the optimal p-choice, and the resulting pre-major-choice value function $W(b,e,r;q)$ using the logsum formula:

$$W(b,e,r;q) = \sigma_\eta \log\left(\sum_{p=1}^{P} \frac{\exp(V_{ja}^{qp}(b,e,r;q,p))}{\sigma_\eta}\right) \tag{65}$$

iii. Use simple grid search to obtain $q^*(b,e,r)$. In the current computational implementation we also use Extreme Value Type 1 shocks to smooth the q-decisions:

$$U(b,e,r) = \sigma_{\eta^q} \log\left(\sum_{q=0}^{q_{max}} \frac{\exp(W(b,e,r;q))}{\sigma_{\eta^q}}\right) \tag{66}$$

Also, store the major choice probabilities $\pi(p|b,e,r;q^*)$.

(d) Using the exogenous distribution of initial resources $\Phi(b|e)$ and optimal q-choices $q^*(b,e,r)$, compute the distribution of college quality $\chi(Q)$ (and also all conditional distributions by student characteristics).

(e) For each q with positive mass in the distribution $\chi(Q)$, compute the corresponding q-specific major distribution[15]:

$$\Phi(p|q) = \sum_r \sum_e \int \pi(p|b,e,r;q^*) d\Phi_0(b,e,r). \tag{67}$$

(f) For each q with positive mass in the distribution $\chi(Q)$, use the respective ability distribution of students to compute the updated ability-specific revenue schedule $\tilde{v}(q,e)$. In the simple two-ability-type case, the relevant object is the share of high-ability students $\eta(e^h|q)$.

The update calculation relies on the college quality production function and the zero-profit condition:

$$\tilde{v}(q,e^h) = \left(1 - (1-\eta(e^h|q))\frac{\theta}{(1-\theta)}\frac{e^h - e^l}{\bar{e}(q)}\right) \cdot i(q) + \sum_p \Phi(p|q)\phi(p) \tag{68}$$

$$\tilde{v}(q,e^l) = \left(1 + \eta(e^h|q)\frac{\theta}{(1-\theta)}\frac{e^h - e^l}{\bar{e}(q)}\right) \cdot i(q) + \sum_p \Phi(p|q)\phi(p). \tag{69}$$

[15] Observe that computing the optimal college tuition schedules requires the distribution of majors, $\Phi(p|q)$, as implied by household decisions. For this reason, the algorithm treats ability-specific tuition (revenue) schedules as equilibrium objects rather than the ability discount. The reason is that recovering ability-specific tuition from the ability discount requires knowledge of the college fixed cost, which itself depends on the major distribution $\Phi(p|q)$. In turn, computing the major distribution requires the tuition schedule.

Note that the q-specific major distribution $\Phi(p|q)$ computed in **Step 3(d)** enters the above calculation.

(g) Evaluate the maximum absolute deviation $\max(|\tilde{v}(q,e^h)-v(q,e^h)|, |\tilde{v}(q,e^l)-v(q,e^l)|)$, if this distance exceeds a chosen tolerance threshold, return to **Step 3(c)**.

E.2 Solution Algorithm: Capital and Labor Market Equilibrium

The optimality condition of the firm with respect to capital $K(p)$ used in the p-specific output production is given by

$$\begin{aligned}
r+\delta &= \Psi Y^{1-\nu}\lambda(p)\alpha(p)\left(\alpha(p)\left(\Psi K(p)\right)^{\rho(p)} + (1-\alpha(p))\left(\Gamma(p)L_t(p)\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} \left(\Psi K(p)\right)^{\rho(p)-1} \\
&= \Psi Y^{1-\nu}\lambda(p)\alpha(p)\left(\alpha(p) + (1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} \left(\Psi K(p)\right)^{\nu-1} \\
&= \Psi\left(\frac{Y}{\Psi K(p)}\right)^{1-\nu}\lambda(p)\alpha(p)\left(\alpha(p) + (1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}},
\end{aligned} \tag{70}$$

where $k(p) = \frac{\Psi K(p)}{\Gamma(p)L(p)}$ is the effective capital-labor ratio Similarly, the first order condition with respect to labor $L(p)$ is

$$\begin{aligned}
w(p) &= \Gamma(p)Y^{1-\nu}\lambda(p)(1-\alpha(p))\left(\alpha(p)\left(\Psi K(p)\right)^{\rho(p)} + (1-\alpha(p))\left(\Gamma(p)L(p)\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} \left(\Gamma(p)L(p)\right)^{\rho(p)-1} \\
&= \Gamma(p)Y^{1-\nu}\lambda(p)(1-\alpha(p))\left(\alpha(p) + (1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} \left(\Psi K(p)\right)^{\nu-1+1-\rho(p)} \left(\Gamma(p)L(p)\right)^{\rho(p)-1} \\
&= \Gamma(p)\left(\frac{Y}{\Psi K(p)}\right)^{1-\nu}\lambda(p)(1-\alpha(p))\left(\alpha(p) + (1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} k(p)^{1-\rho(p)}
\end{aligned} \tag{71}$$

Using (70) in the above equation we obtain:

$$w(p) = \frac{\Gamma(p)}{\Psi}\frac{1-\alpha(p)}{\alpha(p)} \cdot (r+\delta) \cdot k(p)^{1-\rho(p)} \tag{72}$$

which gives the capital intensity $k(p)$ as a function of p-specific wages and interest rates:

$$k(p) = \left(\frac{w(p)}{\frac{\Gamma(p)}{\Psi}\frac{1-\alpha(p)}{\alpha(p)} \cdot (r+\delta)}\right)^{\frac{1}{1-\rho(p)}} \tag{73}$$

This suggests the following algorithm:

1. Guess aggregate interest rate r, wage normalization factor $\Gamma(1)$ and p-specific wages $w(p)$ (for $p > 0$).

2. Solve the household problem (normalization the non-college wage to 1)

   - Compute aggregate assets K^s
   - Compute p-specific labor supply $\{L^s(p)\}_{p=0}^{P}$

3. Use eq. (72) to compute $k^d(p)$:

$$k^d(p) = \left(\frac{w(p)}{\frac{\Gamma(p)}{\Psi} \frac{1-\alpha(p)}{\alpha(p)} \cdot (r+\delta)} \right)^{\frac{1}{1-\rho(p)}} \tag{74}$$

   where $\Gamma(p) = \Gamma(1)\tilde{\Gamma}(p)$ with $\tilde{\Gamma}(p)$ being exogenous scaling parameters.

4. Compute aggregate capital demand: $K^d = \sum_p K^d(p) = \frac{\sum_p k^d(p) L^s(p) \Gamma(p)}{\Psi}$

5. Compute wage normalization factor $\Gamma(1)$:

$$\Gamma(1) = \frac{1}{\left(\frac{Y}{\Psi K^d(0)}\right)^{1-\nu} \lambda(0)(1-\alpha(0)) \left(\alpha(0) + (1-\alpha(0)) \left(\frac{1}{k^d(0)}\right)^{\rho(0)} \right)^{\frac{\nu-\rho(0)}{\rho(0)}} k^d(0)^{\rho(0)-1}} \tag{75}$$

6. For $p > 0$, compute p-specific wages consistent with p-specific household labor supply $L^s(p)$:

$$\tilde{w}(p) = \Gamma(p) \left(\frac{Y}{\Psi K^d(p)}\right)^{1-\nu} \lambda(p)(1-\alpha(p)) \left(\alpha(p) + (1-\alpha(p)) \left(\frac{1}{k^d(p)}\right)^{\rho(p)} \right)^{\frac{\nu-\rho(p)}{\rho(p)}} k^d(p)^{\rho(p)-1} \tag{76}$$

   Observe that both aggregate output Y and p-specific capital demand $K(p)$ are a function of p-specific labor supply by households $L^s(p)$.

7. Iterate on r and $w(p)$ for $p > 0$ and $\Gamma(1)$ until convergence, ensuring that the aggregate capital market clearing condition and p-specific labor market clearing conditions hold.

Special Case of Perfect Substitutes. With $\nu = 1$, equation (70) simplifies to

$$r + \delta = \Psi \lambda(p) \alpha(p) \left(\alpha(p) + (1-\alpha(p)) \left(\frac{1}{k(p)}\right)^{\rho(p)} \right)^{\frac{1-\rho(p)}{\rho(p)}} \tag{77}$$

from which we can pin down in closed form $k(p)$ in each p as

$$k(p) = \left[\frac{1-\alpha(p)}{\left(\frac{r+\delta}{\Psi\lambda(p)\alpha(p)}\right)^{\frac{\rho(p)}{1-\rho(p)}} - \alpha(p)} \right]^{\frac{1}{\rho(p)}}. \tag{78}$$

Likewise, (71) simplifies to

$$w(p) = \Gamma(p)\lambda(p)(1-\alpha(p)) \left(\alpha(p)k(p)^{\rho(p)} + (1-\alpha(p))\right)^{\frac{1-\rho(p)}{\rho(p)}}. \tag{79}$$

and we may now use (78) in (79) to see how wages are pinned down from $r+\delta$.