

Please sign the inside cover of your blue book to indicate you agree to the Honor Pledge: "I have neither given nor received unauthorized aid on this test or assignment." **Graphing calculators are not allowed on this test.** Read and follow directions carefully. Use your time wisely! All communication devices and personal items outside of a water bottle, pencil, eraser, and allowed calculator must be put on the side of the room. Show ALL work in your blue book. Graders reserve the right to take off points when work is not shown. Write your name, your row, and test form on the outside. Good luck!

1. (6 pts) Let E be the region between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$. Set up the definite integral (integral with bounds) for $\int \int \int_E (x^2 + y^2 + z^2)^3 dV$.
2. (6 pts) Consider the rectangular prism (box) with corner vertices $(0, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(0, 1, 1)$, $(1, 0, 0)$, $(1, 1, 0)$, $(1, 0, 1)$, and $(0, 1, 1)$. Compute the flux of $\vec{F}(x, y, z) = \langle x, 2y, 3 \rangle$ from the inside to the outside of the box.
3. (8 pts) Find the global maximum and global minimum values of $f(x, y) = 5x^2 + 3y^2$ on the disc $x^2 + y^2 \leq 4$ and state where they occur.
4. (5 pts) Find the shortest distance between the plane $x + 3y + 2z = 14$ and the origin.
5. (8 pts) Let C be the closed curve formed from the line $y = 0$ and the upper halves of the circles $1 = x^2 + y^2$ and $4 = x^2 + y^2$. Let C be oriented counter-clockwise. Calculate the circulation of $\vec{F}(x, y) = \langle -x^2y, y^2x \rangle$ around C .
6. (7 pts) Exactly one of the following limit exists. Compute the limit that exists and show that the other limit does not exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$$

(b)

$$\lim_{(x,y) \rightarrow (2,3)} x^2 + xy$$

7. (7 pts) Let S be the section of the cone $z = -\sqrt{x^2 + y^2} + 3$ above the xy -plane oriented with the downward facing normal vector. If $\vec{F}(x, y, z) = \nabla \times \langle x^2z + 3, e^z, yz^2 + 2z \rangle$, find

$$\int \int_S \vec{F} d\vec{S}$$

8. (3 pts) Find the angle between the vectors $\langle 0, 4, 3 \rangle$ and $\langle 1, 1, 2 \rangle$. (You don't need to simplify your answer.)

9. (3 pts) If \vec{u} , and \vec{v} are non-zero and $\vec{u} \cdot \vec{v} = 0$, what is the relationship between u and v ? Justify your answer.
10. (3 pts) If \vec{u} , and \vec{v} are non-zero and $\vec{u} \times \vec{v} = 0$, what is the relationship between u and v ? Justify your answer.
11. (7 pts) Calculate the work done by the vector force field $\vec{F}(x, y, z) = \langle e^x, z \cos(y), \sin(y) \rangle$ on a particle that moves along the helix $r(t) = \langle \cos t, t, \sin t \rangle$ for $0 \leq t \leq \pi$.
12. (4 pts) Let $\vec{F}(x, y, z) = \langle x, x^2 \cos(x), 2z \rangle$ describe fluid flow. If the fluid expanding, compressing, or is it incompressible? How do you know?
13. (23 pts) Let $f(x, y) = e^{x^3+y}$.
- (3 pts) What is the domain of $f(x, y)$?
 - (3 pts) What is the range of $f(x, y)$?
 - (3 pts) What is $\Delta f(x, y)$?
 - (3 pts) What is $\nabla f(x, y)$?
 - (4 pts) What is the maximum rate of change of $f(x, y)$ at $(-1, 1)$?
 - (4 pts) Compute the directional derivative of $f(x, y)$ at $(-1, 1)$ in the direction of $\vec{u} = \langle 1, 1 \rangle$
 - (3 pts) Find the equation of the tangent plane to $f(x, y)$ and $(-1, 1)$.
14. (4 pts) Find the equation of the tangent plane to the surface $3xyz + xy = 8$ at $(1, 2, 1)$
15. (6 pts) Consider the curve in the xy -plane given by the quarter circle from $(2, 0, 0)$ to $(0, 2, 0)$. If this curve is the base of a fence who's height is given by $f(x, y) = x^3 + xy^2$, what is the area of the fence?
- Bonus. (5 pts) Let F be an arbitrary vector field whose second partial derivatives are continuous on all of \mathbb{R}^3 . Let S be a sphere, oriented outward. Prove that

$$\int \int_S \text{curl } F \cdot d\vec{S} = 0$$