DSGE Model Applications

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Econ 722 – Part 1

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Applications to Monetary and Fiscal Policy Analysis

1. What is the optimal target inflation rate?
2. Was high inflation and output volatility in the 1970s due to loose monetary policy?
3. Effects of the zero lower bound on nominal interest rates on monetary policy.
4. How large are government spending multipliers?
5. Fiscal policy rules and the effect of a change in the labor tax rate.

Application 1: Optimal Target Inflation Rate

- What Is The Optimal Target Inflation Rate?
- New Keynesian distortion: nominal price adjustments are costly $\Longrightarrow$ firms economize on price-adjustments $\Longrightarrow$ non-zero inflation leads to a loss of output.
- Monetary distortion: nominal interest rates determine the cost of holding money $\Longrightarrow$ if cost of holding money is positive, households economize on transactions that require money as medium of exchange $\Longrightarrow$ welfare loss.
- What is the relative magnitude of these distortions?

References: Aruoba and Schorfheide (2011, *American Economic Journal: Macroeconomics*)

Households

- The households maximize

$$\mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} \beta^{(t-\tau)} \left\{ U(C_t) - \phi L_t + \frac{\chi_t}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} \right\} \right]$$

- Households also hold capital stock and rent it out to firms:

$$K_{t+1} = (1-\delta) K_t + \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) \right] I_t$$

- Budget constraint:

$$\begin{aligned} & P_t C_t + P_t I_t + B_{t+1} + M_{t+1} \\ & \quad \leq \quad P_t W_t L_t + P_t R_t^k K_t + \Pi_t + R_{t-1} B_t + M_t - T_t + \Omega_t \end{aligned}$$

- Real money balance term in utility function allows us to derive money demand function.

Intermediate Goods Production

- Production:

$$Y_t(i) = \max \left\{ Z_t K_t(i)^\alpha H_t(i)^{1-\alpha} - \mathcal{F}, 0 \right\}. \quad (1)$$

- Firms can re-optimize prices with probability $1 - \zeta$.
- A random fraction ι of the firms that are not allowed to re-optimize update their price $P_{t-1}(i)$ according to last period's inflation rate π_{t-1}.
- Remaining $1 - \iota$ firms keep their price constant.
- Price stickiness generates inefficiency:

$$Y_t = \frac{1}{D_t} Z_t K_t^\alpha H_t^{1-\alpha}, \quad D_t = \int \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda}{\lambda}} di \geq 1$$

Monetary Policy

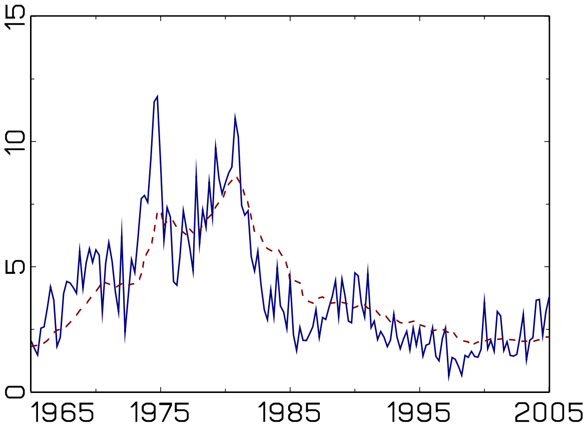
- Monetary Policy Rule:

$$R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \epsilon_{R,t}\}, \quad R_{*,t} = (r_* \pi_{*,t}) \left(\frac{\pi_t}{\pi_{*,t}} \right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2}$$

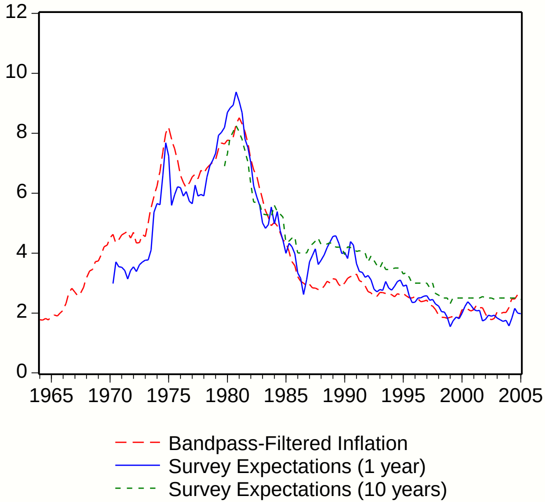
- Agents forecast target inflation according to:

$$\pi_{*,t} = \pi_{*,t-1} + \epsilon_{\pi,t}.$$

Inflation (Blue) and Target Inflation (Red)



Construction of Target Inflation Series



- Data:
  - output (log per capita GDP, detrended)
  - inflation (log differences of GDP deflator)
  - interest rates (federal funds rate)
  - inverse velocity (based on M1)
  - target inflation (see previous slide)

- Use random-walk Metropolis Hastings algorithm to generate draws from posterior $\{\theta^i\}_{i=1}^N$. Store these draws on hard drive.

- Subsequent analysis: for each draw θ^i, $i = 1, 2, \dots, N$ or for a subseqence of draws $i = 1, 11, 21, \dots, N$
  - compute impulse response to target inflation rate shock;
  - compute welfare losses/gain of counterfactual target inflation rate.

Parameter Estimates

Name	Mean	90% Intv
Households		
ν	31.754	[24.76, 38.08]
Firms		
ζ	0.756	[0.728, 0.784]
ι	0.036	[0.000, 0.073]
Central Bank		
ψ_2	1.027	[0.846, 1.224]
ρ_R	0.669	[0.622, 0.719]
σ_R	0.338	[0.284, 0.389]
$\sigma_{R,2}$	0.810	[0.572, 1.020]
$\tilde{\pi}^*_{0,A}$	-0.058	[-3.439, 3.126]
σ_π	0.049	[0.044, 0.053]
Shocks		
ρ_g	0.896	[0.865, 0.931]
σ_g	1.140	[0.989, 1.299]
ρ_χ	0.982	[0.974, 0.991]
σ_χ	1.298	[1.170, 1.415]
ρ_z	0.799	[0.719, 0.887]
σ_Z	2.082	[1.451, 2.696]

Some Parameter Transformations of Interest

- New Keynesian Phillips curve (NKPC):

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f \mathbb{E}_t[\tilde{\pi}_{t+1}] + \kappa \widetilde{MC}_t,$$

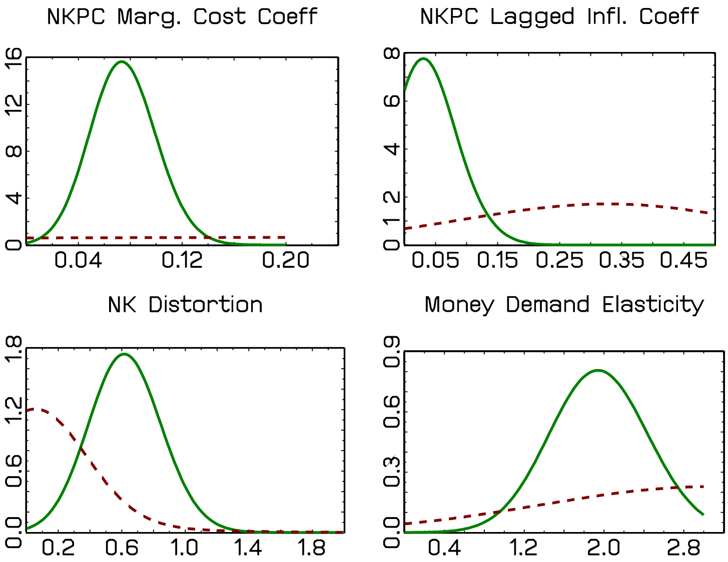
- Percentage loss $100|1/D_* - 1|$ in output due to NK friction, where

$$D_* = \frac{(1-\zeta)(p_*^o)^{-\frac{1+\lambda}{\lambda}}}{1-\zeta\left(\frac{1}{\pi_*}\right)^{-\frac{(1+\lambda)(1-\iota)}{\lambda}}}, \quad p_*^o = \left[\frac{1}{1-\zeta} - \frac{\zeta}{1-\zeta}\left(\frac{1}{\pi_*}\right)^{-\frac{1-\iota}{\lambda}}\right]^{-\lambda}.$$

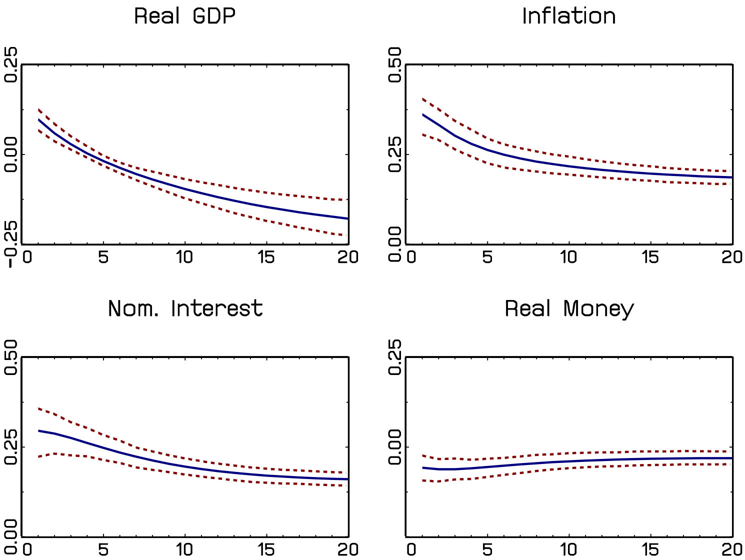
- Money demand function

$$\tilde{\mathcal{M}}_{t+1} = -\frac{1}{\nu(R_* - 1)}\tilde{R}_t + \frac{\gamma}{\nu}\tilde{X}_t - \frac{1-\nu}{\nu}\mathbb{E}[\tilde{\pi}_{t+1}] + \tilde{\chi}_t$$

Posterior (Green) and Prior (Red) Densities



Response to Target Inflation Shock



Welfare Implications

We focus on steady state welfare comparison.

1. Fix benchmark target inflation at π_*. Compute steady state consumption, hours, real money balances and let

$$V_0 = \frac{1}{1-\beta} U(C) - \phi L + \frac{\chi}{1-\nu} \left(\frac{M}{P} \right)^{1-\nu}$$

2. Recompute steady state under counterfactual target inflation rate $\tilde{\pi}$:

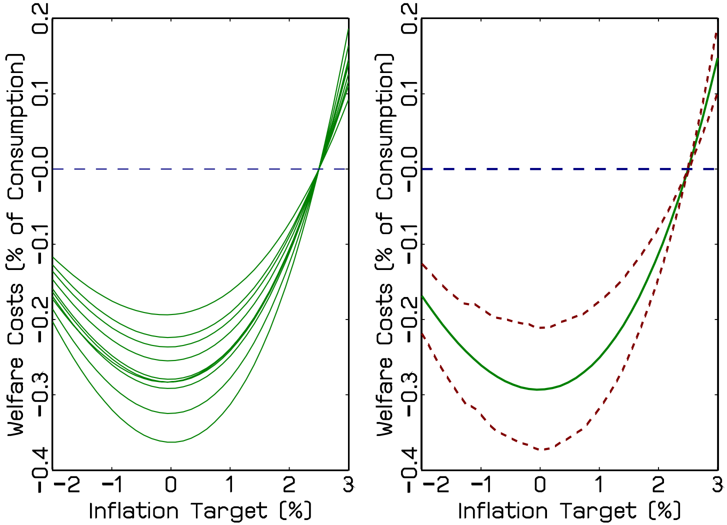
$$V_1 = \frac{1}{1-\beta} U(\tilde{C}) - \phi \tilde{L} + \frac{\chi}{1-\nu} \left(\widetilde{\frac{M}{P}} \right)^{1-\nu}.$$

3. Scale consumption by a factor κ under the benchmark inflation rate π_* and define

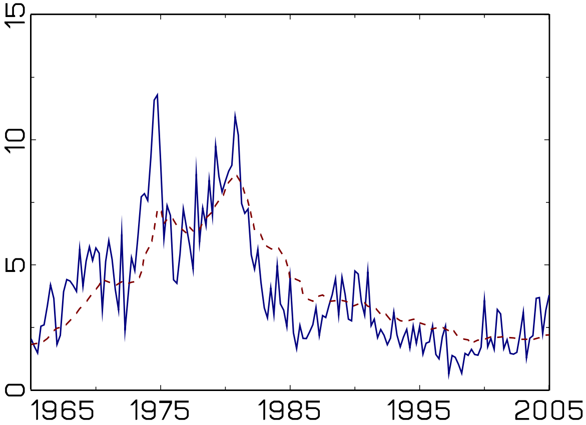
$$\tilde{V}_0(\kappa) = \frac{1}{1-\beta} U\big((1+\kappa)C\big) - \phi L + \frac{\chi}{1-\nu} \left(\frac{M}{P} \right)^{1-\nu}.$$

4. Welfare loss/gain: determine $\tilde{\kappa}$ such that $\tilde{V}_0(\kappa) = V_1$.

Welfare Implications



Application 2: Was High Inflation Volatility in the 1970s Due to Loose Monetary Policy?



References: Lubik and Schorfheide (2004, *American Economic Review*)

Monetary Policy

- Log-linearized Monetary Policy Rule:

$$\widehat{R}_t = \psi_1 \widehat{\pi}_t + \psi_2 (\widehat{y}_t - \widehat{y}_{t-1} + z_t) + \sigma_R \epsilon_{R,t}$$

- If $\psi < 1$, the equilibrium in the NK model becomes indeterminate, meaning that expectations could become self-fulfilling and aggregate outcomes might be affected by "sunspots" (non-fundamental shocks).

Review: How Can One Solve LRE Systems? A Simple Example

Simple model:

$$y_t = \frac{1}{\theta}\mathbb{E}_t[y_{t+1}] + \epsilon_t, \quad \epsilon_t \sim iid(0,1), \quad \theta \in \Theta = [0,2].$$

Let $\xi_t = \mathbb{E}_t[y_{t+1}]$ and $\eta_t = y_t - \xi_{t-1}$. Write:

$$\xi_t = \theta\xi_{t-1} - \theta\epsilon_t + \theta\eta_t.$$

Nonexplosive solutions:

- Determinacy: $\theta > 1$. The only stable solution:

$$\xi_t = 0, \quad \eta_t = \epsilon_t \quad \Longrightarrow \quad y_t = \epsilon_t$$

- Indeterminacy: $\theta \leq 1$ the stability requirement imposes no restrictions on forecast error:

$$\eta_t = \widetilde{M}\epsilon_t + \zeta_t \quad \Longrightarrow y_t = \theta y_{t-1} + \widetilde{M}\epsilon_t + \zeta_t - \theta\epsilon_{t-1}$$

Indeterminacy

- In the small-scale New Keynesian model, the policy rule coefficient ψ_1 affects the determinacy of the RE solution.
- In our simple linearized model:
  - no capital;
  - passive fiscal policy;
  - indexation to trend inflation $\bar{\pi} = \pi_*$;

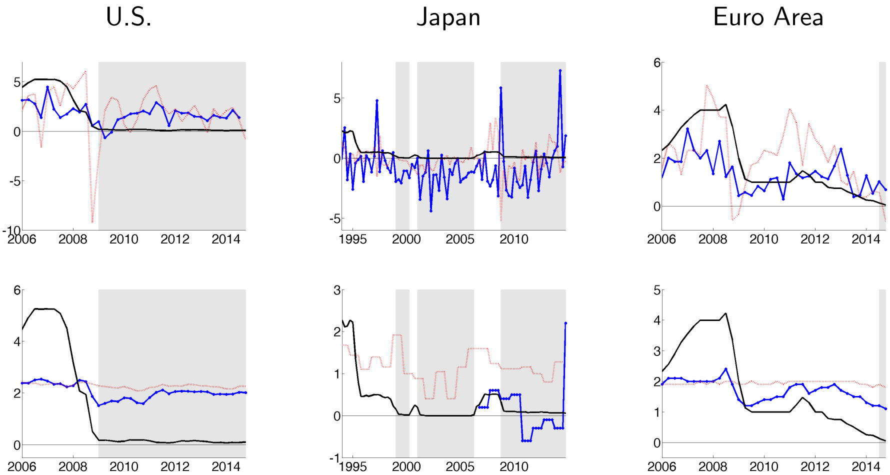
  the RE equilibrium becomes indeterminate if $\psi_1 < 1$.
- Empirical question: historically, was $\psi_1 < 1$?
- Normative question: how should CB set ψ_1? $\Longrightarrow$ keep $\psi_1 > 1$ "Taylor" principle.
- **Lubik and Schorfheide (2004):** estimate model on pre-1980 and post-1980 samples and compute posterior probabilities of *determinacy* vs. *indeterminacy*.

Application 3: Effect of Zero (or Effective) Lower Bound on Nominal Interest Rates

- Because agents can always hold cash, there is a lower limit to the nominal interest rate on bonds.
- ZLB has been binding in U.S., Japan, and Euro Area.
- ZLB constrains monetary policy responses to adverse shocks.
- ZLB in our model:

$$R_t = \max\left\{1,\ R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp\{\sigma_R \epsilon_{R,t}\}\right\}, \quad R_{*,t} = (r_* \pi_{*,t}) \left(\frac{\pi_t}{\pi_{*,t}}\right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_2}$$

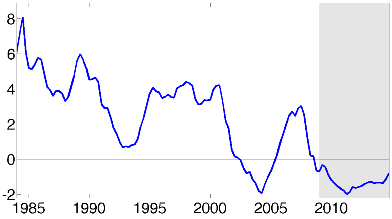
Inflation and Inflation Expectations



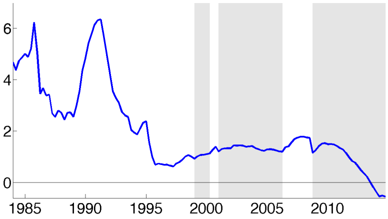
Top Panels: interest rates (black), GDP deflator inflation (blue), CPI inflation (red)
Bottom Panels: interest rates (black), 1-year inflation expecations (blue), 10-year inflation expectations (red)

Ex Ante Real Rates (See Below for $\mathbb{E}_t[\pi_{t+1}]$)

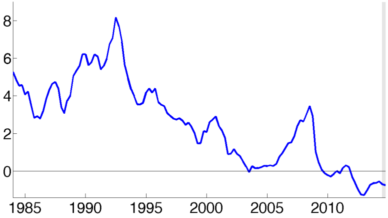
U.S.



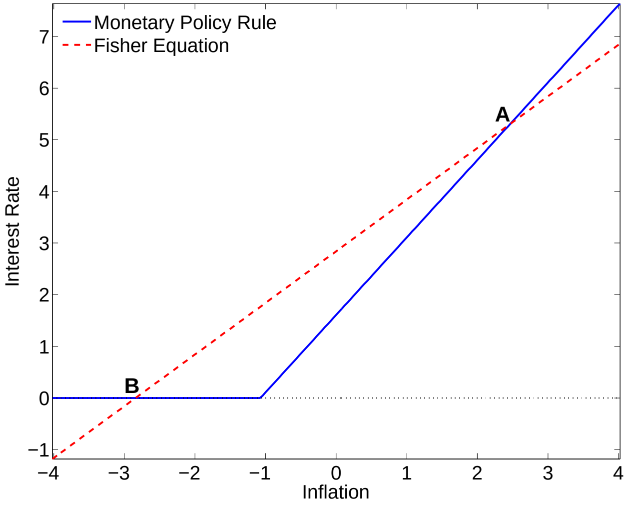
Japan



Euro Area



Steady States in a Model with ZLB Constraint



Bullard (2010) [Seven Faces of “The Peril”]

- The U.S. is closer to a Japanese-style outcome today than at any time in recent history.
- Promising to remain at zero for long time is a double-edged sword. The policy is consistent with the idea that inflation and inflation expectations should rise in response to the promise and that this will eventually lead the economy back toward the targeted equilibrium.
- But the policy is also consistent with the idea that inflation and inflation expectations will instead fall and that the economy will settle in the neighborhood of the unintended steady state, as Japan has in recent years.

Solving and Estimating Models with ZLB Constraint

- ... is much more complicated because of the nonlinearity.
- Requires a more elaborate solution technique;
- and a nonlinear filter, e.g. particle filter, to compute the likelihood function.
- **Reference:** FVRRS on solution and filtering; also HS on Bayesian computations.
- We will just look at some results.

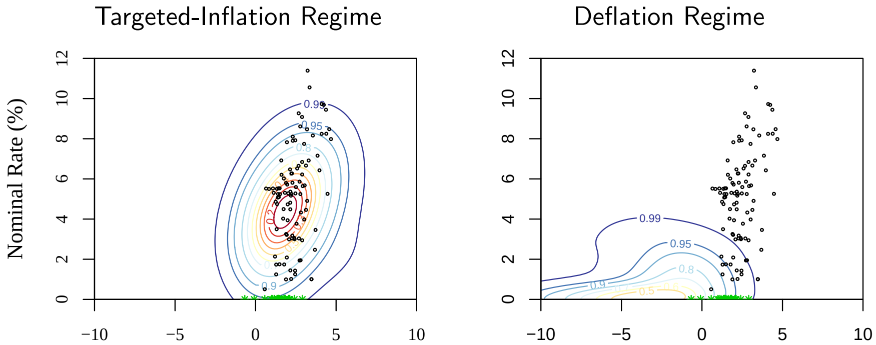
References

- B. Aruoba and F. Schorfheide (2016): "Inflation During and After the Zero Lower Bound," *Proceedings of the 2016 Jackson Hole Economic Policy Symposium.*
- B. Aruoba, P. Cuba-Borda, and F. Schorfheide (2018): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries," *Review of Economics Studies*, **85(1)**, 87-118.

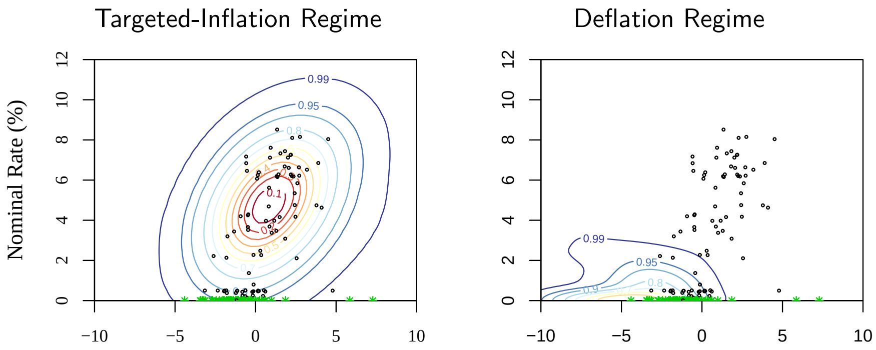
Regime-Switching Equilibrium in a Model with ZLB Constraints

- Assume that agents can coordinate beliefs on exogenous sunspot shock $s_t \in \{0, 1\}$ that follows Markov switching process.
- Model also contains fundamental shocks to: technology growth, government spending, monetary policy, and discount factor.
- We consider an equilibrium with two regimes: targeted-inflation regime and deflation regime.
- Nonlinear model is solved using projection methods; in particular, accounting for ZLB.
- We estimate NK DSGE model based on pre-ZLB output growth, consumption, inflation, and interest rate data assuming that the economies are in the targeted-inflation regime.
- Then conduct nonlinear analysis on ZLB data.

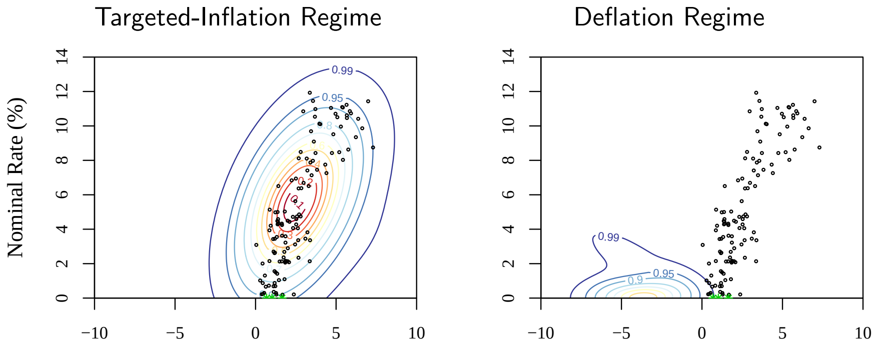
Data and Ergodic Distribution – U.S.



Data and Ergodic Distribution – Japan



Data and Ergodic Distribution - Euro Area



Findings

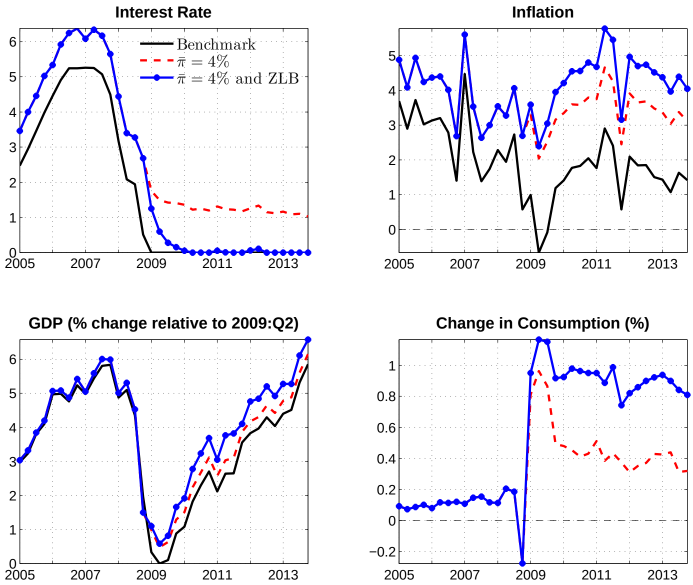
- Under targeted inflation regime reaching ZLB is unlikely.
- But, overlap in regime conditional distributions for low interest and inflation rates.
- Japan: observations appear more likely under deflation regime.
- U.S.: ambiguous
- Euro Area: too soon to tell.
- Contour plots ignore dynamic aspects and other observables.

Papers proceed by using a filter to formally assess the probability that countries enter deflation regime.

Policy Question: Should Inflation Target Be Increased?

- How bad is deflation?
  - Adverse shocks that generate deflation are bad.
  - Welfare costs due to “New Keynesian” distortion.
  - Could be amplified by downward nominal wage rigidity.
- Experiment: change inflation target (in our model it is about 2.5%).

What If... the U.S. Had Targeted 4% Inflation?



What is Behind the Plot?

- We have estimated parameter of the DSGE model. Here we fix θ at posterior mean.
- **Recall:**
  - In our model, fluctuations are generated by exogenous shocks: technology growth z_t, government spending g_t, ...
  - Given θ, we can "invert" the model, and compute values for the exogenous shocks that explain the data.
- Back out historical series of shocks.
- **Counterfactual:**
  - Resolve the model with new policy parameters (here target inflation rate).
  - Feed in the historical shocks and compute counterfactual path for output, inflation, interest rates...
  - In one of the counterfactuals we adjust the monetary policy shocks to keep economy at the ZLB.

Review: Filtering

- State-space representation of linearized DSGE model

$$
\begin{aligned}
y_t &= \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_2(\theta)s_t(+u_t) \quad \text{measurement} \\
s_t &= \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \quad \text{state transition}
\end{aligned}
$$

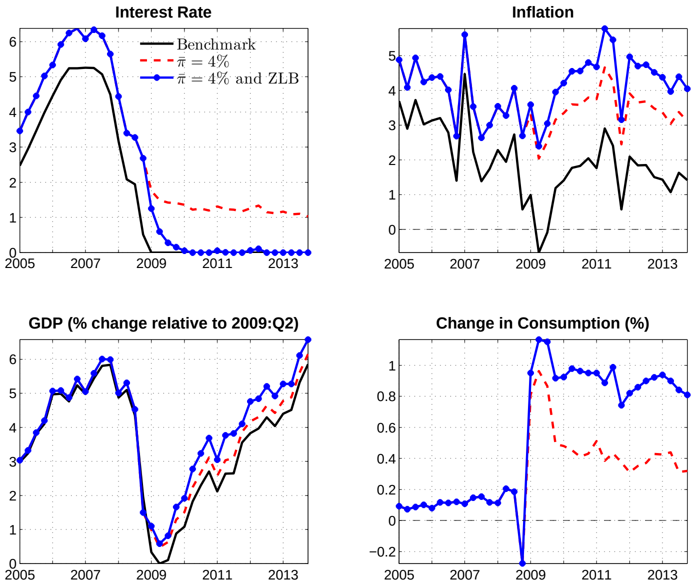
- Likelihood function:

$$
p(Y_{1:T}|\theta) = \prod_{t=1}^{T} p(y_t|Y_{1:t-1},\theta)
$$

- A filter generates a sequence of conditional distributions $s_t|Y_{1:t}$.
- Iterations:
  - Initialization at time $t-1$: $p(s_{t-1}|Y_{1:t-1},\theta)$
  - Forecasting t given $t-1$:
    1. Transition equation: $p(s_t|Y_{1:t-1},\theta) = \int p(s_t|s_{t-1},Y_{1:t-1},\theta)p(s_{t-1}|Y_{1:t-1},\theta)ds_{t-1}$
    2. Measurement equation: $p(y_t|Y_{1:t-1},\theta) = \int p(y_t|s_t,Y_{1:t-1},\theta)p(s_t|Y_{1:t-1},\theta)ds_t$
  - Updating with Bayes theorem. Once y_t becomes available:

$$
p(s_t|Y_{1:t},\theta) = p(s_t|y_t,Y_{1:t-1},\theta) = \frac{p(y_t|s_t,Y_{1:t-1},\theta)p(s_t|Y_{1:t-1},\theta)}{p(y_t|Y_{1:t-1},\theta)}
$$

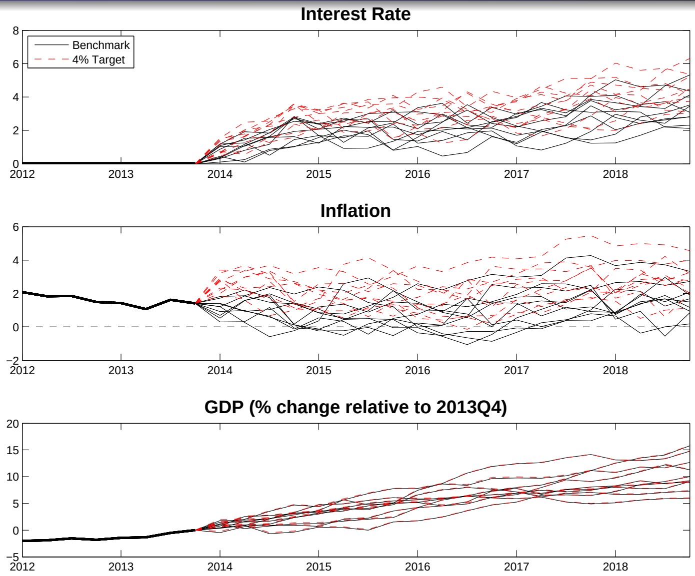
What If... the U.S. Had Targeted 4% Inflation?



What If... the U.S. Had Targeted 4% Inflation?

- Benefit: Higher target inflation rate $\longrightarrow$ ability to conduct conventional expansionary monetary policy.
- Costs:
  - Increased price adjustment costs may lead to welfare loss.
  - Other costs, e.g., holding cash balances.
- Japan: spending long time at ZLB may be unrelated to inflation target.
- From ex ante perspective, costs and benefits have to be weighted by prob of reaching ZLB.
- From *ex ante* perspective, the case for a higher inflation target is not particularly strong.

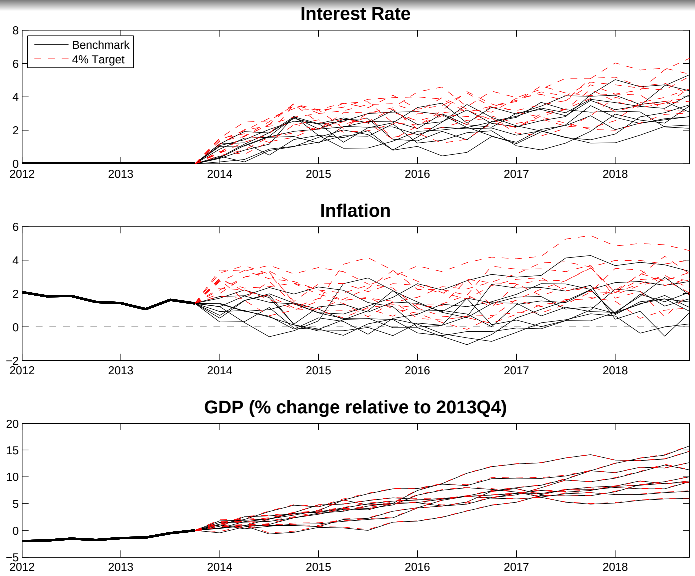
What If... the U.S. Switches to a 4% Target?



What is Behind the Plot?

- We have estimated parameter of the DSGE model. Here we fix θ at posterior mean.
- Back out historical series of shocks up until the end of 2014.
- **Counterfactual:**
  - Resolve the model with new policy parameters (here target inflation rate).
  - Starting from the historical shocks and observations at the end of 2013, simulate model forward by drawing innovations for the exogenous shock processes. Do this multiple times to generate multiple trajectories.

What If... the U.S. Switches to a 4% Target?

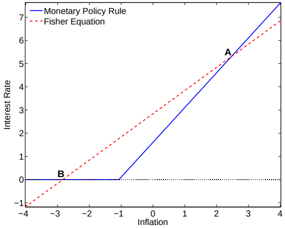


What If... the U.S. Switches to a 4% Target?

- Even if policy is credible, expected real effects of this policy change are essentially zero.
- Only positive effect would be the ability to execute unanticipated expansionary monetary policy actions in response to adverse shocks.
- Raising the target does not eliminate deflation regime.
- Potentially adverse effect on the credibility of the central bank.

Policies That Transcend the Model

- Managing expectations, e.g., through unconventional monetary policies.
- In ACS we argue that the aggressive unconventional monetary policies in the U.S., in contrast to the more measured and possibly contradictory responses of the Bank of Japan, may have prevented a switch to the deflation regime in the U.S.
- Eliminating the deflation steady state / regime: e.g., discontinuous monetary policy rule; active fiscal and passive monetary policy; fiscal authority that responds to level of nominal debt or directly to inflation, signaling that deflationary steady state is unsustainable.



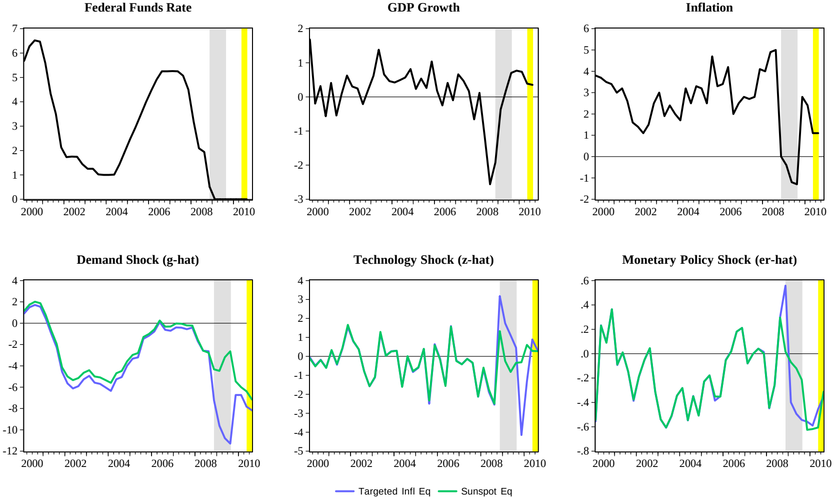
Application 4: Government Spending Multipliers

- Our model allows us to conduct basic fiscal policy experiments. We can examine the effects of an increase in government spending
- **Recall:**

$$\widehat{y}_t = \widehat{c}_t + \widehat{g}_t, \quad \widehat{g}_t = \rho_g \widehat{g}_{t-1} + \sigma_g \epsilon_{g,t}$$

- We can examine an impulse response to $\epsilon_{g,t}$.
- This analysis ignores potential distortions from raising tax revenues.

Data and Historical Shocks



Gray shading indicates deflation regime in sunspot equilibrium.
Yellow shading indicates $0.65 < Pr(s_t = 1) < 1$.

Policy Experiments

- Standing at the beginning of 2009:Q1 and taking **only** the filtered states in 2009:Q1 as given, we consider

  1. a fiscal policy intervention calibrated to ARRA;
  2. a combination of the fiscal policy intervention with an expansionary monetary policy that lasts for one year.

- Mechanics: conditional on time T states we
  - generate draws for future shocks;
  - compute paths $Y_{T+1:T+H}$, $\pi_{T+1:T+H}$, $R_{T+1:T+H}$ without policy intervention;
  - compute paths $Y^I_{T+1:T+H}$, $\pi^I_{T+1:T+H}$, $R^I_{T+1:T+H}$ with policy intervention;
  - inspect the distribution of the intervention effects:
    - $100\ln(Y^I_{T+h}/Y_{T+h})$;
    - $\pi^I_{T+h} - \pi_{T+h}$ (annualized rates);
    - $R^I_{T+h} - R_{T+h}$ (annualized rates).

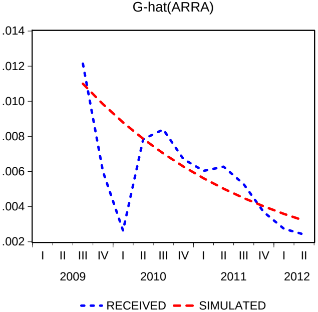
Mechanics of Intervention 1 (Pure Fiscal)

Conditional on time T states we

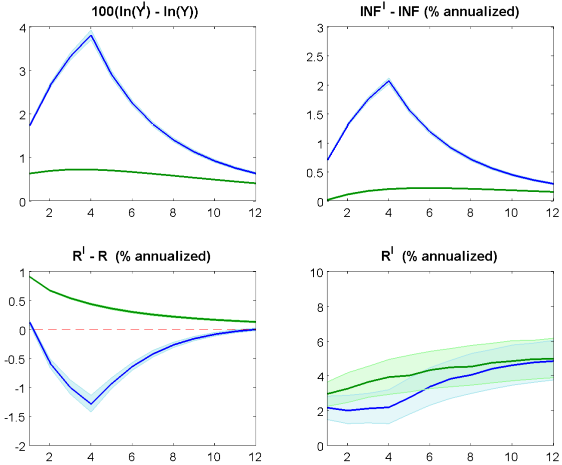
- generate draws for future shocks;
- compute paths $Y_{T+1:T+H}$, $\pi_{T+1:T+H}$, $R_{T+1:T+H}$ without policy intervention;
- compute paths $Y^I_{T+1:T+H}$, $\pi^I_{T+1:T+H}$, $R^I_{T+1:T+H}$ with policy intervention;
- **inspect the distribution of the intervention effects:**
  - $100\ln(Y^I_{T+h}/Y_{T+h})$;
  - $\pi^I_{T+h} - \pi_{T+h}$ (annualized rates);
  - $R^I_{T+h} - R_{T+h}$ (annualized rates).

Calibration of Intervention 1 (Pure Fiscal)

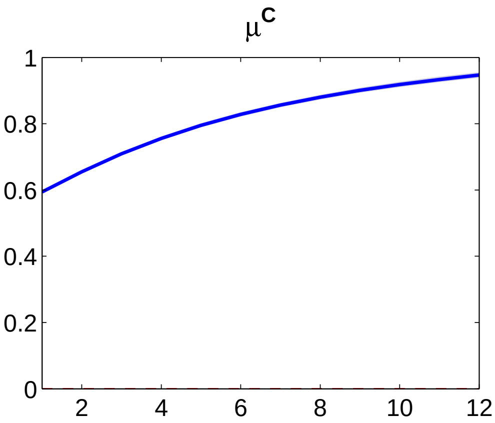
- Fiscal policy intervention is calibrated to portion of the American Recovery and Reinvestment Act (ARRA) of February 2009:
  - Tax cuts and benefits;
  - entitlement programs;
  - funding for federal contracts, grants, and loans;
- Convert expenditures into $\hat{g}_t^{ARRA}$ and construct a demand shock that generates a path comparable to $\hat{g}_t^{ARRA}$



Intervention 1: Pure Fiscal (Green Lines)



- Pointwise medians (solid); 20%-80% percentiles (shaded area) for pure fiscal intervention.



Multiplier: $\mu_t^c = \dfrac{\sum_{\tau=1}^{t}(Y_\tau^I - Y_\tau)}{\sum_{\tau=1}^{t}(G_\tau^I - G_\tau)}$;

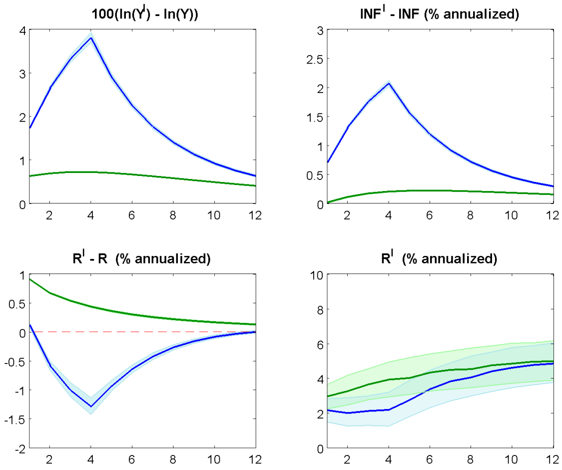
Intervention 2: Fiscal and Monetary Policy

- At the beginning of 2009:Q1 the Fed contemplates to amplify the effect of the expansionary fiscal policy by an expansionary monetary policy that keeps interest rates at or near zero.
- Recall monetary policy rule:

$$R_t = \max\left\{1, \left[r\pi_*\left(\frac{\pi_t}{\pi_*}\right)^{\psi_1}\left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_2}\right]^{1-\rho_R} R_{t-1}^{\rho_R} e^{\sigma_R \epsilon_{R,t}}\right\}.$$

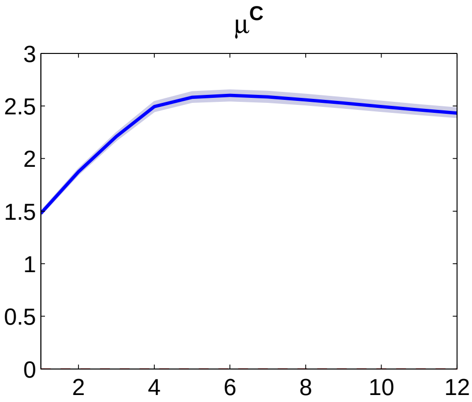
- Un-intervened paths:
  - all $\epsilon_{g,T+h}$ and $\epsilon_{z,T+h}$ are drawn from $N(0,1)$;
  - $\epsilon_{R,T+h} = 0$.
- Intervened paths:
  - $\sigma_g \epsilon_{g,T+1} \sim N(0.011, \sigma_g^2)$;
  - all other $\epsilon_{g,T+h}$ and $\epsilon_{z,T+h}$ shocks are drawn from $N(0,1)$;
  - solve for the $\tilde{\epsilon}_{R,T+1:T+4} \geq -2\sigma_r$ such that for $h = 1,2,3,4$ $R^I_{T+h}(\epsilon_{R,T+h} = 0) - R^I_{T+h}(\epsilon_{R,T+h} = \tilde{\epsilon}_{R,T+h})$ is maximized while $\leq 1\%$ (annualized).

Intervention 2: Fiscal and Monetary Policy (Blue)



- Pointwise medians (solid); 20%-80% percentiles (shaded area) for both interventions.

Intervention 2: Government Spending Multipliers



Multiplier: $\mu_t^c = \dfrac{\sum_{\tau=1}^{t}(Y_\tau^I - Y_\tau)}{\sum_{\tau=1}^{t}(G_\tau^I - G_\tau)}$;

An Alternative Policy Exercise

- Instead of standing at the beginning of 2009Q1, do an ex-post analysis.
- Use the filtered shocks for 2000Q1-2010Q2.
- **Two experiments:**
  1. Fiscal intervention only, calibrated to ARRA: reduce $\widehat{g}_t$ by the amount of ARRA spending.
  2. Both fiscal and monetary intervention $\Longrightarrow$ try to keep interest rates low.
- Intervention dates: 2009Q1 (ZLB) and 2007Q1 (interest rate around 6% p.a.)

Cumulative Government Spending Multipliers

Period	2009Q1 Both	2009Q1 Only Fiscal	2007Q1 Both	2007Q1 Only Fiscal
1	1.74	1.42	1.44	0.59
2	1.69	1.54	1.85	0.66
3	1.65	1.54	2.21	0.73
4	1.76	1.64	2.46	0.78
5	1.79	1.67	2.56	0.83
6	1.79	1.68	2.62	0.88

Application 5: Richer Fiscal Policy Specifications

- Based on Leeper, Plante, and Traum (2010, *Journal of Econometrics*). See also Herbst and Schorfheide (2015, Chapter 6).

- Authors study a model without nominal frictions and monetary policy – but the two components could be combined in a single model $\Longrightarrow$ raises some questions about interaction of fiscal and monetary policy.

Household Budget Constraint

$$(1+\tau_t^c)c_t + i_t + b_t = (1-\tau_t^l)W_t l_t + (1-\tau_t^k)R_t^k u_t k_{t-1} + R_{t-1}b_{t-1} + z_t.$$

- **Distortionary taxes:**
  - on consumption: τ_t^c
  - on labor income: τ_t^l
  - on capital income: τ_t^k
- z_t are transfers (not distortionary).
- Tax rates will vary over time.

Government Budget Constraint

$$B_t + \tau_t^k R_t^k u_t K_{t-1} + \tau_t^l w_t L_t + \tau_t^c C_t = R_{t-1} B_{t-1} + G_t + Z_t.$$

- Capital letters denote aggregate quantities.
- More sources of tax revenue than in our simple model.

Tax Rules

- Tax rates respond to state of the economy...

$$\begin{aligned}
\hat{\tau}_t^k &= \varphi_k \hat{Y}_t + \gamma_k \hat{B}_{t-1} + \phi_{kl} \hat{u}_t^l + \phi_{kc} \hat{u}_t^c + \hat{u}_t^k, \\
\hat{\tau}_t^l &= \varphi_l \hat{Y}_t + \gamma_l \hat{B}_{t-1} + \phi_{lk} \hat{u}_t^k + \phi_{lc} \hat{u}_t^c + \hat{u}_t^l, \\
\hat{\tau}_t^c &= \phi_{ck} \hat{u}_t^k + \phi_{cl} \hat{u}_t^l + \hat{u}_t^c.
\end{aligned}$$

- but they also have an exogenous component:

$$\begin{aligned}
\hat{u}_t^k &= \rho_k \hat{u}_{t-1}^k + \sigma_k \epsilon_t^k, \quad \epsilon_t^k \sim N(0,1), \\
\hat{u}_t^l &= \rho_l \hat{u}_{t-1}^l + \sigma_l \epsilon_t^l, \quad \epsilon_t^l \sim N(0,1), \\
\hat{u}_t^c &= \rho_c \hat{u}_{t-1}^c + \sigma_c \epsilon_t^c, \quad \epsilon_t^c \sim N(0,1).
\end{aligned}$$

Government Spending and Transfers

- The government spending rule is given by

$$\hat{G}_t = -\varphi_g \hat{Y}_t - \gamma_g \hat{B}_{t-1} + \hat{u}^g_t, \quad \hat{u}^g_t = \rho_g \hat{u}^g_{t-1} + \sigma_g \epsilon^g_t, \quad \epsilon^g_t \sim N(0,1).$$

- The transfer rule is given by

$$\hat{Z}_t = -\varphi_z \hat{Y}_t - \gamma_z \hat{B}_{t-1} + \hat{u}^z_t, \quad \hat{u}^z_t = \rho_z \hat{u}^z_{t-1} + \sigma_z \epsilon^z_t, \quad \epsilon^z_t \sim N(0,1).$$

Fiscal Policy Analysis

- We can proceed as in our previous analysis...
- Estimate DSGE models, in particular the policy rule parameters.
- Study effects of tax and spending policies through:
  - impulse response functions – just as we did previously for government spending, we can now study tax cuts;
  - recompute equilibrium under alternative tax and spending rules – new steady state, different dynamics, transitions.

Fiscal Policy Analysis

- **Problem** ...
- Does the data have enough variation to identify all the parameters and policy trade-offs?
- Let us estimate the model and different priors and see what happens.

Prior Distributions for Fiscal Rule Parameters

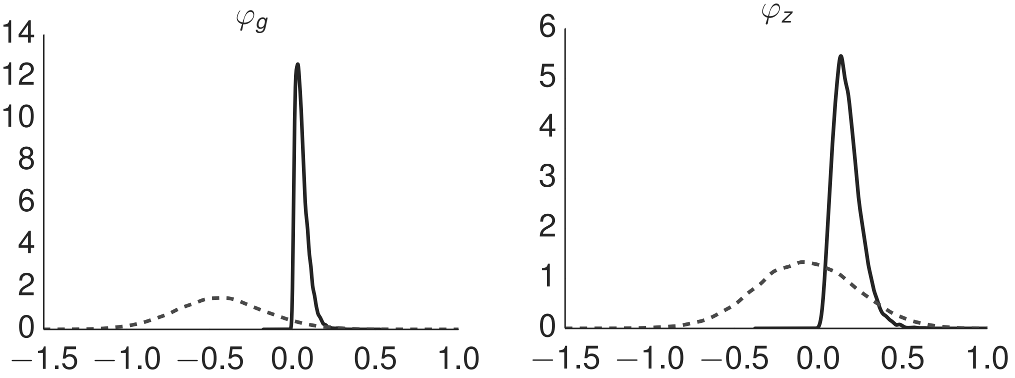
	LPT Prior			Diffuse Prior		
	Type	Para (1)	Para (2)	Type	Para (1)	Para (2)
Debt Response Parameters						
γ_g	G	0.4	0.2	U	0	5
γ_{tk}	G	0.4	0.2	U	0	5
γ_{tl}	G	0.4	0.2	U	0	5
γ_z	G	0.4	0.2	U	0	5
Output Response Parameters						
φ_{tk}	G	1.0	0.3	N	1.0	1
φ_{tl}	G	0.5	0.25	N	0.5	1
φ_g	G	0.07	0.05	N	0.07	1
φ_z	G	0.2	0.1	N	0.2	1
Exogenous Tax Comovement Parameters						
ϕ_{kl}	N	0.25	0.1	N	0.25	1
ϕ_{kc}	N	0.05	0.1	N	0.05	1
ϕ_{lc}	N	0.05	0.1	N	0.05	1

Notes: Para (1) and Para (2) correspond to the mean and standard deviation of the Beta (B), Gamma (G), and Normal (N) distributions and to the upper, lower bounds of the support for Uniform (U) distribution.

Posterior Moments

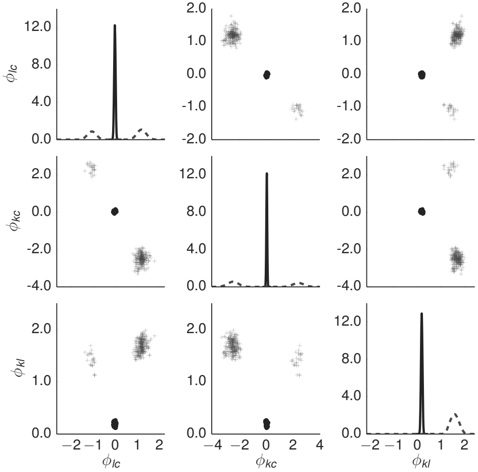
	Based on LPT Prior		Based on Diff. Prior	
	Mean	[5%, 95%] Int.	Mean	[5%, 95%] Int.
Debt Response Parameters				
γ_g	0.16	[0.07, 0.27]	0.10	[0.01, 0.23]
γ_{tk}	0.39	[0.22, 0.60]	0.38	[0.16, 0.62]
γ_{tl}	0.11	[0.04, 0.21]	0.04	[0.00, 0.11]
γ_z	0.32	[0.17, 0.47]	0.32	[0.14, 0.49]
Output Response Parameters				
φ_{tk}	1.67	[1.18, 2.18]	2.06	[1.44, 2.69]
φ_{tl}	0.29	[0.11, 0.53]	0.11	[-0.34, 0.58]
φ_g	0.06	[0.01, 0.13]	-0.43	[-0.87, 0.02]
φ_z	0.17	[0.06, 0.33]	-0.07	[-0.56, 0.41]
Exogenous Tax Comovement Parameters				
ϕ_{kl}	0.19	[0.14, 0.24]	1.57	[1.29, 1.87]
ϕ_{kc}	0.03	[-0.03, 0.08]	-0.33	[-2.84, 2.73]
ϕ_{lc}	-0.02	[-0.07, 0.04]	0.20	[-1.23, 1.40]
Innovations to Fiscal Rules				
σ_g	3.03	[2.79, 3.30]	2.91	[2.66, 3.19]
σ_{tk}	4.36	[4.01, 4.75]	1.26	[1.08, 1.46]
σ_{tl}	2.95	[2.71, 3.22]	2.00	[1.71, 2.33]
σ_{tc}	3.99	[3.67, 4.33]	1.14	[0.96, 1.35]
σ_z	3.34	[3.07, 3.63]	3.34	[3.07, 3.63]

Posterior of Response of Spending and Transfers to Output



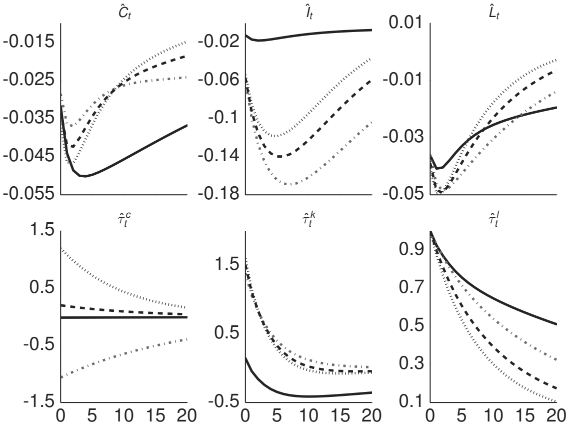
Notes: The figure depicts posterior densities under the LPT prior (solid) and the diffuse prior (dashed).

Posterior of Tax Comovement Parameters



Notes: The plots on the diagonal depict posterior densities under the LPT prior (solid) and the diffuse prior (dashed). The plots on the off-diagonals depict draws from the posterior distribution under the LPT prior (circles) and the diffuse prior (triangles).

Impulse Response to a Labor Tax Innovation



Notes: Figure depicts posterior mean impulse responses under LPT prior (solid); diffuse prior (dashed); diffuse prior with $\phi_{lc} > 0$, $\phi_{kl} < 0$ (dotted); and diffuse prior with $\phi_{lc} < 0$, $\phi_{kl} > 0$ (dots and short dashes). $\hat{C}_t$, $\hat{I}_t$ and $\hat{L}_t$ are consumption, investment, and hours worked in deviation from steady state.

Lesson from Application 5

- It is tempting to specify very rich DSGE models to answer complex policy questions...
- but in large models it is often difficult to identify key parameters and policy trade-offs,
- which may deliver misleading conclusions.

Summary

1. A small-scale DSGE model: specification, steady states, log-linearization, first-order approximation to equilibrium dynamics, state-space representation.
2. Given θ, compute autocovariance, impulse responses, etc. from DSGE model solution; compute the same objects from the data either directly or with VARs.
3. Statistical inference: frequentist versus Bayesian; use the Kalman filter to evaluate likelihood function.
4. Frequentist inference: maximum likelihood, simulated minimum distance approaches, GMM
5. Bayesian inference: priors, posteriors, Metropolis-Hastings algorithm, post-processing draws.
6. Applications to monetary and fiscal policy.