DSGE Model Econometrics

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Cowles Lunch

Papers and software available at **https://web.sas.upenn.edu/schorf/**

A Potted History

1993-1994: Leeper and Sims "Toward a Modern Macroeconomic Model Usable for Policy Analysis" – Bayesian interpretation of likelihood function.

1996-2000: Work by DeJong, Ingram, Whiteman, Otrok (Iowa); Geweke, Landon-Lane (Minnesota); myself (Yale) – Bayesian estimation and model evaluation; posterior simulation.

2000-2003: Christiano, Eichenbaum, and Evans; Smets and Wouters – Bayesian estimation of medium-scale DSGE models.

Early-mid 2000s: Incorporation of Bayesian estimation tools into DYNARE.

Mid 2000s: Central banks (Riksbank in particular) started to use / develop / take seriously DSGE models.

Mid 2000s: Fernandez-Villaverde and Rubio-Ramirez – likelihood-based estimation of *nonlinear* DSGE models.

Subsequently: Widely-used in academia and policy-making institutions.

Not Everybody Is Enthusiastic About DSGE Models

ECONOMICS

Economics Struggles to Cope With Reality

174 JUNE 10, 2016 8:00 AM EDT

By Noah Smith

(...) most people outside the discipline who take one look at these models immediately think they're kind of a joke.

They contain so many unrealistic assumptions that they probably have little chance of capturing reality. Their forecasting performance is abysmal.

Some of their core elements are clearly broken. Any rigorous statistical tests tend to reject these models instantly, because they always include a hefty dose of fantasy.

Not Everybody Is Enthusiastic About DSGE Models

The Trouble With Macroeconomics

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Wednesday 14th September, 2016

(...) To replicate the results from that model (the Smets and Wouters 2007 model), I read the User's Guide for the software package, DYNARE, that the authors used. In listing the advantages of the Bayesian approach, the User's Guide says:
"Third, the inclusion of priors also helps identifying parameters."
This was a revelation. Being a Bayesian means that your software never barfs.
(...) It was news to me that priors are vectors for facts with unknown truth values (FWUTV), but once I understood this and started reading carefully, I realized it was an open secret among econometricians.

"Are you just pissing and moaning, or can you verify what you're saying with data?"

Four Steps

1. Model Solution
2. Model Estimation
3. Model Assessment
4. Substantive Analysis with Estimated Models

Step 1 – Model Solution

Step 1: Model Solution

- Log-linearization of equilibrium conditions: (-) linear, (+) fast, (+) leads to linear state-space model, (+) likelihood is easy to compute.
- Higher-order perturbation solution: (0) a bit slower but numerically stable, (+) *smooth* nonlinear dynamics, good for welfare analysis, (-) likelihood evaluation requires nonlinear filter.
- Global / projection methods: (+) approximate decision rules by flexible fcn $\delta(\mathcal{S}_t; \Theta)$, (+) can handle occasionally-binding constraints, (-) time-consuming, (-) delicate, (-) requires carefully chosen grid, (-) requires nonlinear filter to evaluate likelihood.

Nonlinear Model Solution

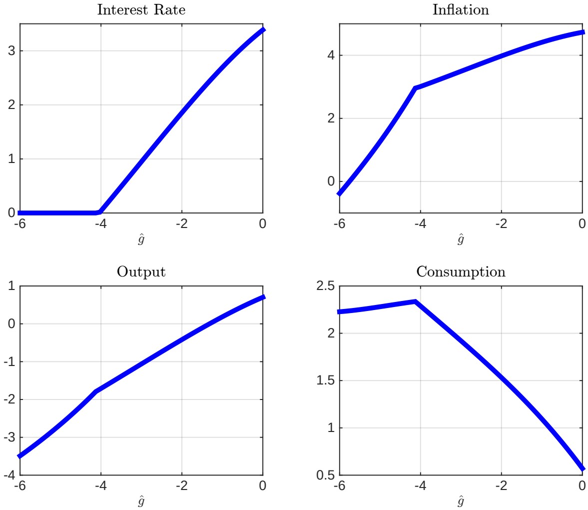
Reference: B. Aruoba, P. Cuba-Borda, and F. Schorfheide (2017): "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries," *Review of Economic Studies*, forthcoming.

- Perturbation solutions capture some nonlinearities but not all $\longrightarrow$ not well suited for occasionally-binding constraints.
- Example: ZLB/ELB for nominal interest rates

$$R_t = \max\left\{1,\ R_t^* e^{\epsilon_{R,t}}\right\}, \quad R_t^* = \left[r\pi_* \left(\frac{\pi_t}{\pi_*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R}.$$

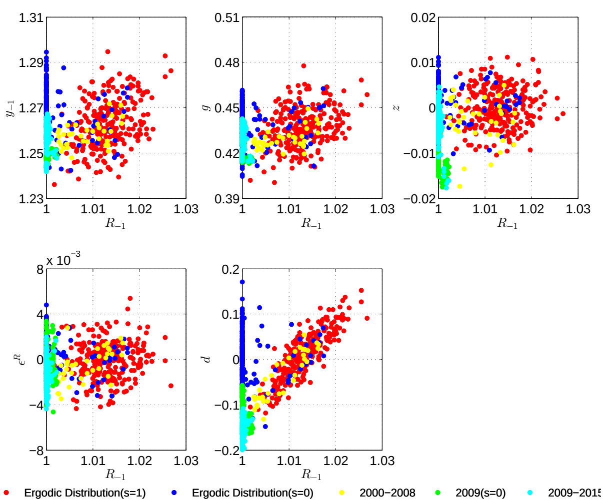
- Three Challenges:
  1. capture "kinks" in decision rules;
  2. solution needs to be accurate in region of state-space that is relevant according to model AND according to data;
  3. multiple equilibria.

Challenge 1: Kinks... Sample Decision Rules - Small-Scale NK Model



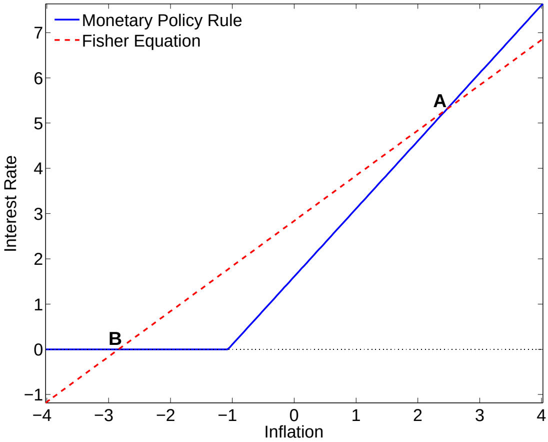
Challenge 2 – Accuracy Where it Matters

Choose Θ to minimize sum squared residuals from the (intertemporal) equilibrium conditions over particular grid of points in state space



Challenge 3 – Multiple Equilibria in NK Models with ZLB Constraint

In a NK model with passive fiscal policy...



Step 2a – Model Estimation
Likelihood Evaluation

- Implemented by sampling draws θ^i from posterior:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}.$$

- Posterior samplers require evaluation of likelihood function: $\theta \longrightarrow$ model solution $\longrightarrow$ state-space representation $\longrightarrow p(Y|\theta)$.

- State-space representation $\longrightarrow p(Y,S|\theta)$:

$$\begin{aligned} y_t &= \Psi(s_t, t; \theta) + u_t, \quad u_t \sim F_u(\cdot;\theta) \\ s_t &= \Phi(s_{t-1}, \epsilon_t; \theta), \quad \epsilon_t \sim F_\epsilon(\cdot;\theta). \end{aligned}$$

- In order to obtain $p(Y|\theta) = \prod_{t=1}^T p(y_t|Y_{1:t-1},\theta)$ we need to integrate out latent states S from $p(Y,S|\theta)$ $\longrightarrow$ use filter:
  - Initialization: $p(s_{t-1}|Y_{1:t-1},\theta)$
  - Forecasting: $p(s_t|Y_{1:t-1},\theta)$, $p(y_t|Y_{t-1},\theta)$
  - Updating: $p(s_t|y_t, Y_{1:t-1},\theta) = p(s_t|Y_{1:t},\theta)$.

Particle Filtering

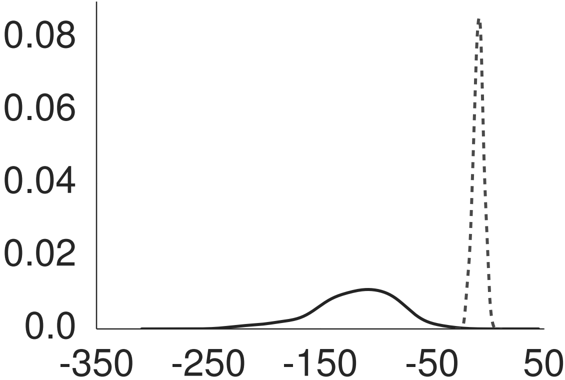
- Particle Filtering: represent $p(s_{t-1}|Y_{1:t-1})$ by $\{s^j_{t-1}, W^j_{t-1}\}_{j=1}^M$ such that

$$\frac{1}{M}\sum_{j=1}^{M} h(s^j_{t-1})W^j_{t-1} \approx \int h(s_{t-1})p(s_{t-1}|Y_{1:t-1})ds_{t-1}.$$

- Example: Bootstrap particle filter
  - Mutation/Forecasting: turn s^j_{t-1} into $\tilde{s}^j_t$: sample $\tilde{s}^j_t \sim p(s_t|s^j_{t-1})$.
  - Correction/Updating: change particle weights to: $\tilde{W}^j_t \propto p(y_t|\tilde{s}^j_t)W^j_{t-1}$.
  - Selection (Optional): Resample to turn $\{\tilde{s}^j_t, \tilde{W}^j_t\}_{j=1}^M$ into $\{s^j_t, W^j_t = 1\}_{j=1}^M$.
- Problem: naive forward simulation of Bootstrap PF leads to uneven particle weights
  $\longrightarrow$ inaccurate likelihood approximation!

Smets-Wouters Model (Linearized)

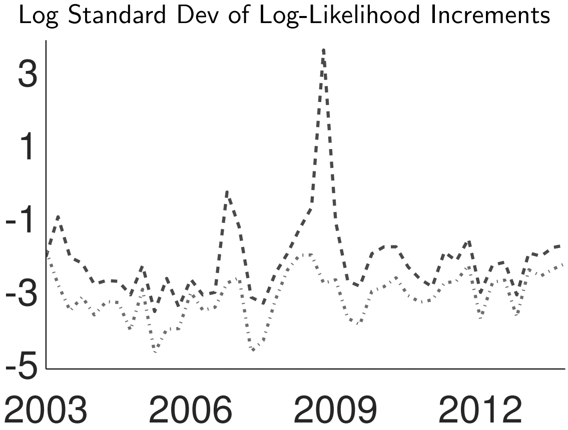
Bootstrap PF ($M = 400,000$) versus Cond. Optimal PF ($M = 4,000$)



Density estimates of $\hat{\Delta}_1 = \ln \hat{p}(Y|\theta) - \ln p(Y|\theta)$ based on $N_{run} = 100$.

Source: Herbst and Schorfheide (2015), *Bayesian Estimation of DSGE Models*, Princeton University Press.

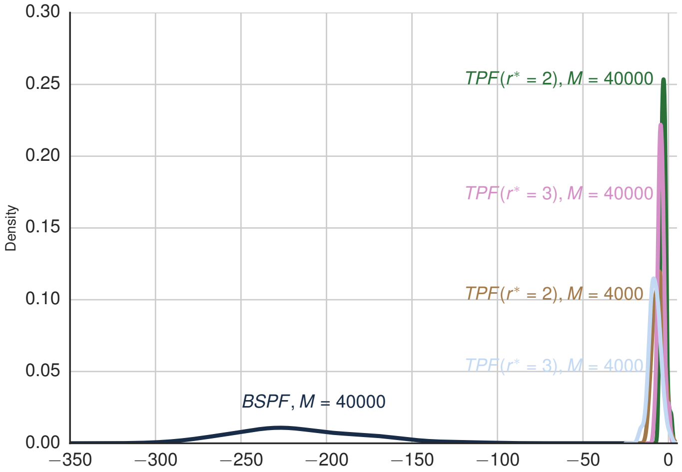
Small-Scale NK DSGE Model (Linearized)



Bootstrap PF ($M = 40,000$) is dashed; Cond-opt. PF ($M = 400$) is dotted.

Source: Herbst and Schorfheide (2015): *Bayesian Estimation of DSGE Models*, Princeton University Press.

Tempered Particle Filtering – Great Recession Sample



Source: Herbst and Schorfheide (2017): "Tempered Particle Filtering," *Manuscript*.

Step 2b – Model Estimation Posterior Inference

Identification

- We are trying to learn the parameters θ from the data.
- Formal definitions... e.g., model is identified at θ_0 if $p(Y|\theta) = p(Y|\theta_0)$ implies that $\theta = \theta_0$.
- In the early DSGE days, lack of identification did not seem an issue.
- Over time, it emerged as an important problem.
- Without identification or with weak identfication:
  - use more/different data to achieve identification;
  - use identification-robust inference procedures.
- Lack of identification does not raise conceptual issues for Bayesian inference (as long as priors are proper), but possibly computational challenges.

Lack of Identification: Two Examples

1. Monetary policy rule coefficients

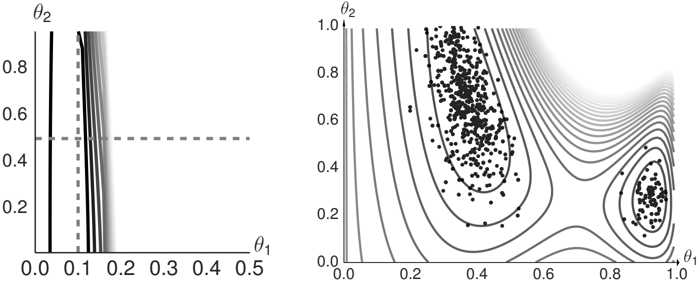
$$\hat{R}_t = \psi \hat{\pi}_t + \epsilon_{R,t}.$$

2. Distinguishing internal propagation (e.g., partial indexation of prices to past inflation) from external propagation (e.g., persistent price mark-up shocks)

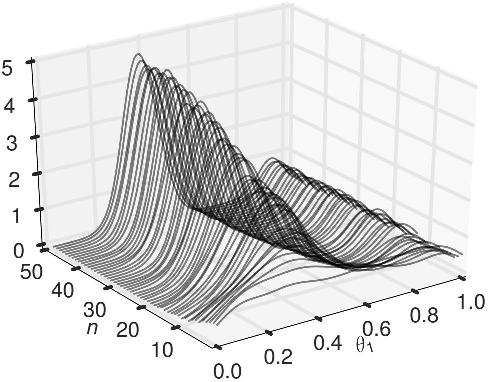
The Role of Priors

- Ideally: probabilistic representation of our knowledge/beliefs before observing sample Y.
- More realistically: choice of prior as well as model are influenced by some observations. Try to keep influence small or adjust measures of uncertainty.
- DSGE model literature: use priors to incorporate information from sources other than estimation sample. Useful to group parameters:
  1. steady state related;
  2. endogenous propagation;
  3. exogenous shock.
- In other literatures:
  1. keep them "uninformative" (???) so that posterior inherits shape of likelihood function;
  2. use them to regularize the likelihood function;

Lack of Identification as Computational Challenge



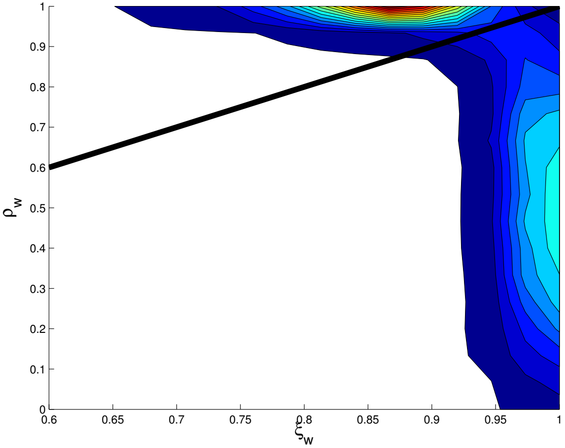
Source: Herbst and Schorfheide (2015): *Bayesian Estimation of DSGE Models*, Princeton University Press.



$$\pi_n(\theta) = \frac{[p(Y|\theta)]^{\phi_n} p(\theta)}{\int [p(Y|\theta)]^{\phi_n} p(\theta) d\theta}, \quad \phi_n = \left(\frac{n}{N_\phi}\right)^\lambda$$

Source: Herbst and Schorfheide (2015): *Bayesian Estimation of DSGE Models*, Princeton University Press.

Smets-Wouters (Diffuse Prior) Posterior: Internal ξ_w versus External ρ_w Propagation



Source: Herbst and Schorfheide (2014), "Sequential Monte Carlo Sampling for DSGE Models," *Journal of Applied Econometrics*

Putting it All Together

- Once a reasonably accurate likelihood approximation has been obtained, it can be embedded in a posterior sampler.
- The Full Monty is a real pain: see Gust, C., E. Herbst, D. Lopez-Salido, and M. E. Smith (2017): "The Empirical Implications of the Interest-Rate Lower Bound," *American Economic Review*, forthcoming.
- Potential shortcuts:
  - less accurate model solution;
  - cruder state extraction / likelihood approximation;
  - non-likelihood-based parameterization of model.
- Schorfheide, Song, Yaron (2017): slight short-cut in model solution $\longrightarrow$ conditionally-linear state-space representation $\longrightarrow$ efficient particle filter approximation of likelihood $\longrightarrow$ full Bayesian estimation.

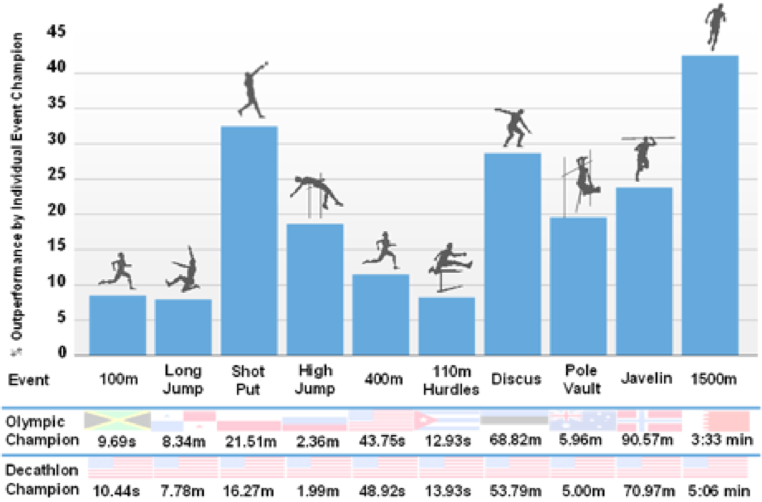
Step III – Model Assessment

Evaluating DSGE Models – A Potted History

- 1980s: can DSGE model reproduce key sample correlations, e.g. between output and hours worked or output and inflation? Compare model-implied correlations and sample correlations computed from actual data.
- 1990s: do impulse responses to, say, unanticipated changes in monetary policy, from a DSGE model look like impulse responses from a vector autoregression (VAR)?
- 2000's: can DSGE models track and forecast key macroeconomic time series?
- The literature has developed numerous econometric tools to provide formalize the evaluation.

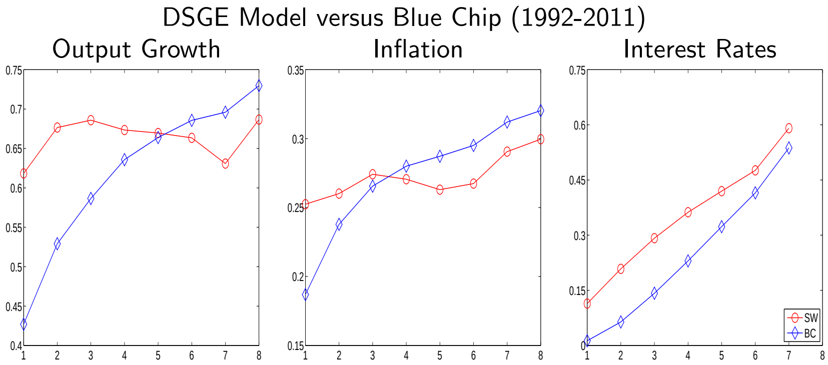
Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016): "Solution and Estimation of DSGE Models," *Handbook of Macroeconomics*, Vol 2., Elsevier

How to think about DSGE models...



Source: BBC, Beijing 2008 Olympic Games, men's event.

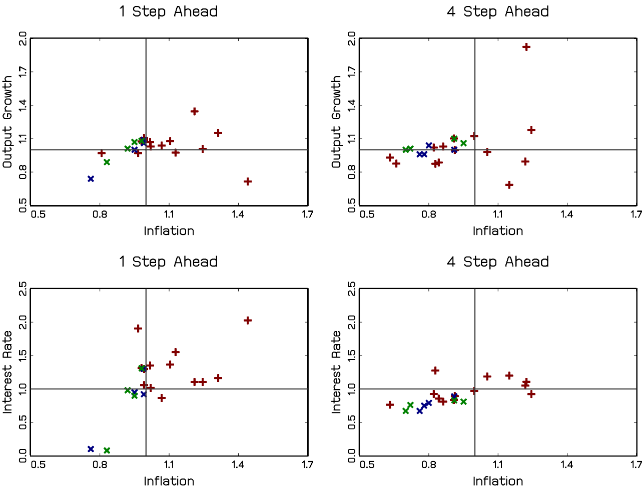
Abysmal Forecasting Performance?



- $h = 1$ is current quarter nowcast.
- Growth rates, inflation rates, interest rates are QoQ %

Source: Del Negro and Schorfheide (2013): "DSGE Model-Based Forecasting," In *Handbook of Economic Forecasting*.

Abysmal Forecasting Performance?



RMSE ratios: DSGE / AR(2)

Source: Del Negro and Schorfheide (2013): "DSGE Model-Based Forecasting," In *Handbook of Economic Forecasting*.

Liberty Street Economics

SEPTEMBER 08, 2017

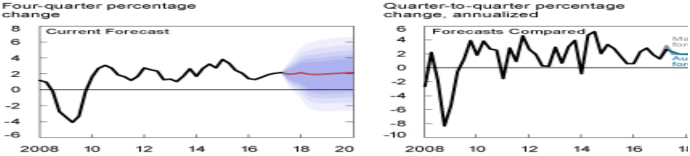
The New York Fed DSGE Model Forecast—August 2017

Michael Cai, Marc Giannoni, Abhi Gupta, Pearl Li, and Argia Sbordone

Model Forecast

	2017		2018		2019		202
	Aug	May	Aug	May	Aug	May	Aug
GDP growth (Q4/Q4)	2.0	1.9	1.9	2.1	2.1	2.2	2.2
Core PCE inflation (Q4/Q4)	1.3	1.5	1.3	1.5	1.5	1.6	1.6
Real natural rate of interest (Q4)	0.5	0.7	0.8	1.0	1.0	1.1	1.2

Forecasts of Output Growth



"New" Models versus "Old" Models

- Macroeconomists/econometricians have been criticized for relying on models that abstract from financial intermediation / frictions.
- With hindsight it turned out that financial frictions were important to understand the Great Recession. But are they also important in normal times?
- We need tools that tell us in real-time when to switch models...
- Linear prediction pool:

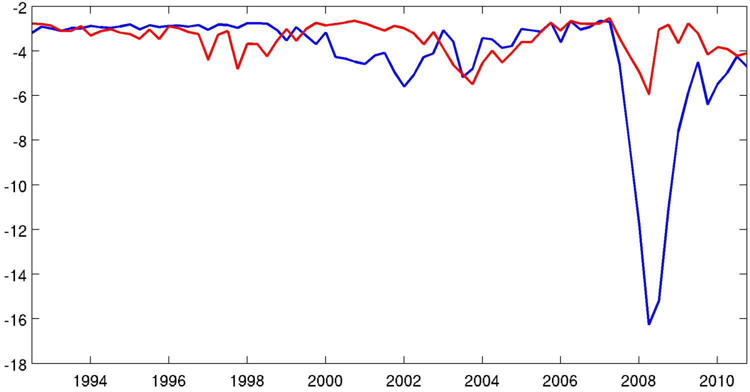
$$\begin{aligned}&\text{Density Forecast}_t\\ &\quad = \lambda_t \cdot \text{Forecast from "Normal" Model}_t\\ &\qquad +(1-\lambda_t) \cdot \text{Forecast from "Fin Frictions" Model}_t\end{aligned}$$

- Determine weight λ_t in real time based on historical forecast performance.

Source: Del Negro, Hasegawa, Schorfheide (2016): "Dynamic Prediction Pools: An Investigation of Financial Frictions and Forecasting Performance," *Journal of Econometrics*.

Relative forecasting performance changes over time

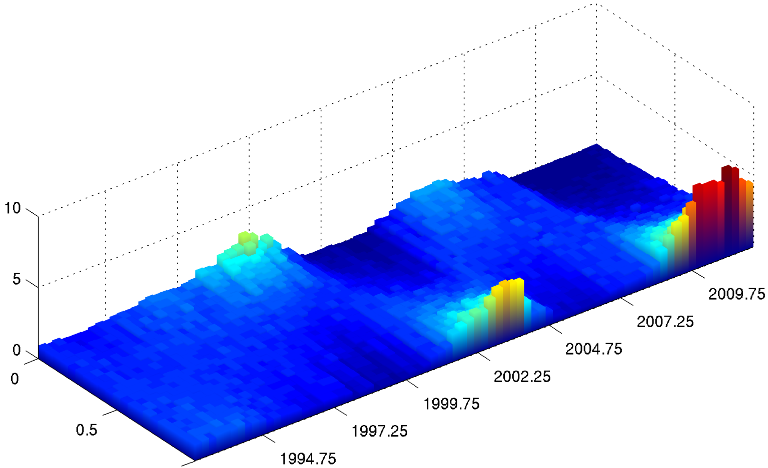
“Old” Smets-Wouters Model vs. “New” DSGE with Financial Frictions



It’s easy to see with hindsight which model we should have used.

"New" Models versus "Old" Models

Time-Varying Weight λ_t (Posterior Distribution) on "New" DSGE with Financial Frictions

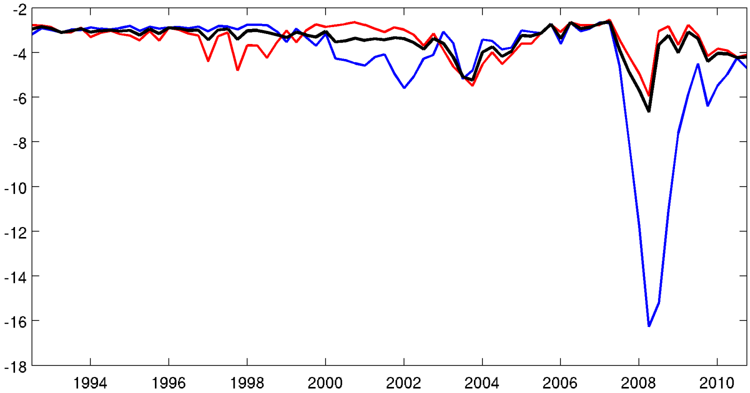


It's more difficult to determine the best model in real time...

"New" Models versus "Old" Models

"Old" Smets-Wouters Model vs. "New" DSGE with Financial Frictions

vs. Dynamic Prediction Pool with Real-Time Weights

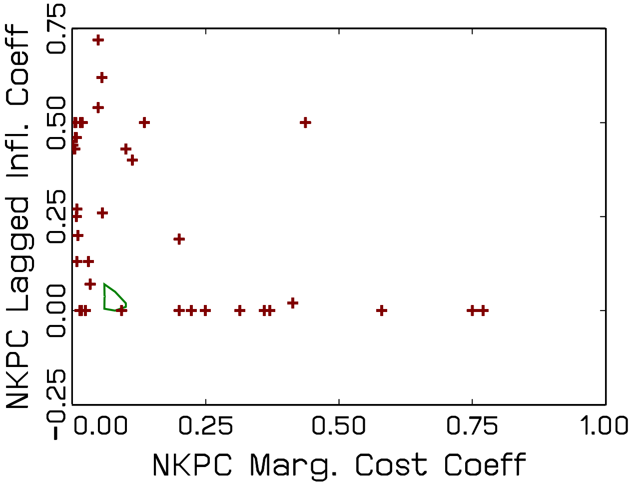


Techniques for determining the best model in real time are available.

Step IV – Substantive Analysis with Estimated Model

NK Phillips Curve

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f \mathbb{E}_t[\tilde{\pi}_{t+1}] + \kappa \widetilde{MC}_t$$



Conclusions

- Literature on methods and applications for DSGE models is well and alive!
- Significant progress in area of model solution and estimation techniques.
- More work needed on the model assessment:
  - Do DSGE models generate the right nonlinearities?
  - Do DSGE models capture the interaction between cross-sectional distributions and macroeconomic aggregates?