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These are review questions. Some are difficult, others are fairly straightforward. You can find most of the answers to these questions in the slides and the Herbst-Schorfheide book. We covered most, but not all of the material in class. Enjoy!

DSGE Models:

Q1. What are the key elements of a New Keynesian DSGE model?

Q2. What generates stochastic fluctuations in a DSGE model?

Q3. What are the key steps to solve a DSGE model with a log-linear approximation?

Q4. What is the difference between a linearization and a log-linearization? Why are DSGE models often log-linearized instead of linearized?

Q5. How does one compute the steady state of a DSGE model?

Q6. Consider the rational expectation difference equation

$$y_t = \frac{1}{\theta}\mathbb{E}_t[y_{t+1}] + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2),$$

where $\theta \in [0, 2]$.

(i) Why does the literature focus on non-explosive solutions of this difference equation?

(ii) What do we mean by *determinacy* and *indeterminacy*?

(iii) Characterize the non-explosive solutions of the above rational expectations equation. Distinguish between the case $\theta < 1$ and $\theta > 1$.

(iv) What mechanism can generate indeterminacy in New Keynesian DSGE models?

Q8. What is the state-space representation of a DSGE model?

Q9. Why is the Kalman filter needed to compute the likelihood function of a log-linearized DSGE model?

Q10. What are the key steps in a filtering algorithm? Characterize the various conditional distributions of states and observations that are computed by the filter.

Q11. How would you elicit a prior distribution for DSGE model parameters?

Introduction to Bayesian Inference:

Q1. Suppose y_t is given by the AR(1) process

$$y_t = \phi_0 + \phi_1 y_{t-1} + u_t.$$

Derive the following formulas:

- Mean: $\mathbb{E}[y_t] = \phi_0/(1-\phi_1)$.
- Variance: $\mathbb{V}[y_t] = \gamma(0) = \sigma^2/(1-\phi_1^2)$.
- Autocovariance: $\gamma(h) = \phi_1^h \gamma(0)$.
- Autocorrelation: $\rho(h) = \phi_1^h$.

Q2. Suppose you are forecasting with an AR(1) model. What are the two sources of uncertainty that you should account for if you want to construct an interval prediction for, say, y_{T+h}?

Q3. Consider the following AR(1) process, initialized in the infinite past:

$$y_t = \phi y_{t-1} + u_t, \tag{1}$$

where $u_t \sim iidN(0,1)$.

(i) Suppose you have a sample of observations $Y_{1:T} = \{y_0, y_1, \ldots, y_T\}$. Derive the conditional likelihood function $p(Y_{1:T}|\phi, y_0)$ for θ.

(ii) Consider the following prior for ϕ: $\phi|y_0 \sim N(0,\tau^2)$. Show that the posterior distribution of ϕ is of the form

$$\phi|Y_{0:T} \sim N(\bar{\phi}_T, \bar{V}_T), \tag{2}$$

and provide expressions for $\bar{\phi}_T$ and $\bar{V}_T$.

Q4. Suppose that θ is a scalar parameter. We want to construct a point estimator. Let $p(\theta|Y)$ be he posterior density and $(\delta,\theta) = (\delta-\theta)^2$ be the quadratic loss function under which the point estimator δ is evaluated. Show that the posterior mean is the optimal point estimator under this loss function as it minimizes the posterior expected loss.

Q5. Suppose hat $y = \theta + u$. The prior distribution is $\theta \sim N(0,\tau^2)$ and $u \sim N(0,1)$. Here the sample size is $T = 1$. Derive the marginal data density

$$p(y) = \int p(y|\theta)p(\theta)d\theta.$$

Q6. Consider the two (nested) models:

$$\begin{aligned} \mathcal{M}_1: &\quad y = u, \quad u \sim N(0,1) \\ \mathcal{M}_2: &\quad y = \theta + u, \quad u \sim N(0,1), \quad \theta \sim N(0,\tau^2). \end{aligned}$$

Assign prior probability λ to $\mathcal{M}_1$. Derive a formula for the posterior odds of $\mathcal{M}_1$ versus $\mathcal{M}_2$. What is Bayesian model averaging?

MCMC

Q1. Why is an algorithm such as the Metropolis-Hastings algorithm needed to sample from the posterior distribution of a DSGE model?

Q2. Describe the steps of the Metropolis-Hastings algorithm.

Q3. Does the MH algorithm generate a sequence of independent draws θ^i from the posterior distribution $p(\theta|Y)$?

Q4. Suppose $\theta = [\theta_1, \theta_2]'$ and you have used the MH algorithm to generate a sequence of draws θ^i, $i = 1, \ldots, N$. How do you approximate the mean and the standard deviation of the marginal posterior $p(\theta_1|Y)$? How would you approximate the 5th and 95th percentile of the posterior distribution?

Q5. Why do practitioners often drop the first 20-50% of draws generated by an MH algorithm?

Q6. Derive the transition kernel K on Page 57 of Herbst and Schorfheide for the discrete example. How does the K change if $\pi_1 > \pi_2$?

Q7. Derive the eigenvalues of K.

Q8. Show that $\pi' K = \pi'$, where $\pi' = [\pi_1, \pi_2]$. Interpret this equation.

Q9. TRUE or FALSE. The more persistent the Markov chain, the more accurate the Monte Carlo approximation of posterior moments computed from the output of this chain. Explain your answer.

Q10. What is a *random walk* MH algorithm?

Q11. Suppose you are using the MH algorithm to approximate the posterior distribution of DSGE model parameters. Describe how you could/would choose the proposal density $q(\cdot)$. What is the rationale for this choice?

Q12. Suppose you find that the acceptance rate of your MH algorithm is close to zero. Would you increase or decrease the variance of the proposal distribution $q(\cdot)$ to raise the acceptance rate? Explain your reasoning.

Q13. After running the MH algorithm, how would you compute the posterior mean of an impulse response function to, say, a monetary policy shock in a New Keynesian DSGE model?

Q14. How are marginal likelihoods (also often called marginal data density) used to update model probabilities?

Q15. How can you numerically approximate the marginal likelihood of a DSGE model based on the output of a posterior sampler, such as the random walk MH algorithm?

Sequential Monte Carlo

Q1. What are the three steps in each iteration of a SMC algorithm? Why is the selection step needed? Why is the mutation step needed?

Q2. What is the difference between likelihood tempering and data tempering? What are the advantages and disadvantages of the two tempering schemes?

Q3. Does the the tuning parameter λ affect the number of required likelihood evaluations?

Q4. What would be the advantage of executing multiple Metropolis-Hastings steps during the mutation phase?

Q5. In the book we recommend to initialize the SMC with draws from the prior distribution. What are potential advantages and disadvantages of this initialization?

Q6. Provide an outline for a recursive proof that shows that the SMC approximation $\bar{h}_N$ converges almost surely to $\mathbb{E}_\pi[h]$.

Particle Filters

Q1. What is the difference between the bootstrap particle filter and the conditionally-optimal particle filter?

Q2. Can the conditionally-optimal particle filter be implemented in a nonlinear DSGE model?

Q3. Why is the resampling step needed in the particle filter?

Q4. TRUE or FALSE: the smaller the measurement error in the state-space representation, the more accurate the particle filter approximation. Explain.

Q5. How do outliers in your sample affect the accuracy of particle filter approximations?

Q6. Is the particle filter approximation of the likelihood function unbiased?