Bayesian Estimation of DSGE Models[1]
Chapter 1: DSGE Modeling

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[1]The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Board of Governors or the Federal Reserve System.

Small-Scale DSGE Model

- Textbook treatments: Woodford (2003), Gali (2008)
- Intermediate and final goods producers
- Households
- Monetary and fiscal policy
- Exogenous processes
- Equilibrium Relationships

Final Goods Producers

- Perfectly competitive firms combine a continuum of intermediate goods:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}.$$

- Firms take input prices $P_t(j)$ and output prices P_t as given; maximize profits

$$\Pi_t = P_t \left(\int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}} - \int_0^1 P_t(j) Y_t(j) dj.$$

- Demand for intermediate good j:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t.$$

- Zero-profit condition implies

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}.$$

Intermediate Goods Producers

- Intermediate good j is produced by a monopolist according to:

$$Y_t(j) = A_t N_t(j).$$

- Nominal price stickiness via quadratic price adjustment costs

$$AC_t(j) = \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 Y_t(j).$$

- Firm j chooses its labor input $N_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits:

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s Q_{t+s|t}\left(\frac{P_{t+s}(j)}{P_{t+s}}Y_{t+s}(j) - W_{t+s}N_{t+s}(j) - AC_{t+s}(j)\right)\right].$$

Households

- Household derives disutility from hours worked H_t and maximizes

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s\left(\frac{(C_{t+s}/A_{t+s})^{1-\tau}-1}{1-\tau}\right.\right.$$
$$\left.\left.+\chi_M \ln\left(\frac{M_{t+s}}{P_{t+s}}\right)-\chi_H H_{t+s}\right)\right].$$

- Budget constraint:

$$\begin{aligned} & P_t C_t + B_t + M_t + T_t \\ & \quad = \quad P_t W_t H_t + R_{t-1}B_{t-1} + M_{t-1} + P_t D_t + P_t SC_t. \end{aligned}$$

Monetary and Fiscal Policy

- Central bank adjusts money supply to attain desired interest rate.
- Monetary policy rule:

$$
\begin{aligned}
R_t &= R_t^{*\,1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}} \\
R_t^* &= r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{Y_t}{Y_t^*}\right)^{\psi_2}
\end{aligned}
$$

- Fiscal authority consumes fraction of aggregate output: $G_t = \zeta_t Y_t$.
- Government budget constraint:

$$
P_t G_t + R_{t-1} B_{t-1} + M_{t-1} = T_t + B_t + M_t.
$$

Exogenous Processes

- Technology:

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}.$$

- Government spending / aggregate demand: define $g_t = 1/(1-\zeta_t)$; assume

$$\ln g_t = (1-\rho_g) \ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t}.$$

- Monetary policy shock $\epsilon_{R,t}$ is assumed to be serially uncorrelated.

Equilibrium Conditions

- Consider the symmetric equilibrium in which all intermediate goods producing firms make identical choices; omit j subscript.
- Market clearing:

$$Y_t = C_t + G_t + AC_t \quad \text{and} \quad H_t = N_t.$$

- Complete markets:

$$Q_{t+s|t} = (C_{t+s}/C_t)^{-\tau}(A_t/A_{t+s})^{1-\tau}.$$

- Consumption Euler equation and New Keynesian Phillips curve:

$$\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{A_t}{A_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \\
1 &= \phi(\pi_t - \pi) \left[\left(1 - \frac{1}{2\nu} \right) \pi_t + \frac{\pi}{2\nu} \right] \\
&\quad - \phi\beta \mathbb{E}_t \left[\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t} \right)^{-\tau} \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} (\pi_{t+1} - \pi)\pi_{t+1} \right] \\
&\quad + \frac{1}{\nu} \left[1 - \left(\frac{C_t}{A_t} \right)^{\tau} \right].
\end{aligned}$$

Equilibrium Conditions – Continued

- In the absence of nominal rigidities ($\phi = 0$) aggregate output is given by

  $$Y_t^* = (1-\nu)^{1/\tau} A_t g_t,$$

  which is the target level of output that appears in the monetary policy rule.

Steady State

- Set $\epsilon_{R,t}$, $\epsilon_{g,t}$, and $\epsilon_{z,t}$ to zero at all times.
- Because technology $\ln A_t$ evolves according to a random walk with drift $\ln \gamma$, consumption and output need to be detrended for a steady state to exist.
- Let

$$c_t = C_t/A_t, \quad y_t = Y_t/A_t, \quad y_t^* = Y_t^*/A_t.$$

- Steady state is given by:

$$\begin{aligned} \pi &= \pi^*, \quad r = \frac{\gamma}{\beta}, \quad R = r\pi^*, \\ c &= (1-\nu)^{1/\tau}, \quad y = gc = y^*. \end{aligned}$$

Other Models Considered in the Book

- A medium-scale DSGE model with capital and nominal wage rigidities: Smets and Wouters (2003, 2007)

- DSGE model for fiscal policy analysis: Leeper, Plante, and Traum (2010)