

Exercises for Lecture 8**Exercise 1: Some Questions**

- Q1.** What are the three steps in each iteration of a SMC algorithm? Why is the selection step needed? Why is the mutation step needed?
- Q2.** What is the difference between likelihood tempering and data tempering? What are the advantages and disadvantages of the two tempering schemes?
- Q3.** Does the tuning parameter λ affect the number of required likelihood evaluations?
- Q4.** What would be the advantage of executing multiple Metropolis-Hastings steps during the mutation phase?
- Q5.** In the book we recommend to initialize the SMC with draws from the prior distribution. What are potential advantages and disadvantages of this initialization?
- Q6.** Provide an outline for a recursive proof that shows that the SMC approximation \bar{h}_N converges almost surely to $\mathbb{E}_\pi[h]$.

Exercise 2: SMC

Replicate the SMC estimation for the stylized state-space model in Chapters 5.1 of Herbst and Schorfheide (2015). To get you started, you can find some MATLAB code on the companion website for the book.

- (i) Describe the identification problems that arise in this model
- (ii) Conduct an experiment that highlights the potential bimodality of the posterior distribution.
- (iii) Conduct an experiment that highlights the potential weak identification (nearly flat posterior surface).
- (iv) Choose a simple time series model that is rich enough so that it can be estimated on actual data and that you can fairly easily evaluate the likelihood function. Pick a data set and use the SMC sampler to conduct posterior inference. Write a summary of your findings that explains the model, the prior, the data, and the results.