# Aggregate Multi-commodity Stochastic Models for Collaborative Trajectory Options Program (CTOP)

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***Abstract*— Collaborative Trajectory Options Programs (CTOP) is a Traffic Management Initiative (TMI) which controls the air traffic flow rates into Flow Constrained Areas (FCAs). CTOP can handle multiple FCAs within a single program, and allows flight operators to submit a set of desired reroute options (called a Trajectory Options Set or TOS) to express their conditional preference for different route choices for each flight. One of the core research questions in CTOP is FCA Planned Acceptance Rates (PARs) optimization under uncertainty. We will first discuss some characteristics of CTOP rate optimization, including the multi-commodity flow nature of the problem caused by multiple constrained resources, TOS-induced demand variability, and the concept of Rate Computation Loop (RCL). In this paper, we are focused on multi-resource rate planning given route assignment for each flight. Three novel aggregate multi-commodity stochastic programming models are proposed: a two-stage static model, in which the ground delays are assigned at the beginning of the planning horizon; a semi-dynamic model, in which the ground delays are assigned at flights' scheduled departure times to take advantage of the latest capacity and scenario tree structure information and a dynamic model, in which a flight can be ground-delayed multiple times by exploiting not only scenario tree structure but also flights' en route time information. The performance of these three models is tested on a realistic CTOP use case. The results are very promising in terms of system delay costs reduced and computational efficiency. These three models are not limited to be used in CTOP, but can be applied to solve the general multiple constrained airspace resource optimization problem as well.**

***Keywords*— CTOP; Multi-commodity Flow, Stochastic Programming; Static Model; Dynamic Model; Aggregate; Rate Planning; Traffic Management Initiative; Air Traffic Flow Management**

## Nomenclature

**Notations Used in All Three Models**

- $P$ — Number of PCAs
- $k$ — $k$-th PCA, $k = 1, \cdots, P$
- CONN — Set of ordered pairs of PCAs. $(k, k') \in$ CONN iff $k$ is connected to $k'$ in the directed graph
- $\Delta^{k,k'}$ — Number of time periods to travel from PCA $k$ to $k'$. Defined for all pairs $(k, k') \in$ CONN
- $T$ — Number of time periods in CTOP planning horizon
- $Q$ — Number of scenarios
- $p_q$ — Probability that scenario $q$ occurs
- $M^k_{t,q}$ — Physical capacity of PCA $k$ at time period $t$ under scenario $q$
- $D^k_{t,r}$ — Scheduled direct demand at PCA $k$ (from airports) at time period $t$ from flights with same path $r$
- $P^k_{t,r}$ — Planned direct demand at PCA $k$ at time period $t$ from flights with same path $r$
- $G^k_{t,r}$ — Number of flights with same path $r$ whose arrival time at PCA $k$ is adjusted from time interval $t$ to $t + 1$ or later using ground delay at their point of origin
- $L^k_{t,r,q}$ — Number of flights with same path $r$ that actually pass PCA $k$ during time $t$ in scenario $q$
- $A^{k,q}_{t,r}$ — Number of flights with same path $r$ taking air delay before PCA $k$ during time $t$ in scenario $q$

**Notations in Semi-Dynamic and Dynamic Models**

- $t_s$ — Time period at which stage $s$ begins
- $S^k_{s,t,r}$ — Number of flights with same path $r$ originally scheduled to depart in stage $s$ and arrive in PCA $k$ (direct demand) in interval $t$
- $X^{k,q}_{s,t,t',r}$ — Number of flights with same path $r$, originally scheduled to depart in stage $s$ arrive in PCA $k$ (direct demand) in interval $t$, rescheduled to arrive in interval $t'$ under scenario $q$
- $P^{k,q}_{t,r}$ — Planned direct demand at PCA $k$ in time interval $t$ from flights with same path $r$ in scenario $q$
- $B$ — Total number of branches in the scenario tree
- $N_b$ — The number of scenarios corresponding to branch $b \in \{1, \ldots, B\}$

**Notations in Dynamic Model**

- $D^k_{t,L,r}$ — Number of flights with same path $r$, flight time $L$ (to the first PCA $k$) and with original departure time $t$
- $G^{k,q}_{t,L,r}$ — Number of flights with same path $r$, flight time $L$ (to the first PCA $k$) receiving ground delay at $t$ in scenario $q$
- $P^{k,q}_{t,L,r}$ — Number of flights with same path $r$, flight time $L$ (to the first PCA $k$) and departure time $t$ in scenario $q$

## I. Introduction

Traffic Management Initiatives (TMIs) are tools that air traffic managers use to balance demand and capacity in congested airports and airspace regions. Collaborative Trajectory Options Programs (CTOP) is a new TMI which assigns ground delays and/or reroutes around one or more Flow Constrained Area (FCA)-based airspace constraints in order to manage demand through capacity. CTOP has the ability to handle multiple congested areas with a single program, and allows flight operators to submit a set of desired reroute options (called a Trajectory Options Set or TOS) to express the conditional preference for different route choices for each flight [1]. CTOP combines many features of its predecessors including Ground Delay Program (GDP), Airspace Flow Program (AFP), and Reroutes and is more automated, efficient, and flexible.

There has been moderate research on TMI optimization in the past three decades. On the problem side, most of the literature is focused on Single Airport Ground Hold (SAGH) problem. On the methodology side, stochastic optimization has been the dominant decision making under uncertainty approach in this area. The first model describing the strategy of holding flights on the ground to avoid costly and unsafe air holding was proposed by Odoni in 1987 [2]. The first two-stage and multi-stage stochastic programming models for SAGH were formulated by Richetta et al. in early 1990s [3][4]. Since then, the Federal Aviation Administration (FAA) has made significant changes in doing Air Traffic Flow Management (ATFM), moving from a centralized system to one called Collaborative Decision Making (CDM) that gives more decision making responsibilities to airspace users [5][6][7]. In the past two decades, nearly all efforts have been guided by this CDM philosophy. In the CDM paradigm, FAA will set the Planned Acceptance Rates (PARs) for the constrained resources, then resource allocation algorithms will be run to assign the ground delays and/or reroutes to the affected flights. After that the airlines are allowed to reassign the flights to the slots they own to minimize their respective objectives. The first CDM compatible stochastic model for SAGH was proposed by Ball et al. [8]. That model, called Static Stochastic Model (SSM), is a two-stage high aggregate model that directly computes PARs for an weather impacted airport. In the aforementioned models, once a ground delay decision is made, it cannot be revised, even though the flight is still on the ground and further ground holding is possible. Mukherjee formulated a flight-level multistage model that allows a flight to take ground delays multiple times based on the latest capacity information [9]. Estes et al. proposed an aggregate version of Mukherjee's model and showed that it is more computationally tractable [10]. Mukherjee also formulated a dynamic model for managing air traffic inbound to airport where both the airport and its approach routes are subject to uncertainties [11]. There are only three references in literature on the CTOP rate-planning problem up to the present: in [12], we present a highly aggregate CDM compatible stochastic model called ESOM which is an extension to SSM in [8]; in [13], we formulate six centralized disaggregate stochastic models to benchmark against the models in [12] and in this work; in [14], we introduce a heuristic called saturation technique and discuss its important usage in GDP and CTOP rate planning.

In this paper, we explore several approaches to the CTOP rate-planning problem, each with varying degree to which traffic managers can modify or revise flights' controlled departure times. This paper is organized as follows: to give the reader the basic background knowledge, in section II we will first give a brief introduction to CTOP. In section III we will discuss several key properties of CTOP and the specific problem we will solve in this paper. From section IV to section VI, we will explain in detail the three stochastic models we propose. In section VII we will introduce the experiment setup and discuss the main results. In section VIII, we summarize the findings of this paper and point out the future work.

**Flight ID**

| ACID | ORIG | DEST | IGTD | TYPE | ERTD |
|---|---|---|---|---|---|
| ABC123 | LAX | ATL | 05/1945 | LJ60 | 05/1945 |

**Trajectory Option Set**

| RTC | Route | ALT | SPEED |
|---|---|---|---|
| 0 | TRM PKE DRK J6 IRW FSM MEM ERLIN9 | 350 | 435 |
| 30 | TRM PKE DRK J134 LBL SGF BNA RMG4 | 350 | 435 |
| 50 | TRM PKE DRK J134 BUM FAM BNA RMG4 | 350 | 430 |
| 60 | TRM BLH J169 TFD J50 SSO J4 EWM J66 ABI J4 MEI LGC2 | 350 | 425 |
| 70 | TRM BLH J169 TFD ELP J2 JCT J86 IAH J2 LCH J590 GCV LGC2 | 310 | 430 |

Fig. 1. TOS Example of a Flight from LAX to ATL [15]

## II. CTOP in a Nutshell: TOS, Adjusted Cost and CTOP (Allocation) Algorithm

TOS is the most important concept in CTOP and is what the entire idea of CTOP is built upon. As can be seen from Fig 1, a TOS consists of a flight's ID, origin and destination airports, earliest runway time of departure, candidate routes information and restrictions.

FAA allocate the routes to flights on a flight by flight basis according to their earliest Initial Arrival Times (IATs). A flight's IAT is the earliest ETA (Estimated Time of Arrival) at any of a CTOP's FCAs using any of the flight's TOS options. We can consider IAT as a flight's CTOP capture time. This is the CTOP version of Ration by Schedule (RBS).

For a given flight, CTOP allocation algorithm will calculate the adjusted cost for each candidate route and assign the route with the minimum adjusted cost to this flight. The key equation here is:

$$\text{Adjusted Cost} = \text{RTC} + \text{Required Ground Delay}$$

Relative Trajectory Costs (RTCs) are values submitted by the flight operator to express his/her preference over route options. Required ground delay is calculated by the CTOP algorithm given current available slots, which is the ground hold time this flight will need to bear in order to take a specific route. As shown in Fig 2, this flight will be allocated with route 2, which has the smallest adjusted cost among all route options. For a more detailed introduction to CTOP algorithm, the readers are referred to [1] or [16].

## III. Key Characteristics of CTOP, Rate Planning Given Route Assignments

One characteristic of CTOP rate optimization is that it is in nature a multi-commodity problem, since flights will traverse different congested airspace and reach different destinations. One the other hand, the SAGH models [2]-[10] and single airport rerouting model are essentially single commodity flow models, since the air traffic is bound for a single airport.

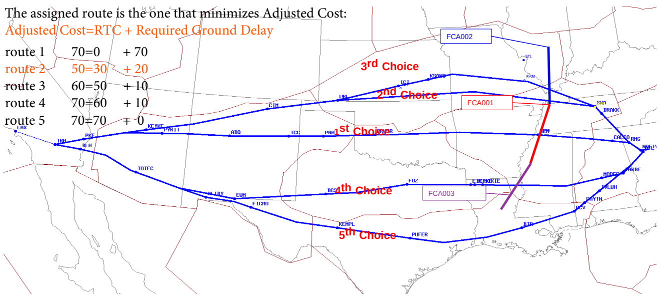


Fig. 2. Flight Routes in the TOS and the Adjusted Cost [15]

In this paper, we will differentiate two concepts: FCA, which is an artificial line or region in the airspace and serves like a valve to control the traffic flows in a region; and Potentially Constrained Area (PCA), which is the actual troubled area and whose future capacity realization is represented by a finite set of scenarios (Figure 3). In this paper, we will directly set the rates into the PCA, rather than through controlling FCA.

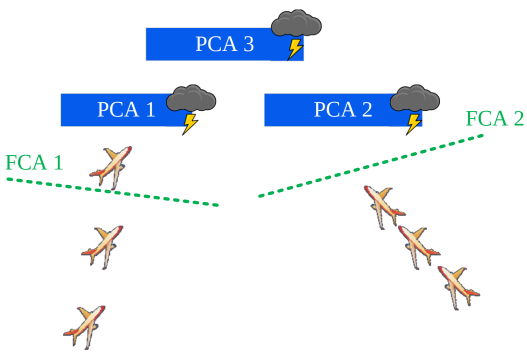


Fig. 3. PCAs and Candidate FCAs

The introduction of TOS brings two difficulties to CTOP planning: firstly, one flight now may have more than one route option, therefore it is not straightforward to estimate the demand to the FCAs; secondly, for a given demand estimation, if we do the planning accordingly, after we run the the CTOP allocation algorithm, the demand may shift and invalidate the proposed PARs. This problem is known as TOS-induced demand variability [12]. We have shown in [14] that even in a simpler GDP problem, we have to consider the demand information, not just capacity information, in order to find an optimal solution.

There are two possible solutions to this problem. The first solution is to solves the rate planning and TOS allocation as a single optimization problem. This approach is very likely computationally intractable, not just because of its disaggregate nature, but also because it has to model the CTOP resource allocation algorithm, which is nonlinear, in order to be CDM compatible. The second approach, which we adopt in [12], is an iterative algorithm. In the first iteration, we will start with an initial demand estimation, which can come from flights' most preferred routes. Based on this demand estimation and capacity information, we will run the rate optimization model and compute the PARs for the FCAs. Then we will run the resource allocation algorithm, get the new route and ground delay for each flight, and compute the new demand on the resources. Now it can be seen that we have a computation loop, since the demand will possibly change if PARs change and the computation of PARs is dependent on the demand estimation (Figure 4). We will exit the computation loop if the PARs have stabilized, i.e. the PARs or objective values are the same for two consecutive iterations.

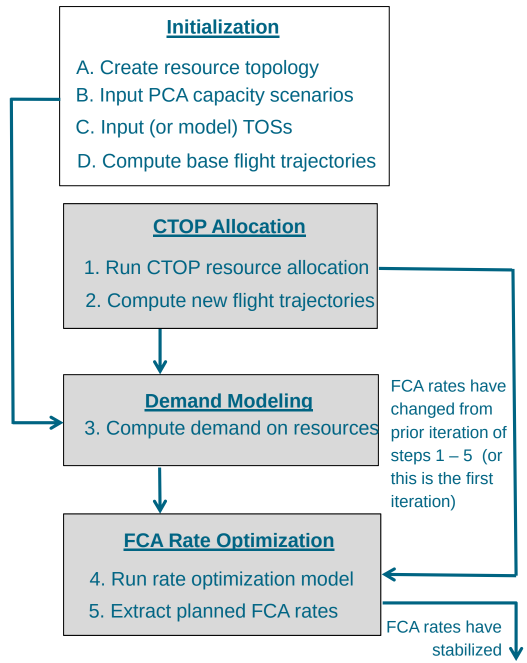


Fig. 4. Rate Computation Loop (RCL)

This paper is not going to investigate the convergence issue of the rate computation loop, but rather we will study at each specific iteration, assuming the route for each flight has been assigned and we are only allowed to dynamically assign delays to flights, what are the minimal ground and air delay costs needed for flights to traverse the affected airspace in the CTOP planning horizon. In other words, we will be focused on an important subproblem of CTOP optimization: given a demand estimation, how can we best manage this demand, in terms of system delay costs, through the congested regions.

## IV. Two-stage Static Model

In this section, we introduce the two-stage aggregate multi-commodity flow model. We want aggregate models for both equity and efficiency reasons. The time bins used during the planning horizon are 15 minutes in length. The first stage decisions are the ground delays assigned to the flights. The second stage decisions are the air delays the flights need to take in response to the realization of the weather scenarios. In the two-stage model, flights are grouped by "path", which is the sequence of PCAs flights traverse. A path uniquely determines a commodity in a multi-commodity flow model. Note that a path is not the same as a TOS route; the latter starts and ends at airports and is composed of waypoints, rather than PCAs.

For each path $r$, the demand of commodity $r$ will enter the PCA system via the first PCA $k = r_1$ on that path. we have the following relationship for PAR, originally scheduled demand and ground delays:

$$P_{t,r}^{k} = D_{t,r}^{k} - (G_{t,r}^{k} - G_{t-1,r}^{k}) \quad k = r_1, \forall r, t \tag{1}$$

To elaborate, let's consider a path in Figure 8: PCA1 $\rightarrow$ PCA_EWR. The group of flights correspond to this path will enter the PCA system through the first PCA on this path, which is PCA1.

The total demand at PCA $k$ with same path $r$ at time $t$ includes either direct flights from airports (first PCA on the path) or flights from upstream PCA whose number is scenario dependent. For each group of flights we have:

$$L_{t,r,q}^{k} = \begin{cases} \text{if } k = r_1 & P_{t,r}^{k} - (A_{t,r,q}^{k} - A_{t-1,r,q}^{k}) \\ \text{else} & \text{UpPCA}_{t,r,q}^{k} - (A_{t,r,q}^{k} - A_{t-1,r,q}^{k}) \end{cases} \tag{2}$$

$$\text{UpPCA}_{t,r,q}^{k} = L_{t-\Delta^{k',k},r,q}^{k'} \quad (k', k) \in r \tag{3}$$

Constraint (2) says the number of flights which actually pass PCA $k$ equals to total demand minus the incremental number of flights taking air delay. Constraint (3) means that, when calculating the traffic flow from the upstream PCA, the average travel time between PCAs is also considered.

In the PCA1 $\rightarrow$ PCA_EWR path example, the first case in (2) corresponds to PCA1 and the second case corresponds to PCA_EWR.

Different groups of flights are related through the capacity constraints at each PCA:

$$M_{t,q}^{k} \geq \sum_{r} L_{t,r,q}^{k}, \quad \forall t, q, k \tag{4}$$

The nonnegativity constraints are very straightforward:

$$D_{t,r}^{k}, P_{t,r}^{k}, G_{t,r}^{k}, L_{t,r,q}^{k} \geq 0 \quad \forall t, r, q, k \tag{5}$$

It is worth noting that the following boundary condition, which are commonly seen in GDP models,

$$G_{0,r}^{k} = A_{T,r}^{k} = 0 \quad \forall k, r \tag{6}$$
$$G_{T,r,q}^{k} = A_{T,r,q}^{k} = 0 \quad \forall k, r, q$$

cannot ensure all flights will land at the end of the planning horizon in the CTOP model. This is an important difference between GDP and CTOP models. The cause of this difference is that in CTOP we have multiple constrained resources and it takes time to travel from one resource to a downstream resource.

If we do want all the flights to land/exit the PCA network at the end of the planning horizon, we will need to explicitly enforce that, for each commodity and for each scenario, the total demand of commodity $r$ equals to the cumulative amount of commodity $r$ which exits the PCA system via the last PCA on path $r$:

$$\sum_{t=1}^{T} D_{t,r}^{k=r_1} = \sum_{t=1}^{T} L_{t,r,q}^{k=r_{-1}} \quad \forall r, q \tag{7}$$

$r_{-1}$ means the last PCA on path $r$. The objective function minimizes the ground delay and expected air delay cost:

$$\min \quad c_g \sum_{t=1}^{T} \sum_{r} G_{t,r}^{k=r_1} + c_a \sum_{q=1}^{Q} p_q \sum_{k \in \text{PCAs}} \sum_{t=1}^{T} \sum_{r} A_{t,r,q}^{k} \tag{8}$$

The above two-stage model is an explicit multi-commodity flow model. We treat flights taking different paths as different commodities. One obvious drawback of this model compared with ESOM model in [12] is the computation time, since we have more decision variables and coupled constraints. One benefit of this two-stage model is that we can enforce integrality constraint, whereas in ESOM we have to round the solutions to the nearest integers.

There is no FCA in the above model and we do not try to directly optimize PARs for FCAs. However, once we solve the model and know the flights' departure times from first stage decisions, we could calculate the Acceptance Rates (ARs) into a certain region. This allows our decision support tools to explore a range of FCA locations to control the flows through the PCAs. Once the locations of the FCAs are set, then the filters and rates can be determined.

## V. Multistage Semi-Dynamic Model

A drawback of the static model is that we do not take advantage of the updated weather information or the structure of a scenario tree. In this section, we will introduce a multistage semi-dynamic model which could partially overcome this limitation. We can also call it Richetta's type dynamic model because, like Richetta et al. did for the GDP problem [4], we will assign delays at flights' original scheduled departure times. The reason we call the model semi-dynamic model is because compared with Mukherjee's type dynamic model (section VI), the ground delay, once assigned, cannot be revised and thus the model is not "fully" dynamic.

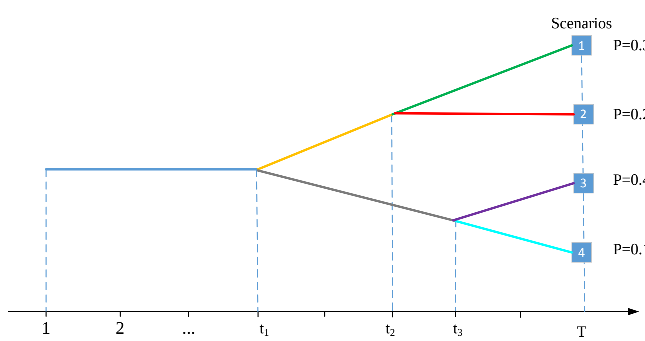


Fig. 5. A Scenario Tree with Four Scenarios [9]

In this model, we will use the concept of stage. A stage can comprise several time periods, at which we have the same weather information. For example in Figure 5, there are four stages, and dotted vertical lines indicate the starting times of each stage. Because we have multiple PCAs in CTOP, the branching point in a scenario tree means we have new weather information for at least one PCA. This time the ground delay decisions are also scenario dependent.

Like in the previous section, we differentiate direct demand from airports and demand from the upstream PCA. $k = r_1$ in decision variables $X^{k,q}_{s,t,t',r}, S^k_{s,t,r}, P^{k,q}_{t,r}$. The first set of constraints is the conservation of flow constraints for each group of flights:

$$\sum_{t'=t}^{T} X^{k,q}_{s,t,t',r} = S^k_{s,t,r} \quad \forall s, \forall q, \forall r$$

For variable $X^{k,q}_{s,t,t',r}$, we must have $t' \geq t \geq t_s$. If $t' = t_s$, then the departure airport is very close to the PCA $k$ and the en route time is within 15 minutes. In general it is not likely to happen.

From $X^{k,q}_{s,t,t',r}$ we can calculate the direct demand at a PCA $k$ with same path $r$, which is now scenario dependent:

$$P^{k,q}_{t,r} = \sum_{s} \sum_{t' \leq t} X^{k,q}_{s,t',t,r} \quad \forall q, \forall r \tag{9}$$

The other constraints are very similar:

$$\begin{aligned}
&L^k_{t,r,q} = \begin{cases} \text{if } k = r_1 & P^{k,q}_{t,r} - (A^k_{t,r,q} - A^k_{t-1,r,q}) \\ \text{else} & \text{UpPCA}^k_{t,r,q} - (A^k_{t,r,q} - A^k_{t-1,r,q}) \end{cases} \\
&\text{UpPCA}^k_{t,r,q} = L^{k'}_{t-\Delta^{k',k},r,q} \quad (k', k) \in r \\
&M^k_{t,q} \geq \sum_r L^k_{t,r,q} \quad \forall t, q, k \\
&0 \leq X^{k,q}_{s,t,t',r}, P^k_{t,r}, G^k_{t,r}, L^k_{t,r,q} \\
&G^k_{0,r} = A^k_{0,r} = 0 \\
&\sum_s \sum_t S^{k=r_1}_{s,t,r} = \sum_{t=1}^{T} L^{k=r_{-1}}_{t,r,q} \quad \forall r, q
\end{aligned} \tag{10}$$

For dynamic model, we also have a set of nonanticipativity constraints, which ensure that decisions made at time $t$ are solely based on the information available at that time [17].

$$X^{k,q^b_1}_{s,t,t',r} = \cdots = X^{k,q^b_{N_b}}_{s,t,t',r} \tag{11}$$

The constraints mean if a set of scenarios are on the same branch, we should take exactly the same actions with respect to the set of scenarios. The branch(es) information is determined by $s$, the original scheduled departure time. For example, if we are at time period $t_1 + 1$ (stage 2), we should have:

$$\begin{aligned}
X^{k,1}_{2,t_1+1,t',r} &= X^{k,2}_{2,t_1+1,t',r} \\
X^{k,3}_{2,t_1+1,t',r} &= X^{k,2}_{4,t_1+1,t',r} \quad \forall k, t', r
\end{aligned} \tag{12}$$

The objective function minimizes the expected ground delay and air delay cost:

$$\begin{aligned}
\min \quad & \sum_{q=1}^{Q} p_q \{ \sum_s \sum_{t'=t}^{T} \sum_r c_g (t' - t) X^{k,q}_{s,t,t',r} + \\
& \sum_{k \in \text{PCAs}} \sum_{t=1}^{T} \sum_r c_a A^k_{t,r,q} \}
\end{aligned} \tag{13}$$

## VI. Multistage Dynamic Model

In this section, we will introduce the "fully" dynamic model or Mukherjee's type model. The idea of this model is that when making ground delay decisions, we will consider the fact this flight may be further ground delayed later on ("plan to replan").

One major difference between Mukherjee's type model with the previous two models is that we will group flights not only by path but also by en route time. This is because in a "true" dynamic model, we enforce the nonanticipativity constraints at a flight's actual departure time. Therefore we will directly model the release time, not the arrival time. And we need to know, if we let these flights take off now, how long will it take for these flight get into the system and become real demands to the PCAs.

Like in the two-stage static model, we have conservation of flow constraints at departure airports for all groups of flights:

$$P^{k,q}_{t,L,r} = D^k_{t,L,r} - (G^{k,q}_{t,L,r} - G^{k,q}_{t-1,L,r}) \quad \forall t, L, r \tag{14}$$

The direct demand for PCA $k$ at time $t$ from flights with same path $r$ and en route time $L$ under scenario $q$:

$$\sum_L P^{k,q}_{t-L,L,r}$$

The other constraints are very similar:

$$\begin{aligned}
&L^k_{t,r,q} = \begin{cases} \text{if } k = r_1 & \sum_L P^{k,q}_{t-L,L,r} - (A^k_{t,r,q} - A^k_{t-1,r,q}) \\ \text{else} & \text{UpPCA}^k_{t,r,q} - (A^k_{t,r,q} - A^k_{t-1,r,q}) \end{cases} \\
&\text{UpPCA}^k_{t,r,q} = L^{k'}_{t-\Delta^{k',k},r,q} \quad (k', k) \in r \\
&M^k_{t,q} \geq \sum_r L^k_{t,r,q} \\
&0 \leq P^{k,q}_{t,L,r}, G^{k,q}_{t,L,r}, L^k_{t,r,q} \\
&G^{k,q}_{0,L,r} = A^{k,q}_{0,r} = 0 \\
&\sum_{t=1}^{L} D^{k=r_1}_{t,L,r} = \sum_{t=1}^{T} L^{k=r_{-1}}_{t,r,q} \quad \forall r, q
\end{aligned} \tag{15}$$

We have nonanticipativity constraints for the number of flights which are planned to release at time $t$ for all groups:

$$P^{k,q^b_1}_{t,L,r} = \cdots = P^{k,q^b_{N^b}}_{t,L,r} \tag{16}$$

Objective function:

$$\min \quad \sum_{q=1}^{Q} p_q \{ c_g \sum_{t=1}^{T} \sum_L \sum_r G^{k=r_1,q}_{t,L,r} + c_a \sum_{k \in \text{PCAs}} \sum_{t=1}^{T} \sum_r A^k_{t,r,q} \} \tag{17}$$

## VII. Experimental Results

To demonstrate the performance of proposed models, we create an operational use case based on actual events from July 15, 2016 [18].


Fig. 6. Weather Forecast for 2210z, Taken at 1522z on July 15, 2016

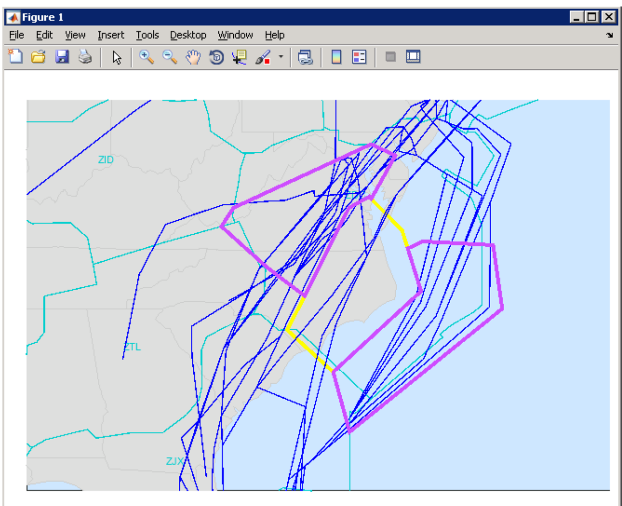


Fig. 7. Traffic Routing Around the Original PCA

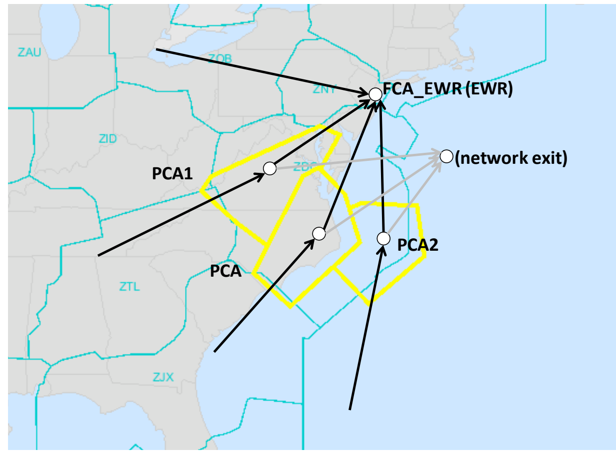


Fig. 8. Geographical display of a PCA Network

### A. Southern ZDC and EWR with Convective Activity

This use case primarily addresses convective weather activity in southern Washington Center (ZDC). Figure 6 shows the pattern of convective weather activity for that day. It can be seen that southern ZDC is adversely impacted by the weather. We further assume there is demand-capacity imbalance at EWR airport. In principle, the EWR imbalance could be addressed by an isolated GDP. However, much of the traffic bound for EWR is passing through southern ZDC; therefore, we show how the EWR arrival traffic can be folded into the same CTOP that addresses southern ZDC. Note that the traffic congestion at southern ZDC is comparable to an AFP with two wing FCAs added, shown in Figure 7. The PCA network is shown in Figure 8. We assume there is a four-hour capacity reduction in ZDC/EWR from 2000Z to 2359Z.

### B. Creating Capacity Profiles

Translating ensemble weather forecasts to probabilistic capacity information is an important line of research in the aviation weather community [19][20][21]. These developed techniques have the potential to be applied to CTOP research. For simplicity, in this work we directly manipulated the capacity profiles from the base forecast to create the alternate capacity profiles. This gives us full control over the capacity profiles for experimental purposes.

For illustration, we use a very simple scenario tree in this paper, shown in Figure 9. Nevertheless this tree has more than one branching point, which is more complex than a scenario bush. We expect the dynamic models will take advantage of the structure information and outperform the static model. The three scenarios corresponds to the good, average and bad scenarios. The capacity information is listed in Table I.

Note in GDP optimization, we usually add one extra time period to make sure all flights will land at the end of the planning horizon. Because CTOP has multiple constrained resources, we need to add more than one time period depending on the topology of the PCA network. In this use case, we add four extra time periods, because the maximum average travel time between the three en route PCAs and KEWR is around 1 hour (4 time periods). For any time periods outside the CTOP start-end time, e.g. the extra four time periods in Table I, we assumed nominal capacity.

### C. Traffic Demand

For flight data, we used historical flight data pulled from September 8, 2016 as a representative clear weather day for traffic demand. We avoided using the actual flight data from July 15, 2016 because flight plans and airline operational schedules were likely influenced by weather forecasts and related ATFM events. We only keep flights which pass through one of the 3 PCAs created in ZDC plus all EWR arrivals. The resulting set contained 1098 flights. To form the base (preferred) route for each flight, we drew historical filed flight plans (from Sept. 8, 2016) from the System Wide Information Management (SWIM) data.

A typical TOS package that might be submitted for this day would have one route for each FCA, and one route-out route to avoid all FCAs. To model the TOS sets that airlines might submit in response to a CTOP, we can draw from

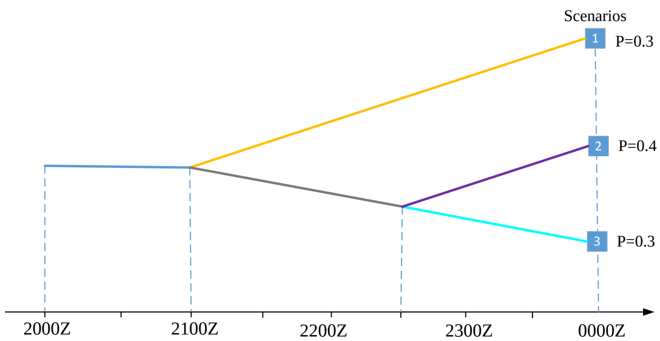


Fig. 9. Scenario Tree Used in the Experiment

a combination of reroute TMIs from SWIM database and routes in the Coded Departure Route (CDR) database.

Because our goal in this paper is to investigate the minimal delay costs for flights to traverse the affected airspace given assigned routes, rather than running the RCL to get stable PARs, we will only use the most preferred routes.

### D. Model Comparisons

From Figure 8 we know there are in total 7 possible paths: direct demand to KEWR, pass one of the three PCAs then land at EWR or pass one of the PCAs then exit the system. We require all the CTOP captured flights to land at KEWR/exit the PCA network at the end of the planning horizon.

The optimization models are solved using Gurobi 7.5.2 on a laptop with 2.6 GHz processors and 16 GB RAM [22]. The main results are listed in Table II. There are some key observations from this table:

- The two-stage solution outperforms the deterministic policy (SCEN1-3 and EEV), as it should, since it explicitly considers the uncertainty when making holding decisions. EEV is the expected result of using the EV solution. EV is the expected value problem [17].
- The semi-dynamic model solution is better than the two-stage model solution and dynamic model in turn performs better than semi-dynamic model, which are also expected, because dynamic models uses more weather evolution information than two-stage static model.
- The computation times for three stochastic models are all very short. Actually in this use case we can directly get the integer solution from the LP relaxation of the problem. The authors are investigating whether LP relaxation can always give produce integer solution in the general case for the three formulations.

## VIII. Conclusions

In this paper, we propose three stochastic programming models for CTOP. En route and airport capacity uncertainty is represented by a finite number of scenarios arranged in a scenario tree. We have demonstrated that stochastic solutions are superior to deterministic solutions, and stochastic dynamic model is better than stochastic static/semi-dynamic models in terms of total expected cost. Our experiment shows that the fully dynamic model can achieve 28.5% lower expected cost than the deterministic model and 26.4% lower costs than the static model.

The ongoing work includes testing on more realistic capacity data, investigating the impact of cost ratio, air holding limit, lead time, etc. on the model solutions, and comparing the three models with ESOM and disaggregate models [12][13].

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## X. Appendix

### A. Acronyms

| | |
|---|---|
| **ATFM** | Air Traffic Flow Management |
| **TMI** | Traffic Management Initiative |
| **GDP** | Ground Delay Program |
| **AFP** | Airspace Flow Program |
| **FCA** | Flow Constrained Area |
| **PCA** | Potential Constrained Area |
| **PAR** | Planned Acceptance Rate |
| **CTOP** | Collaborative Trajectory Options Program |
| **TOS** | Trajectory Options Set |
| **FAA** | Federal Aviation Administration |
| **RTC** | Relative Trajectory Cost |
| **RCL** | Rate Computation Loop |
| **RBS** | Ration by Schedule |

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| | Resource/Time Bin | 20:00 | 15 | 30 | 45 | 21:00 | 15 | 30 | 45 | 22:00 | 15 | 30 | 45 | 23:00 | 15 | 30 | 45 | 00:00 | 15 | 30 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SCEN1 | PCA | 13 | 13 | 13 | 13 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| | PCA1 | 44 | 44 | 44 | 44 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| | PCA2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| | EWR | 8 | 8 | 8 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| SCEN2 | PCA | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| | PCA1 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| | PCA2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| | EWR | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| SCEN3 | PCA | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 13 | 25 | 25 | 25 | 25 |
| | PCA1 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 44 | 50 | 50 | 50 | 50 |
| | PCA2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| | EWR | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 10 | 10 | 10 | 10 |

TABLE I

CAPACITY SCENARIOS

| | Ground Delay Periods If This Scenario Occurs: | | | Air Holding Periods If This Scenario Occurs: | | | Total Cost If This Scenario Occurs: | | | Expected Cost | Running Time |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | SCEN1 | SCEN2 | SCEN3 | SCEN1 | SCEN2 | SCEN3 | SCEN1 | SCEN2 | SCEN3 | | Seconds |
| SCEN1 | 88 | 88 | 88 | 0 | 198 | 392 | 88.0 | 484.0 | 872.0 | 481.6 | $\ll 1.0$ |
| SCEN2 | 280 | 280 | 280 | 0 | 0 | 209 | 280.0 | 280.0 | 698.0 | 405.4 | $\ll 1.0$ |
| SCEN3 | 470 | 470 | 470 | 0 | 0 | 0 | 470.0 | 470.0 | 470.0 | 470.0 | $\ll 1.0$ |
| EV | | | | | | | | | | 192.0 | $\ll 1.0$ |
| EEV | 192 | 192 | 192 | 0 | 89 | 279 | 192.0 | 370.0 | 750.0 | 430.6 | $\ll 1.0$ |
| Two-Stage Model | 280 | 280 | 280 | 0 | 0 | 190 | 280.0 | 280.0 | 660.0 | 394.0 | 0.109 |
| Semi-Dynamic Model | 156 | 280 | 403 | 0 | 0 | 67 | 156.0 | 280.0 | 537.0 | 319.9 | 0.612 |
| Dynamic Model | 116 | 280 | 463 | 0 | 0 | 7 | 116.0 | 280.0 | 477.0 | 289.9 | 1.480 |
| Perfect Information | 88 | 280 | 470 | 0 | 0 | 0 | 88.0 | 280.0 | 470.0 | 279.4 | |

TABLE II

DETERMINISTIC VS. STOCHASTIC SOLUTIONS COMPARISION ($c_a/c_g = 2$)


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