Seminar on Stochastic Processes 2022

List of *In person* Poster Presentations
(ordered alphabetically by LAST name)

- **Presenter**: Ini Adinya (University of Ibadan, Nigeria)
  Title: Options Valuation with Stochastic Interest rate and Recession-induced Stochastic Volatility.
  Abstract: This study seeks to formulate mathematical model for options valuation with stochastic interest rate and Recession-induced Stochastic Volatility. We simulated the derived model and used it to forecast future returns of stock asset in a recessed economy. The figures obtained and the model parameter estimated for the model showed the effectiveness of volatility control function $c(t)$ which is dependent on the state of the economy.

- **Presenter**: Daniel Blanquicett (University of California, Davis)
  Title: An isotropic Bootstrap Percolation in d-dimensions.
  Abstract: Consider a p-random subset A of initially infected vertices in the discrete cube $[L]^d$, and assume that the neighbourhood of each vertex consists of the a_i nearest neighbors in the $\pm e_i$ directions for each $i \in \{1, 2, ..., d\}$, where $a_1 \leq a_2 \leq \ldots \leq a_d$. Suppose we infect any healthy vertex already having r infected neighbours, and that infected sites remain infected forever. We will discuss recent progress on the determination of the critical length for percolation (full infection) in these models.

- **Presenter**: Marco Carfagnini (University of Connecticut)
  Title: Small Deviations and Chung's law of the iterated logarithm for hypoelliptic diffusions.
  Abstract: A small ball problem and Chung's law of iterated logarithm for two hypoelliptic diffusions are considered. The first one is a Brownian motion on the Heisenberg group and the second one a Kolmogorov diffusion of step n.

- **Presenter**: Carsten Chong (Columbia University)
  Title: The stochastic heat equation with multiplicative Lévy noise: Existence, moments, and intermittency.
  Abstract: We study the stochastic heat equation (SHE) $\partial_t u = \frac{1}{2}\Delta u + \beta u \xi$ driven by a multiplicative Lévy noise ξ with positive jumps and coupling constant $\beta > 0$, in arbitrary dimension $d \geq 1$. We prove the existence of solutions under an optimal condition if $d = 1, 2$ and a close-to-optimal condition if $d \geq 3$. Under an assumption that is general enough to include stable noises, we further prove that the solution is unique. By establishing tight moment bounds on the multiple Lévy integrals arising in the chaos decomposition of u, we further show that the solution has finite pth moments for $p > 0$ whenever the noise does. Finally, for any $p > 0$, we derive upper and lower bounds on the moment Lyapunov exponents of order p of the solution, which are asymptotically sharp in the limit as $\beta \to 0$. One of our most striking findings is that the solution to the SHE exhibits a property called strong intermittency for any non-trivial Lévy measure, at any disorder intensity $\beta > 0$, in any dimension $d \geq 1$. This behavior contrasts with that observed for the SHE on $\mathbb{Z}^d$ and for the SHE on $\mathbb{R}^d$ with Gaussian noise, for which intermittency does not occur in high dimensions for small β.

- **Presenter**: Dimitrios Diamantidis (Indiana University)
  Title: The effect of the pedigree on pairwise coalescent times.
  Abstract: We consider a population model with discrete, non-overlapping generations for a fixed size population. In our model, each generation reproduces as in the Wright Fisher model with high probability and with low probability we have a high reproduction event. Conditional on the pedigree of the population we study the effect of high reproduction events as well as the pairwise coalescent times for a sample of two genes.

- **Presenter**: Ella Hiesmayr (University of California, Berkeley)
  Title: Large deviations of the largest eigenvalue of sparse random graphs with non-Gaussian edge weights.
  Abstract: Large deviations of the spectral properties of random matrices have been an active area of research for several decades. These questions were first addressed for matrices with independent Gaussian entries relying on an analysis of the joint density of the eigenvalues available in this setting. Subsequently there has been a lot of activity around similar questions for other random matrix models, in particular for matrices with non-Gaussian entries and more recently those arising from random weighted networks. Although there are methods to study such examples when the matrices are not too sparse the arguments break down, say, when the graph has constant average degree, an example of central interest.

  While in recent work the large deviation behavior of the spectral edge of random graphs of Gaussian matrices with such sparsity was studied, the case of non-Gaussian distributions remained open. In this talk we will discuss recent work with Shirshendu Ganguly and Kyeongsik Nam addressing this. In particular, we will present a surprising universality result showing that the rate function does not depend on the exact tail behavior when the tails are lighter than Gaussians. The analysis in the case of weights with heavier tails relies on a new bound of the largest eigenvalue of a matrix using its entrywise L_p norm, proved using a generalization of the classical Motzkin-Strauss theorem, that could be of independent interest.

- **Presenter**: Aidan Howells (University of Wisconsin, Madison)
  Title: Stochastic Reaction Networks in Dynamic Compartments.
  Abstract: The theory of stochastic reaction networks with mass-action kinetics is a useful tool for understanding processes, chemical and otherwise, in homogeneous environments. One generalization of these ideas to dynamic heterogeneous environments like biological cells inside tissue was recently proposed by Duso and Zechner. In five minutes I will attempt to convey the idea of their model, why it is interesting, and how I am building on it.

- **Presenter**: Tomoyuki Ichiba (University of California, Santa Barbara)
  Title: Degenerate Competing Three-Particle Systems.
  Abstract: We study systems of three interacting particles, in which drifts and variances are assigned by rank. These systems are degenerate: the variances corresponding to one or two ranks can vanish, so the corresponding ranked motions become ballistic rather than diffusive. Depending on which ranks are allowed to "go ballistic"" the systems exhibit markedly different behavior, which we study here in some detail. Also studied are stability properties for the resulting planar process of gaps between successive ranks. This is joint work with Ioannis Karatzas.

- **Presenter**: Adam Quinn Jaffe (University of California, Berkeley)
  Title: Virtual Markov Chains.
  Abstract: We construct the so-called "virtual Markov chains (VMCs)" as a projective limit of finite state space Markov chains, in the same way that the virtual permutations are a projective limit of the permutations of finite sets. First, we show the representation theorem that every VMC is uniquely determined by a pair of a "virtual initial distribution (VID)" and a "virtual transition matrix (VTM)" that are also "compatible" in a certain sense. Second, we show the decomposition theorem that every VMC can be decomposed into an infinite sequence of independent "staircase Markov chains (SMCs)", and we use this to prove a zero-one law for VMCs.

- **Presenter**: Kei Kobayashi (Fordham University)
  Title: Spectral heat content for time-changed killed Brownian motions.
  Abstract: The spectral heat content is investigated for time-changed killed Brownian motions, where the time change is given by either a subordinator or an inverse subordinator, with the underlying Laplace exponent being regularly varying at infinity. In the case of subordinators, depending on whether the index of

regular variation is greater than 1/2, equal to 1/2, or less than 1/2, the small-time asymptotic behavior of the spectral heat content changes substantially. In contrast, when the time change is given by an inverse subordinator, we obtain a single statement that is valid for all values of the index. This is joint work with Hyunchul Park.

- **Presenter**: Bhumesh Kumar (University of Wisconsin, Madison)
  Title:A Law of Iterated Logarithm for Distributed Stochastic Approximation.
  Abstract: We consider a family of distributed nonlinear stochastic approximation schemes useful in multi-agent reinforcement learning and derive a novel law of iterated logarithm. In particular, our result describes the convergence rate on almost every sample path where the algorithm converges. This result is the first of its kind in the distributed setup and provides deeper insights than the existing ones, which only discuss convergence rates in the expected or the CLT sense. Importantly, our result holds under significantly weaker assumptions: neither the gossip matrix needs to be doubly stochastic nor the stepsizes square summable. As an application, we show that, for the stepsize $n^{-\gamma}$ with $\gamma \in (0,1)$, the distributed TD(0) algorithm with linear function approximation has a convergence rate of $O(\sqrt{n^{-\gamma} \ln n})$ a.s.; for the $1/n$ type stepsize, the same is $O(\sqrt{n^{-1} \ln \ln n})$ a.s. These decay rates do not depend on the graph depicting the interactions among the different agents.

- **Presenter**: Wenjian Liu (City University of New York)
  Title: Phase Transition of the Reconstructability on Regular Grids
  Abstract: The multidimensional grids mixed with Boolean processing functions would better model the real-world communication and social networks, since vertices often receive multiple input signals in these scenarios. The broadcasting problem on Regular Grids is to understand whether it is possible to propagate information starting from the source specifically in regular grids, with noise adding in each edge as an asymmetric Ising channel. The Bayesian reconstruction problem on a directed acyclic graph with the structure of a regular grid is to collect and analyze massive data samples at the nth layer of the grid to identify whether there is non-vanishing information of the root, as n goes to infinity. This project is to explore the critical conditions of the noise level of the asymmetric channel, and the Boolean processing functions that allow the reconstruction of the source from deep layers.

- **Presenter**: Phanuel Mariano (Union College)
  Title: Spectral bounds for exit times on doubling Dirichlet spaces with applications.
  Abstract: We consider a diffusion process X_t on a metric measure space associated to a local regular Dirichlet form. Now let τ_D be the first exit time of X_t from a domain $D \subset M$ and let $\lambda_1(D)$ be the bottom of the spectrum for the related diffusion operator L acting on $L^2(D)$. Assuming the heat kernel exists and has sub-Gaussian upper bounds, we prove an asymptotically sharp spectral upper bound on $\mathbb{P}_x(\tau_D > t)$. Moreover, we can show that the supremum of the exit time over all starting points $x \in D$ is finite if and only if $\lambda_1(D) > 0$ by proving that their product is uniformly bounded from above and below for all domains $D \subset M$. We verify our results hold for many interesting sub-Riemannian manifolds. This includes sub-Riemannian manifolds with transverse symmetries satisfying a generalized curvature-dimension inequality and Carnot groups. Another application of our results is a new characterization of sub-Gaussian heat kernel upper bounds that gives a partial answer to a conjecture of Grigor'yan, Hu and Lau. This talk/poster is based on joint work with Jing Wang.

- **Presenter**: Hugo Panzo (Technion – Israel Institute of Technology)
  Title: Improved upper bounds for the Hot Spots constant.
  Abstract: The Hot Spots constant was recently introduced by Steinerberger as a means to control the global extrema of the first nontrivial eigenfunction of the Neumann Laplacian by its boundary extrema. We use a probabilistic technique to derive a general formula for a dimension-dependent upper bound that can be tailored to any specific class of bounded Lipschitz domains. This formula is then used to compute upper bounds for the Hot Spots constant of the class of all bounded Lipschitz domains in $\mathbb{R}^d$ for both small d and for asymptotically

large d that significantly improve upon the existing results. Joint work with Phanuel Mariano and Jing Wang.

- **Presenter**: Ruxiao Qian (Lehigh University)
  Title: Intermittency property of SHE with Dobric-Ojeda fractional-type noise.
  Abstract: We managed to estimate that the order of n-th moment expectation of the solution is coincide with the Stochastic Heat Equation (SHE) with fractional Brownian motion time type noise, Stochastic Wave Equation(SWE) works as well. Also, we are going to generalized the Dobric-Ojeda noise and fractional Brownian motion type noise into one with linear interpolation, which also gives us the same order of the n-th moment expectation of the solution.

- **Presenter**: Xiao Shen (University of Utah)
  Title: The size of the largest hole in the infected region of first-passage percolation.
  Abstract: We establish upper and lower bounds for the size of the largest hole in the infected region $B(t)$ in first-passage percolation (FPP). The lower bound "$\log(t)$" holds for general edge weight τ_e with the minimum assumptions that τ_e is non-deterministic and satisfies $\mathbb{P}(\tau_e = 0) < p_c(d)$. The upper bound "$\log(t)^C$" holds for planner FPP under the additional assumptions that τ_e has an exponential moment and the uniform curvature condition is satisfied by the the limit shape of $B(t)$. This is a Joint work with Michael Damron, Julian Gold, and Wai-Kit Lam.

- **Presenter**: Daniel Slonim (Purdue University)
  Title: Ballisticity of One-Dimensional Random Walks in Random Environments with Jumps.
  Abstract: We examine a class of random walks in random environments on the one-dimensional integer lattice with bounded jumps, a generalization of the classic one-dimensional model. The environments we study have i.i.d. transition probability vectors drawn from Dirichlet distributions. For this model, we characterize recurrence and transience, and in the transient case we characterize ballisticity (non-zero limiting speed). For ballisticity, we define two parameters; one that governs finite trapping effects, and another that governs repeated traversals of arbitrarily large regions of the graph. We show that a right-transient walk is ballistic if and only if both parameters are greater than 1.

- **Presenter**: Evan Sorensen (University of Wisconsin, Madison)
  Title: Busemann process and semi-infinite geodesics in non-discrete spaces.
  Abstract: In the last 10-15 years, Busemann functions have been a key tool for studying semi-infinite geodesics in planar first and last-passage percolation. We study Busemann functions in non-discrete models and use these to derive geometric properties of the full collection of semi-infinite geodesics. This includes a characterization of uniqueness and coalescence of semi-infinite geodesics across all asymptotic directions. To deal with the uncountable set of points, we develop new methods of proof and uncover new phenomena, compared to discrete models. For example, in Brownian last-passage percolation, for each asymptotic direction, there exists a random countable set of initial points out of which there exist two semi-infinite geodesics in that direction. Further, there exists a random set of points, of Hausdorff dimension 1/2, out of which, for some random direction, there are two semi-infinite geodesics that split from the initial point and never come back together. Based on joint work with Timo Seppäläinen.

- **Presenter**: Jinwoo Sung (University of Chicago)
  Title: Minkowski Content of Liouville Quantum Gravity Metric Spaces.
  Abstract: A Liouville Quantum Gravity (LQG) surface is a "canonical" random two-dimensional surface, initially formulated as a random measure space and later as a random metric space. We show that the LQG metric almost surely determines the LQG measure, answering a question of Gwynne and Miller. More precisely, we prove that the LQG measure is almost surely a deterministic multiple of the Minkowski content measure

for the LQG metric. Our primary tool is the continuum mating-of-trees theory for space-filling SLE. This is a joint work with Ewain Gwynne.

- **Presenter**: Ran Tao (University of Maryland, College Park)
  Title: Gaussian fluctuations of a nonlinear stochastic heat equation in $d = 2$.
  Abstract: We study the Gaussian fluctuation of a nonlinear stochastic heat equation in $d = 2$. The equation is driven by a Gaussian multiplicative noise that is white in time, colored in space, and tuned logarithmically in its strength. By using the Malliavin-Stein's method, we prove the central limit theorem for an amplified fluctuation of the solution field. We also give a functional version of this CLT.

- **Presenter**: Adam Waterbury (University of California, Santa Barbara)
  Title: Approximating Quasi-Stationary Distributions with Interacting Reinforced Random Walks.
  Abstract: We propose two numerical schemes for approximating quasi-stationary distributions (QSD) of finite state Markov chains with absorbing states. Both schemes are described in terms of certain interacting chains in which the interaction is given in terms of the total time occupation measure of all particles in the system and has the impact of reinforcing transitions, in an appropriate fashion, to states where the collection of particles has spent more time. The schemes can be viewed as combining the key features of the two basic simulation-based methods for approximating QSD originating from the works of Fleming and Viot (1979) and Aldous, Flannery and Palacios (1998), respectively. The key difference between the two schemes studied here is that in the first method one starts with $a(n)$ particles at time 0 and number of particles stays constant over time whereas in the second method we start with one particle and at most one particle is added at each time instant in such a manner that there are $a(n)$ particles at time n. We prove almost sure convergence to the unique QSD and establish Central Limit Theorems for the two schemes under the key assumption that $a(n) = o(n)$.

- **Presenter**: Johnny Yang (Indiana University, Bloomington)
  Title: The extinction probability of stochastic FKPP on metric graphs.
  Abstract: We consider the extinction probability for the stochastic FKPP equation on metric spaces, with a focus on the circle and metric graphs. The results are compared with those on the real line.

Seminar on Stochastic Processes 2022

List of <u>*Remote*</u> Poster Presentations
(ordered alphabetically by LAST name)

- **Presenter**: Bishal Diyali (Central University of Haryana, India) *(Remote)*
  Title: Discriminating between the lognormal and log-logistic distributions in the presence of type-II censoring.
  Abstract: The lognormal and log-logistic distributions are two of the most commonly used distributions for studying positively skewed lifetime data. Both the distributions share number of interesting properties but selecting a more appropriate distribution and discriminating among them for a given data to best fit is an important issue. Further when the data are observed in the presence of some censoring scheme the problem becomes more challenging. In this paper, we address the problem of selecting a more appropriate distribution by discriminating based on the random samples drawn in the presence of type-II censoring. We consider the difference of the maximized log-likelihood functions, and compute the asymptotic distribution of the discrimination statistic. We further propose a modified discriminating approach, and compute the probabilities of correct selection to check the performance of the discrimination procedure. Finally, simulation study is conducted, and two real data sets are analyzed for the illustration purpose.

- **Presenter**: Zhaohu (Jonathan) Fan (University of Cincinnati) *(Remote)*
  Title: Partial Association between Mixed Data: Assessing the Impact of COVID-19 on College Student Well-being.
  Abstract: The outbreak of COVID-19 has lowered the well-being of college students across the world according to existing studies. In this paper, we study the association between well-being and common psychological factors. We analyze the data from two cohorts of first year undergraduates (in New Zealand) in April 2019 and 2020 (early pandemic), which enables a counterfactual to explore the impact of COVID-19. We found that by controlling for age and gender, the other covariates (students' healthiness, loneliness and accommodation) account for more of the association between well-being and anxiety in 2020 than that in 2019, implying an increased moderating effect of these covariates on the association after the strike of COVID-19. Our empirical findings may deliver various insights to domain experts and lead to more specific studies to assist university policy makers and healthcare providers in decision making.

  The empirical analysis in this paper is based on our proposed framework of partial association analysis for mixed data. Specifically, we propose to assess partial association using the rank-based measure, Kendall's tau, based on a unified residual that can be obtained from any general parametric model for continuous, binary and ordinal outcome. We show that the conditional independence between two outcome variables is equivalent to the independence between the corresponding pair of unified residuals. We also show several useful statistical properties of the proposed partial association measure. A practical guide that covers estimation and inference is provided.

- **Presenter**: Jeffrey Kuan (Texas A&M University) *(Remote)*
  Title: Joint moments of multi-species q-Boson.
  Abstract: The Airy2 process is a universal distribution which describes fluctuations in models in the Kardar-Parisi-Zhang (KPZ) universality class, such as the asymmetric simple exclusion process (ASEP) and the Gaussian Unitary Ensemble (GUE). Despite its ubiquity, there are no proven results for analogous fluctuations of multi-species models. Here, we will discuss one model in the KPZ universality class, the q-Boson. We will show that the joint multi point fluctuations of the single-species q-Boson match the single-point fluctuations of the multi-species q-Boson. Therefore, the single-point fluctuations of multi-species models in the KPZ class ought to be the Airy2 process. The proof utilizes the underlying algebraic structure of the multi-species q-Boson, namely the quantum group symmetry and Coxeter group actions.

- **Presenter**: Sefika Kuzgun (University of Kansas) *(Remote)*
  Title: Convergence of densities for stochastic heat equation.
  Abstract: Consider the one-dimensional stochastic heat equation driven by a space-time white noise with constant initial condition. The purpose of this poster is to present a recent result on the uniform convergence of the density of the normalized spatial averages of the solution u on an interval $[-R, R]$, as R tends to infinity, to the density of the standard normal distribution, assuming some non-degeneracy and regularity conditions on the diffusion coefficient. These results are based on the combination of Stein method for normal approximations and Malliavin calculus techniques. This is a joint work with David Nualart.

- **Presenter**: Yanghui Liu (City University of New York) *(Remote)*
  Title: Numerical stochastic integration with respect to Gaussian noise.
  Abstract: Rough paths techniques give the ability to define solutions of stochastic differential equations driven by low-regularity signals which are not semi-martingales. In this context, rough integrals are usually Riemann-Stieltjes integrals with correction terms that are sometimes seen as unnatural. As opposed to those somewhat artificial correction terms, in this talk I will introduce a trapezoid rule for rough integrals driven by general d-dimensional Gaussian processes. Namely we shall approximate a generic rough integral by Riemann sums avoiding the usual higher order correction terms, making the expression easier to work with and more natural. Our approximations apply to all controlled processes and to a wide range of Gaussian processes including fractional Brownian motion with a Hurst parameter $H > 1/4$. This is a joint work with Zachary Selk and Samy Tindel.

- **Presenter**: Oussama Rida (National school of applied sciences - University Hassan First) *(Remote)*
  Title: Two-Parameters Stochastic Chapman-Richards Diffusion Process: Application to Bike-Sharing Growth in Europe.
  Abstract: A stochastic diffusion process based on two-parameter Chapman-Richards curve is proposed in this study. From the corresponding Itô's stochastic differential equation (SDE), firstly we establish the probabilistic characteristics of the studied process, such as the solution to the SDE, the probability transition density function and their distribution, the moments function in particular the conditional and unconditional trend functions. Secondly, we treat the parameters estimation problem by using the maximum likelihood method in basis of the discrete sampling, thus we obtain non-linear equation that can be solved by Grasshopper Optimization Algorithm, finally we apply the model to forecast bike-sharing growth in Europe.

- **Presenter**: Qiang Wu (University of Illinois at Urbana-Champaign) *(Remote)*
  Title: Mean field spin glass under weak external field.
  Abstract: We study the fluctuation and limiting distribution of free energy in mean-field spin glass models with Ising spins under weak external fields. We prove that at high temperature, there are three sub-regimes concerning the strength of external field $h \approx \rho N^{-\alpha}$ with $\rho, \alpha \in (0, \infty)$. In the super-critical regime $\alpha < 1/4$, the variance of the log-partition function is $\approx N^{1-4\alpha}$. In the critical regime $\alpha = 1/4$, the fluctuation is of constant order but depends on ρ. Whereas, in the sub-critical regime $\alpha > 1/4$, the variance is $\Theta(1)$ and does not depend on ρ. We explicitly express the asymptotic mean and variance in all three regimes and prove Gaussian central limit theorems. Our proofs mainly follow two approaches. One utilizes quadratic coupling and Guerra's interpolation scheme for Gaussian disorder, extending to many other spin glass models. However, this approach can prove the CLT only at very high temperatures. The other one is a cluster-based approach for general symmetric disorders, first used in the seminal work of Aizenman, Lebowitz, and Ruelle (Comm. Math. Phys. 112 (1987), no. 1, 3–20) for the zero external field case. It was believed that this approach does not work if the external field is present. We show that if the external field is present but not too strong, it still works with a new cluster structure. In particular, we prove the CLT up to the critical temperature in the Sherrington-Kirkpatrick (SK) model when $\alpha \geq 1/4$. We further address the generality of this cluster-based approach. Specifically, we give similar results for the multi-species SK model and diluted SK model.

- **Presenter**: Zhengye Zhou (Texas A&M University) *(Remote)*
  Title: Orthogonal polynomial duality functions for multi-species SEP($2j$) and ASEP(q, j).
  Abstract: In this poster, I present the orthogonal polynomial self dualities for the multi-species symmetric exclusion process (SEP($2j$)) and asymmetric exclusion process (ASEP(q, j)), which allow up to $2j$ particles to occupy a site. We are interested in orthogonal duality because the expectations of orthogonal polynomials duality functions give all moments of the process.

  We show that multi-species SEP($2j$) is self-dual with orthogonal duality functions given by homogeneous products of multivariate Krawtchouk polynomials and the multi-species ASEP(q, j) is self-dual with nested products of multivariate q-Krawtchouk polynomials. We use different methods for each process, yet both of the methods use representations of Lie algebras. For multi-species SEP($2j$), the duality functions come from unitary intertwiners between different $*$-representations of Lie algebra $\mathfrak{sl}_{n+1}$, while for multi-species ASEP(q, j), we find unitary symmetries in $U_q(\mathfrak{gl}_{n+1})$.