4 Macroeconomic patterns and contemporary models

Most macroeconomic texts start with an elaborate description of complex historical economic patterns, then jump to abstract theoretical constructs describing the economic system. We will instead start with describing how complex historical patterns can be partitioned into organized parts that can be tied to economic models explaining growth and cyclical behavior. There are many ways to partition a complex pattern, although not all are useful for creating models that identify sensitive entry points aimed at policy actors for system change. This chapter outlines a partitioning process that should facilitate creating generic models without disconnecting symbiotic relationships underlying parts of the decomposed historical pattern. It additionally attempts to tie dominant parts of the decomposed pattern to contemporary models of economic growth and cycles. In the following chapters, we will also visit classical models of growth, limits and cycles, which are relatively more systemic than the contemporary models, as well as operational representations of theories that embody managers as role players, making them amenable to intervention through managerial policies. All presented models use the stock and flow consistent system dynamics representation.

Composition of complex historical patterns

Figure 4.1 shows historical time series for GDP per capita in the US from 1871 to 2009 along with the trend it follows. A visual examination of the complex pattern shows several ups and downs superimposed on an exponential growth trend. After the growth trend is separated, the remaining pattern will still not reduce to a simple periodicity. We know however, that empirical economists have found many cyclical trends in the historical records of market economies. Notable among these are the business cycles that have a periodicity of 5-10 years, the Kuznets cycles discovered by Russian economist Simon Kuznets with 20-30 year periodicity, and a long wave discovered by another Russian economist Nikolai Kondratieff with 50-70 year periodicity. Thus, another way to understand the composition of the pattern in Figure 4.1 is to combine those known periodicities with the observed long term exponential growth trend, which is attempted in Figure 4.2. Note there is a remarkable similarity between the historical pattern of Figure 4.1 and the synthesized pattern of Figure 4.2 not only giving credence to the existence of the three types of cycles recorded in market economies but also suggesting that the complex historical pattern can be sliced into its stylized components which should be the starting point to build economic theory.

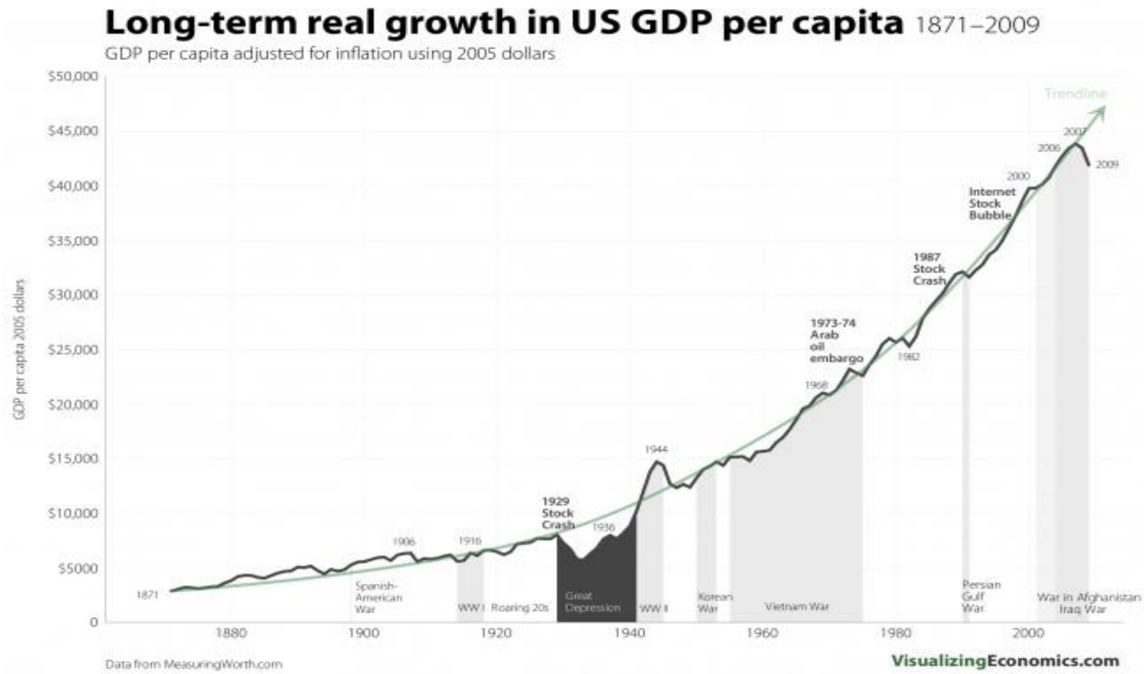

Figure 4.1 **Historical time series of US GDP/capital in 2005 dollars**

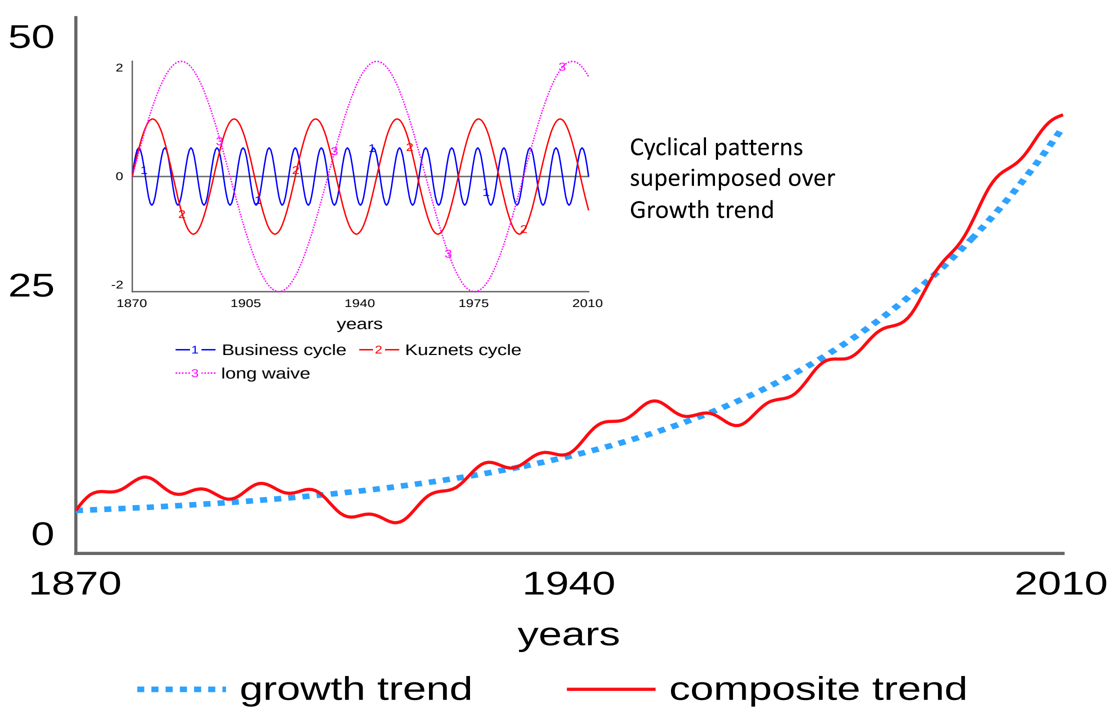

Figure 4.2 **Synthesized composite trend combining long term growth and known cyclical tendencies.**

Before I outline the key structural elements of the contemporary models attempting to explain the growth and cyclical components of the historical trend shown in Figure 4.2, I'll try to explain the partitioning philosophy that should create patterns for constructing generalizable theories rather than situation-specific forecasting models.

Partitioning philosophy

A precise definition of a pattern of behavior bounds a theory explaining that pattern. Many types of behavioral patterns occurring in reality can be visualized in the conceptual space illustrated in Figure 4.3 that extends in three dimensions labeled Historical patterns, Institution-specific patterns, and Period specific patterns.

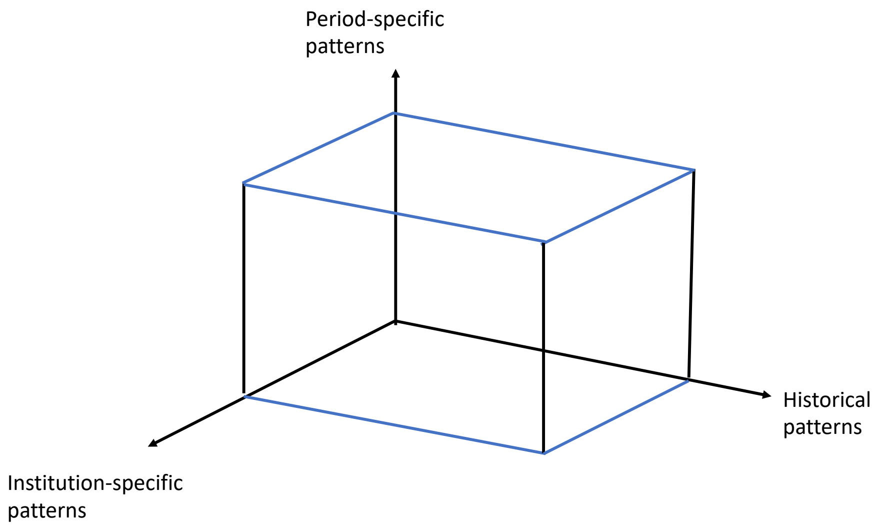

Figure 4.3 Conceptual space for visualizing economic patterns to determine system boundary.

A historical pattern recorded for a specific economy (country or region) will contain multiple modes of behavior simultaneously occurring in the system over a selected time frame, which will create a complex and unique historical pattern as illustrated in Figure 4.1. Several stylized country or region- specific patterns might also be recorded for a class of institutions, like market economies. The economic cycles of various periodicities shown in Figure 4.2 would fall in this category. Last but not least, stylized patterns of behavior, including dynamic equilibria, might be recorded in various extended periods of history, such as urban decay in the cities of industrialized countries in the 1970s (1); egalitarian land distribution among farmers in medieval India (2); worker capitalism in artisan economies (3), Feudalism in colonial India (2) and czarist Russia (4), etc.

When viewed simultaneously as a composite historical trend, multiple patterns can identify the boundary of a unique and complex model that, when appropriately calibrated, can replicate a specific history but can be used for little else. Such a conceptual slice of the observed patterns of behavior is shown in Figure 4.4.

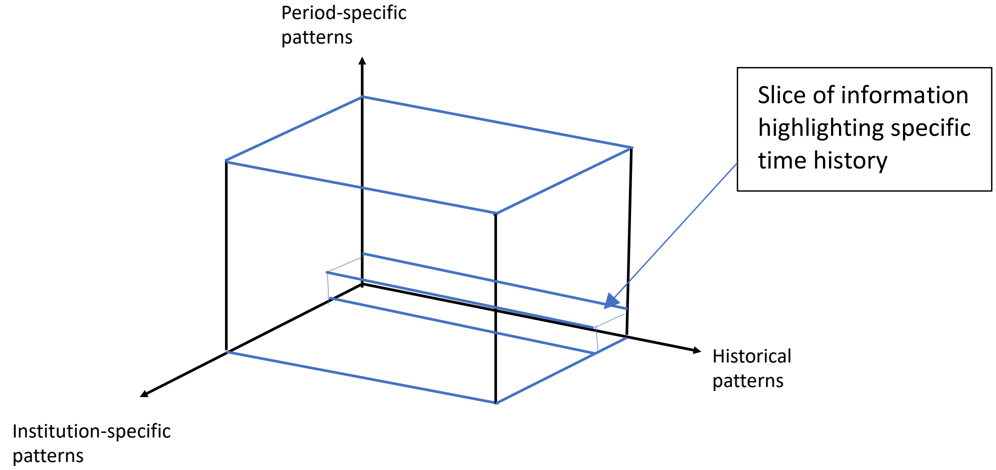

Figure 4.4 Slice of information subsuming multiple patterns simultaneously existing in economy-specific historical data.

A historical time series is often far too complex to be replicated by a parsimonious model, although a complex model might be able to replicate and extrapolate it into future. Such a model might appear to forecast future, but the validity of this forecast may only be argued on basis of the shock and awe of its complexity and the sophistication of its calibration process, which is how forecasting models are often defended. However, when stylized slices subsuming institution-specific and period-specific patterns are viewed as multiple manifestations of a ubiquitous phenomenon, they help to create a generic theory not only explaining those manifestations but also pointing to the policies needed for changing one manifestation to another. Thus, real world data visualized as multiple institution-specific and period-specific manifestations create a slice of reality shown in Figure 4.5, which can serve a sound bases for developing a generalizable theory.

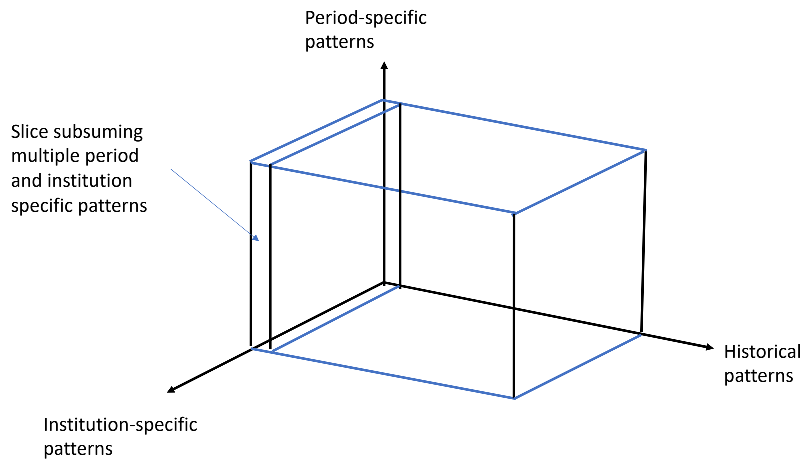

Figure 4.5 Slice of information subsuming multiple stylized patterns existing in several institutions over multiple periods.

Such a model does not track any specific history and cannot be calibrated using historical data. Nor can it forecast future. Instead, it captures a set of ubiquitous patterns contingent upon policies active at pertinent times and places. It thus represents a theory explaining a range of manifestations of a specific slice of the complex historical pattern addressed. Since this theory subsumes many manifestations of a stylized pattern, it can also be experimented with to identify changes in policies for achieving changes in system behavior. All stylized models of economic behavior, classical and contemporary, fall in this category, although not all are useful policy tools.

Chapter 3 has outlined how managerial role play creates feedback processes that may endogenously lead to stylized patterns of behavior, like growth, limits and oscillations. In this chapter, we'll attempt to reconstruct the contemporary theories of economic growth and cycles as computable stock and flow models and attempt to tie them to the respective stylized components of the historical economic trends illustrated in Figure 4.2.

Contemporary economic growth models

Contemporary economic growth models can be divided into two broad categories: 1) demand driven and 2) supply driven. Variations on these themes may include technological progress and growth in productivity that are often discussed in the context of endogenous growth. We revisit these models with focus on the circular information paths or feedback loops driving the growth process in each and how their momentum might get limited. The contemporary growth models are however described below in the chronological order they appeared.

1. Keynes's concept of demand driving supply through multiplier effect

The earliest model that can be placed in the contemporary category is Keynes's concept of multiplier, which is a demand driven growth process. It explained how an exogenous stimulus such as an autonomous increase in government spending may cause the economy to grow its output to a new plateau through repeated but diminishing rounds of additional income generation. A simple macroeconomic growth model based on Keynes (5) is shown in Figure 4.6.

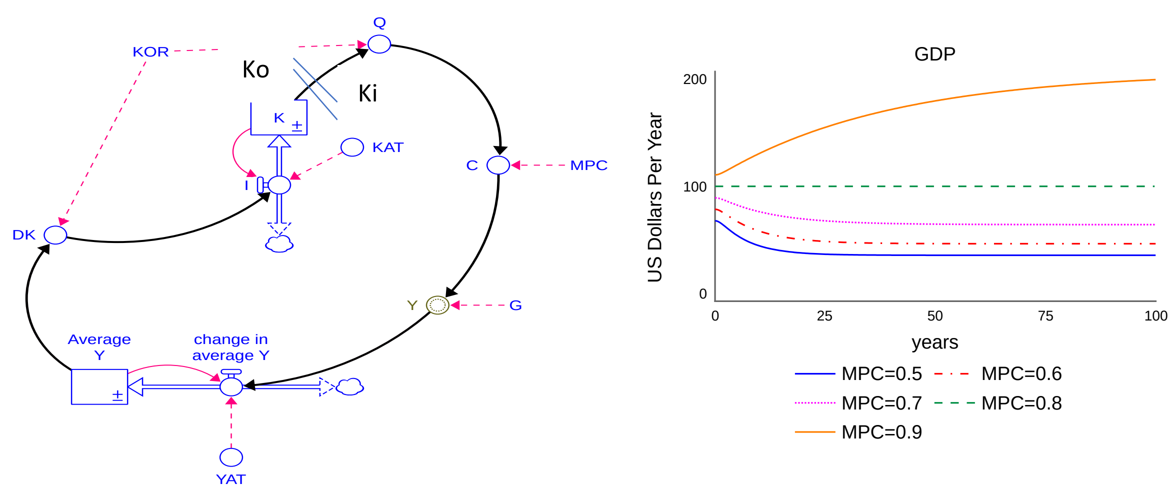

Figure 4.6 Growth from Keynesian multiplier

Table 4.1 describes the computational logic of this model. Y represents the gross domestic product, C is consumption, I investment, G government spending, MPC marginal propensity to consume, K capital, DK desired capital, Q production and KOR capital output ratio.

Table 4.1 Computational logic of Keynes's multiplier model

Variable	Equation
Average Y(t)	Average Y(t - dt) + (change in average Y) * dt
C	Q * MPC
change in average Y	(Y-Average Y)/YAT
DK	(Average Y)*KOR
I	(DK-K)/KAT
K(t)	K(t - dt) + (I) * dt
Q	K/KOR
Y	C+G

In this system an autonomous increase ΔG in G will yield much larger cumulative addition to Y through subsequent but diminishing rounds of additional income generation. Thus:

$$\Delta Y = \Delta G * (1 + MPC + MPC^2 + MPC^3 \ldots\ldots\ldots)$$

$$= \Delta G / (1 - MPC)$$

Now, we can also represent above model using stocks and flows as in Figure 5.1 and compute the OLSSG of the major positive feedback loop created by the computational process. Note I = 0 in steady state, hence the link between I and Y can be omitted for computing the gain of the multiplier. If we break the link from K to Q and enclose the string of computations so created in a black box, K_o will be the output of the black box and box K_i the input.

In steady state,

Average Y = Y , YAT is averaging time for calculating average Y.

$$K_o = DK$$

$$= Y*KOR = (C+G)*KOR$$

$$= (Q*MPC + G)*KOR$$

$$= (K_i /KOR) *MPC + G)*KOR$$

$$= K_i *MPC + G*KOR$$

$$K_o/ K_i = MPC + (G*KOR/ K_i)$$

$$= MPC + G/Q$$

When there is no growth or decline in the economy, the OLSSG, K_o/K_i =1. Hence, MPC = 1 – G/Q, or G = Q*(1-MPC). When G is autonomously increased, OLSSG > 1, growth happens, but in each subsequent round, Q rises and hence G/Q diminishes until OLSSG of the multiplier loop =1. This process creates diminishing rounds of growth until OLSSG becomes 1.When an autonomous change in MPC or G reduces income Y, OLSSG < 1. Subsequent diminishing rounds of decline lead to another lower equilibrium where OLSSG =1.

Figure 4.6 also shows the growth behavior for different values of MPC ranging from 0.5 to 0.9, with equilibrium set at G = (1-MPC)*Q. This system can come to equilibrium with OLSSG =1 with any starting value created by changing MPC or G, but it does not oscillate. Any growth or decline trend initiated by the change will attenuate until the change becomes zero.

2. Harrod-Domar model of savings/investment driving economic growth

The earliest supply side growth model is due to Professors Roy Harrod and Evsey Domar, who working autonomously in Cambridge, England and Cambridge, Massachusetts came to the same conclusion. This model linked economic growth to the rate of saving that drove investment and the productivity of capital defined by capital output ratio (KOR). Figure 4.7 renders this model with stocks and flows, adding also the autonomous population growth structure so we can track per capita income and unemployment rate. Table 4.2 shows the computational logic of the model. S is saving rate, and s is fraction of income saved, which is equal to (1-MPC). KLR is the capital labor ratio.

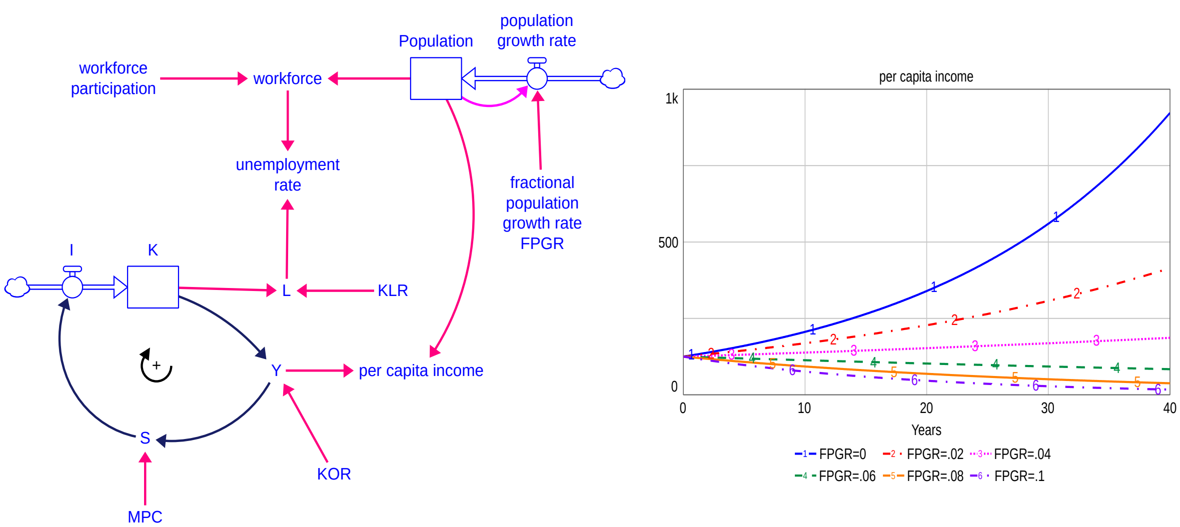

Figure 4.7 **Harrod-Domar model of saving driven economic growth**

$\Delta K = I = S = s*Y = (1\text{-}MPC)*Y$

Since, $Y = K/KOR$

$\Delta K/K = (1\text{-}MPC)/KOR = \Delta Y/Y$

Which is the equilibrium rate of growth in a closed economy when all savings are invested and there is no depreciation. Note labor and unemployment rate do not factor into the calculation, emphasizing capital theory of value. Note also that if population growth rate exceeds (1-MPC)/

KOR, per capita income will decline. The accompanying graph shows per capita income for different values of fractional population growth rate. MPC is kept constant at 0.9 yielding s = 0.1, and KOR is pegged at 2.

Table 4.2 Computational logic of Harrod-Domar model

Variable	Equation
K(t)	K(t - dt) + (I) * dt
Population(t)	Population(t - dt) + (population growth rate) * dt
I	S
population growth rate	Population*fractional population growth rate FPGR
L	K/KLR
per capita income	Y/Population
S	Y*(1-MPC)
unemployment rate	1-(L/workforce)
workforce	Population*workforce participation
Y	K/KOR

The open-loop gain of the minor positive feedback loop driving growth = (1+(1-MPC)/KOR)), which can be calculated as illustrated in chapter 3. A low value of MPC and a high value of KOR can increase growth in theory, but both have not been achievable in the poor countries the model was used in. In fact, the simplicity of this growth model led to its overuse with disastrous results. KOR often turned out to be lower than expected and low-income populations of the poor countries really could not increase saving rates. To be able to meet their investment targets, these countries often took loans from World Bank and IMF. As a result, a large proportion of these countries ended up with unprecedented debt burden along with widespread poverty and economic stagnation. Furthermore, unprecedented population growth has worsened their per capita income.

3. Solow-Swan model of labor and productivity driving economic growth

The Solow-Swan model of economic growth proposed by professors Robert Solow and Trevor Swan in mid 1950s, also working independently, extended the Harrod-Domar model by factoring labor contribution and productivity growth, both exogenously determined, into the creation of output Y. This was done using a Cobb Douglas function with constant returns to scale. They also accounted for capital decay over time, requiring maintenance investment even when there is no growth. Figure 4.8 shows the structure of this model using stocks and flows, Table 4.3 lists its computational logic.

The model is initialized in a growth mode in which Δ Y > Δ Population, hence per capita income shows exponential growth. MPC is subsequently increased from .9 to .95 at time 100, yielding reduction of s from 0.10 to 0.05. Figure 4.8 also shows a simulation of the model with those settings.

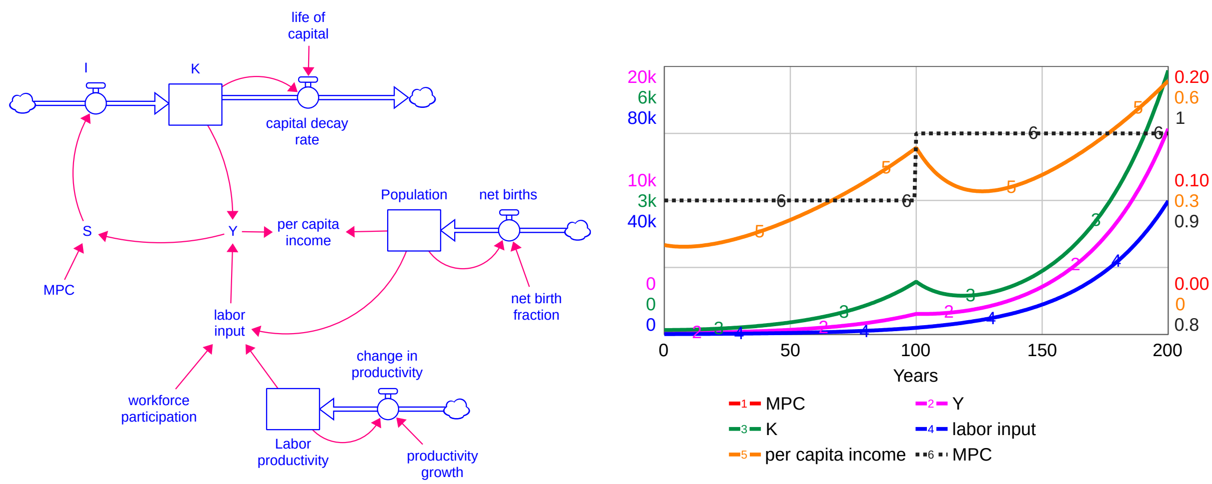

Figure 4.8 **Solow-Swan model simulated with 50% reduction in s at time 10**

Table 4.4 **Computational logic of the Solow-Swan Model**

	Equation
K(t)	K(t - dt) + (I - capital decay rate) * dt
Labor productivity(t)	Labor productivity(t - dt) + (change in labor productivity) * dt
Population(t)	Population(t - dt) + (net births) * dt
capital decay rate	K/life of capital
change in labor productivity	Labor productivity*productivity growth
I	S
net births	Population*net birth fraction
labor input	workforce participation*Population*Labor productivity
MPC	0.90+STEP(0.05,100)
per capita income	Y/Population
S	Y*(1-MPC)
Y	(K^0.5)*(labor input^0.5)

A reduction in saving rate causes only a momentary decline in capital as investment falls below the level necessary to compensate for decay, but rising workforce and productivity buoy it back. Note however that the externalities arising from productivity growth, especially environmental costs, are not considered.

4. Romer model of endogenous growth

Although widely taught in macro-economic texts, the Solow-Swan model assumes autonomous technological growth driving production (6). Romer pointed out in the late 1980s using cross country data that technological growth in fact depends on the labor engaged in the knowledge sector of an economy (7, 8). Figure 4.9 extends the Solow-Swan model of Figure 4.8 by relating growth in labor productivity to the knowledge labor, although the fraction of workforce in the knowledge sector is exogenously specified. Table 4.5 lists the computational detail of this model.

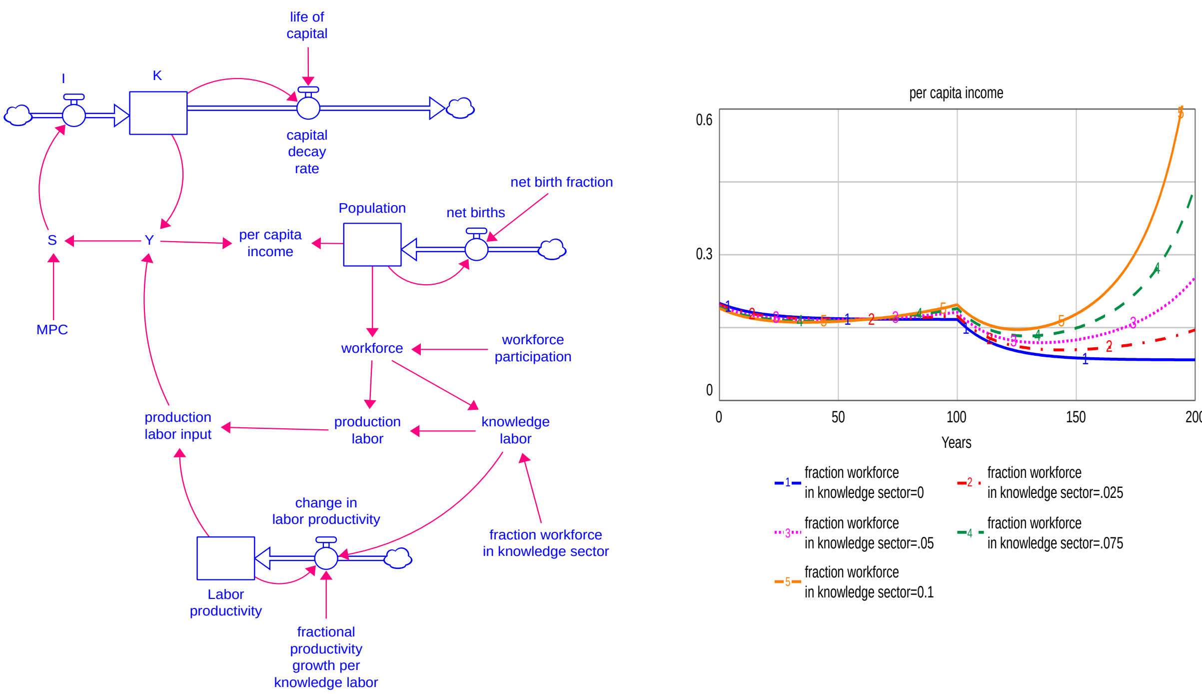

Figure 4.9 **Romer's knowledge driven endogenous growth process added to Solow model**

Note also that while the knowledge sector labor creates growth in productivity, it does not contribute to production. Thus, in the short run, income per capita will be lower when a larger fraction of workforce is allocated to the knowledge sector. The subsequent increases in productivity will however create higher growth in the long run, also accelerating recovery when saving rate is halved as in the experiment with Solow-Swan model. This is borne out by the simulations of the model shown in Figure 4.9. The model was simulated with various fractions of workforce in the knowledge sector ranging between 0 and 0.1. Additionally, as in Figure 4.8, saving rate is reduced by 50% at time 100 to understand the recovery process as driven by population and productivity growth mechanisms.

Table 4.5 Computational logic of Romer's endogenous growth process added to Solow-Swan model.

variable	Equation
K(t)	K(t - dt) + (I - capital decay rate) * dt
Labor productivity(t)	Labor productivity(t - dt) + (change in labor productivity) * dt
Population(t)	Population(t - dt) + (net births) * dt
capital decay rate	K/life of capital
change in labor productivity	fractional productivity growth per knowledge labor*knowledge labor*Labor productivity
knowledge labor	workforce*fraction workforce in knowledge sector
per capita income	Y/Population
production labor	workforce-knowledge labor
production labor input	production labor*Labor productivity
S	Y*(1-MPC)
workforce	Population*workforce participation
Y	(K^0.5)*(production labor input^0.5)

Romer's endogenous growth model answered the question why productivity growth is higher in some economies than the other. It also operationalized the productivity growth process by pointing to the policy of increasing allocation of resources to the knowledge sector either through public expenditure or incentives to the private sector.

5. Variations on Romer Model

Variations on this model attempt to endogenize some of the parameters, notably the saving rate, which was tied to average income creating what is known as Ramsey-Cass-Koopmans model (9) outlined in Figure 4.10. Note that while the Koopmans model links saving rate to an explicit computation of an optimal value considering multiple periods, in reality, people would achieve an optimal value through iterative corrections, which create a dynamic optimization process. Hence MPC is modeled in the dynamic model of Fig 4.10 as a function of a multi-period average of per capita income, which returns the following logic for the determination of MPC:

$$MPC = f(\text{average per capita income}) \qquad f' < 0$$

Where expected per capita income is an exponential average of past per capita income averaged over 2 years for this model but this time constant can be flexibly specified. The behavior of this model is still sensitive to the fraction of work force in the knowledge sector as in case of Tomar's mode of endogenous growth as shown in the simulations in Figure 4.10. Since growth in income per capital decreases MPC – hence increasing savings and investment, growth rates are accelerated.

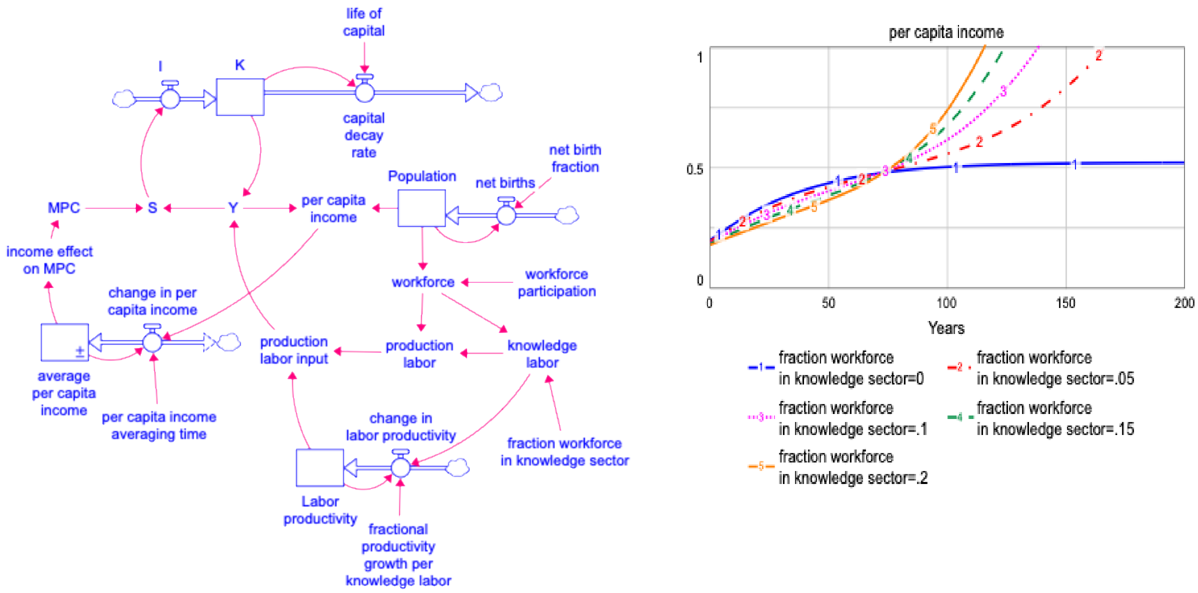

Figure 4.10 Endogenization of MPC as suggested by Ramsey, Cass and Koopmans.

Contemporary growth models do not seem however to treat the environmental and social limits to growth as the classical models did. In fact, neoclassical economics mostly excluded environmental, demographic and social limitations from its formal analyses until early 1970s, although it extensively addressed the periodic constraints to growth arising out of the stagnation caused by imbalances in the market. As an exception, Hotelling (10) dealt with exhaustible resources with concerns that the market may not be able to return optimal rates of exhaustion, but without pessimism about the technology to bring to fore new sources as old ones are exhausted. These early concerns have been followed by a blissful confidence in the ability of the technological developments and prices to provide access to unlimited supplies of resources (11).

There have however been eloquent arguments for integrating depletion of resources into the models of economic growth (12). Albeit, the bulk of the work in contemporary economics has not deviated much from its earlier focus on optimal rates of depletion and pricing of resources (13) without concerns for environmental capacity, which are mostly expressed in passing. There have been some concerns also expressed about intergenerational equity, but its treatments remain tied to arbitrary rates of discount (14, 15). Environmental analysis seems to have appeared as an add-on in response to the environmental movement spearheaded by the infamous *Limits to Growth* study (16). In this add on, the neoclassical economic theory has mostly continued to assume mineral resources to be unlimited and to expect prices and technological developments to continue to unearth richer mines so existing mines may be abandoned (17). The reality of political power, the creation and resolution of social conflict and the psychological and behavioral factors also remain excluded from the contemporary models, although they contribute significantly to the performance of the economies as well as limiting their growth (18). We will discuss those factors later in this book.

Contemporary models of economic cycles

The term "economic cycle" has sometimes been used interchangeably with business cycle, but the former refers to a wide range of periodic ups and downs superimposed on growth history as discussed earlier in this chapter, while the latter usually implies a 5-7 year cyclical trend observed in market economies. The business cycle has traditionally been attributed to investment dynamics (19), although capital formation lead-times and capital output ratios existing in reality would in fact generate cycles of much longer periodicity (20). The real business cycle theory attempted to explain deviations from normal business cycle periodicity by attributing them to the rational responses of the economic actors to external events (21). It even suggested that depending on external events, each deviation from the business cycle periodicity will have a different explanation. Parsimonious models of these approaches are discussed below:

1. Cyclical behavior from Investment dynamics

Samuelson extended the Keynesian multiplier concept to include an acceleration process for explaining the 5-8 year business cycle as a demand-driven investment dynamic. Figure 4.11 isolates the acceleration process from Samuelson's multiplier-accelerator model and shows its behavior with different values of KOR (19).

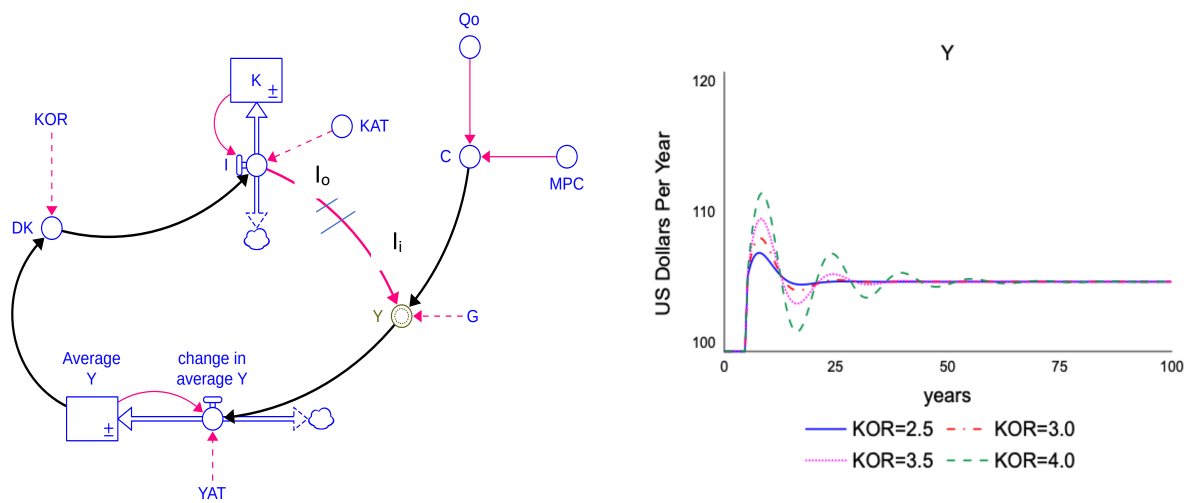

Figure 4.11 The accelerator loop isolated from multiplier.

To calculate OLSSG of this loop, we assume K and Y are in steady state and break the loop as shown:

$I_o \quad = 1/KAT * (DK-K)$

$\quad =1/KAT *KOR * (I_i + C + G) – 1/KAT *(K)$

In steady state, I = 0 , Hence K = KOR * (C + G)

Therefore, $I_o / I_i = KOR/KAT$

This feedback arrangement does not, however, create growth/decline behavior typical to positive feedback due to the peculiar way it is coupled with the minor negative feedback loop adjusting investment. As Graham (22) points out, oscillations can occur when a series of delays (first-order negative loops) are interconnected to form a positive loop, which is the case in the accelerator structure that connects the adjustments of K and Average Y through first order negative feedback loops.

A high gain created by the ratio KOR/KAT can lead to overly high adjustments in 'I' that may drive K above DK. This may create an overshoot subsequently calling for negative adjustment, which is borne out by the sensitivity simulations of the isolated accelerator loop in Figure 4.10. Since, the gain of the accelerator positive feedback loop is given by KOR/KAT, lower values of this ratio lead to lower gain and hence less overshoot.

This pattern is perpetuated when the multiplier and accelerator loops are combined as shown in the sensitivity runs in Figure 4.12, with one caveat: the periodicity of the combined system is much longer than the one shown only by accelerator. Note that the system consists of two positive feedback loops: the multiplier – a major loop, and the accelerator – a minor loop. Additionally, there are two minor negative feedback loops that were assumed to be in steady state when we calculated the gains of the positive feedback loops. The combination of the multiplier and accelerator loops creates an overshoot and oscillatory behavior only when the gain of the accelerator loop given by KOR/KAT is large enough.

With KOR = 4, which, corresponds to the US economy and KAT of about 3 years, the model creates a periodicity of about 23 years as shown in Figure 4.12, which is much longer than the recorded business cycle. Indeed, Low (20) challenged Samuelson's explanation of business cycle using his multiplier-accelerator model saying that reasonable capital adjustment times create cycles with much longer periodicity than the business cycle. Yet, variations of this model appear in most macroeconomic texts to explain the so-called business cycle.

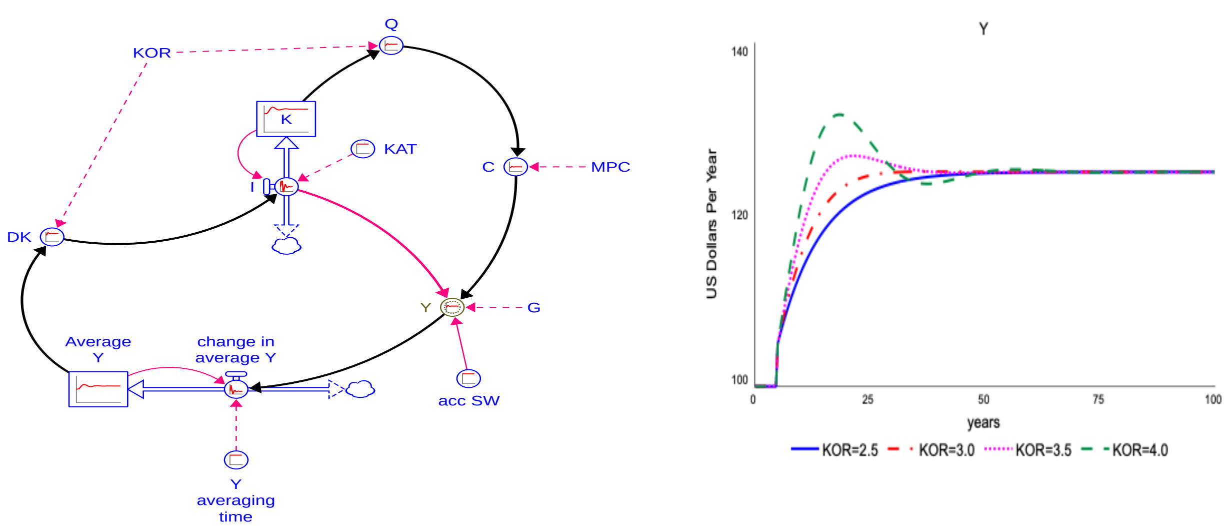

Figure 4.12 **Sensitivity of oscillatory behavior to changes in the gain of accelerator loop.**

2. Real business cycle models

The term real business cycle emphasizes the absence of money in the description of economic instability. The related models often add external shocks to endogenous growth logic, demand as well as savings driven, to explain ups and downs over the course of growth (23).

It should be noted that if a model subsumes logic for oscillations such as in the variations on multiplier accelerator and the generic oscillatory structures described in Chapter 3, it will show systematic instability in response even to random shocks. On the other hand, endogenous growth structures lacking oscillatory logic will require systematic external shocks to exhibit cyclical behavior which is entrained to the shocks thus making each cycle different per Lucas's explanation (21). Furthermore, while the periodicity of the former will depend on their internal structure, that of the later will depend on the systematic shocks. Hence, real business cycle models cannot provide endogenous explanations of the recorded economic cycles of various periodicities. This is demonstrated in Figure 4.13(a) in which normally distributed random noise was added to KOR in the multiplier accelerator model of Fig 4.12 initially set at equilibrium.

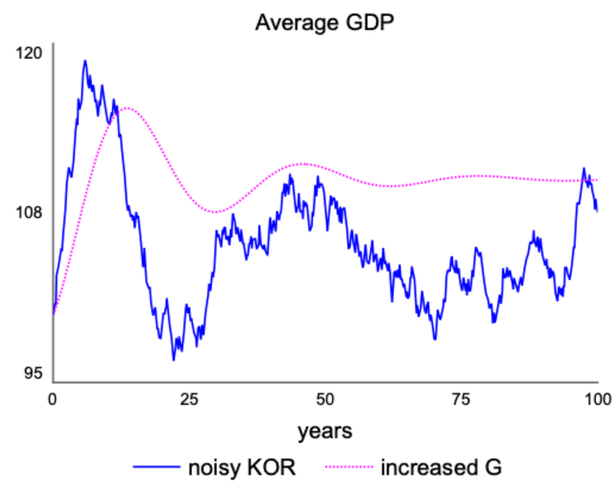

Figure 4.13(a): Randomness added to KOR in the Multiplier accelerator model of Figure 4.11

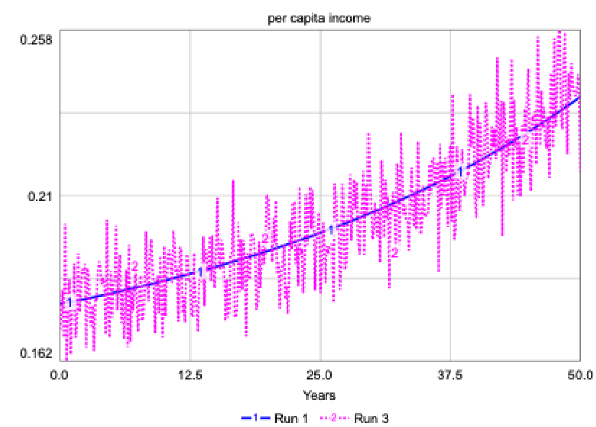

Figure 4.13(b): Randomness added to productivity in the endogenous growth model of Figure 4.9

Figure 4.12 Randomness added to models with and without oscillatory structure.

The noise creates oscillations of periodicity roughly corresponding to that created by a one-time step disturbance applied to G. Figure 4.13(b) shows behavior when similar random noise is added to productivity in the endogenous growth model of Figure 4.9. As internal structure to generate oscillatory behavior does not exist in the endogenous growth model, the outcome is entrained to the exogenously applied noise rather than exhibiting a unique periodicity.

Business cycle models are mostly used for forecasting various phases of a cycle, so interventions could be devised for slowing downturns and recovering from recessions. Cycles are not seen as continuums of ups and downs so policies to dampen them could be explored. Policies also do not target managerial roles. Instead, they often target changing important indicators like interest rates and money velocity that bring important messages to drive managerial decision-making. There is a need to develop operational models that can explain the cyclical behavior of recorded periodicities, then formulate policies to dampen the cycles and create stable paths to postulated targets. This will be discussed in Chapter 6.

Summary

Complex historical patterns must be decomposed into their simpler parts for understanding endogenous relationships generating them. The simpler parts include growth as well as cycles of various periodicities. Contemporary models attempting to explain growth and cyclical behavior are discussed. While growth behavior generally occurs due to gain in a positive feedback loop, while major negative feedbacks create instability as discussed in Chapter 3, there are exceptions to these rules. Under certain contingencies, a positive feedback loop can generate goal seeking behavior; and a positive feedback loop, combined with a minor negative feedback loop, can even lead to oscillations. A case in point is Samuelson's multiplier-accelerator model often used to explain business cycle, which may generate overshoot and oscillatory behavior with certain parameter sets, which this chapter has also attempted to explore. In this author's view, contemporary real business cycle models that often modify growth models to explain cyclical economic behavior do not adequately explain recorded economic cycles of various periodicities. Predominant themes of these models are described, and their limitations discussed.

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