Exploring Relationship Between Wave Speed, Bulk Modulus, and Density of a Medium

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Abstract

The speed of a longitudinal wave through any medium is a constant, regardless of the force causing the wave. The goal of this lab is to observe the relationship between the density and bulk modulus of a liquid and the speed of the longitudinal waves that travel through it. The wave speed of waves traveling through a liquid medium of known density can be measured, which can then be used to calculate the bulk modulus of the fluid. It was found that the calculated bulk modulus of the fluid did not match the theoretical value, and thus a change to the model used or experimental design may be necessary for future experimentation.

Background

The Newton-Laplace Equation models the speed of sound through liquids at low frequencies at a given temperature [1]. The speed of a longitudinal wave in a fluid is dependent on the density and bulk modulus of the liquid it is traveling through, as modeled by the equation

$$v = \sqrt{\frac{B}{\rho}} \qquad \text{(Eq. 1)}$$

where B is the bulk modulus of the fluid, ρ is the density of the fluid and c is the speed of the wave. Common solutions, such as water, have been studied and have known bulk moduli at various temperatures. The bulk modulus of water is 1.96×10^9 [2]. We can use the above model to determine the bulk modulus of a fluid that does not have a textbook value, and by testing the bulk modulus of water, we can test the accuracy of our model and experiment.

Method

Materials

- At least 1 liquid with a known bulk modulus
    - Water
- Variety of liquids with unknown bulk modulus
    - Maple syrup
    - Gel hand soap
    - Hand sanitizer
    - Etc.
- Video recording device (ie. smartphone)
- 2 Rulers
- Cup or other containment vessel
- Graduated cylinder
- Triple beam balance

Procedure

Fill a styrofoam cup with one of the liquids listed above and drop a small metal ball from 2.5 inches above the cup while recording from above, as seen in Figure 1. For dark liquids, it may be helpful to use a flashlight to make the waves more visible. It may also be helpful to use a larger vessel and a higher amplitude to be able to see clearer waves. It is recommended to measure the diameter of the liquid's surface within the cup, as well as measure the density (mass divided by volume) of the liquid, and the mass of the ball. Use the time it took for the wave to travel from the center of the cup to the edge of the cup using the recording, as well as the diameter, to calculate the wave speed, which can then be converted to the bulk modulus using Equation 1.

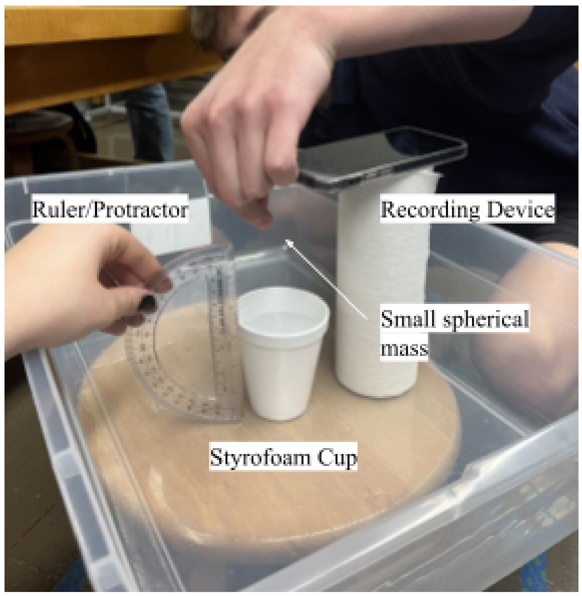

Figure 1) Example set up for a lab testing the relationship between the density of a medium, wave speed and bulk modulus.

Results and Analysis

Trial Information	Trial #	Time (s)	Measured Speed of Wave (m/s)	Experimental Bulk Modulus (Pa)
Water:				
ρ=1.0 g/mL = 1000 kg/m^3	1	1.06	0.031	0.961
Diameter of surface = 7.5 cm	2	0.17	0.191	36.5
3.25 cm = 0.0325 m	3	Bad Trial	N/A	N/A
Diameter of surface = 0.121 m	1	0.13	0.465	217
Radius = 0.0605 m	2	0.16	0.378	143
	3	0.22	0.275	75.6
			Average:	38.3 Pa ± 3.4 Pa
Maple Syrup:				
ρ=1.34 g/mL = 1340 kg/m^3	1	0.08	0.375	188
Diameter of surface = 6.0 cm	2	0.06	0.500	335
3.0 cm = 0.03 m	3	0.08	0.375	188
ρ=1032.85 kg/m^3	1	0.22	0.341	120
Diameter of surface = 0.15 m	2	0.24	0.313	101
Radius = 0.075 m	3	0.25	0.3	93.0
			Average:	154 Pa ± 17.1 Pa

Gel Soap:				
ρ=1.21 g/mL = 1210 kg/m^3	1	0.13	0.250	75.6
Diameter of surface = 6.5 cm	2	0.36	0.090	9.80
3.25 cm = 0.0325 m	3	0.14	0.232	65.1
ρ=1057.59 kg/m^3	1	0.27	0.278	81.7
Diameter of surface = 0.15 m	2	0.25	0.300	95.2
Radius = 0.075 m	3	0.28	0.268	76.0
			Average:	55.3 Pa ± 4.5 Pa
80% Ethanol Hand Sanitizer:				
ρ=0.86 g/mL = 860 kg/m^3	1	0.11	0.318	87.0
Diameter of surface = 7.0 cm	2	0.15	0.233	46.7
3.5 cm = 0.035 m	3	0.13	0.269	62.2
			Average:	63.2 Pa ± 7.1 Pa

Table 1) Table containing experimental bulk moduli of various liquids tested by dropping a mass into a vessel and measuring wave speed. The bulk modulus was calculated using Equation 1.

One of the major systematic sources of error is the size of the vessel. Larger vessels reduce error by increasing the time between the start and end of a wave. Initially, we used a smaller vessel, and the smaller distance meant that small inaccuracies in time resulted in large differences in wave speed. This information was corrected during the lab peer review, and the data improved for the second set of trials.

The sources of random error for the wave speed resulted from the measurement of distance (±0.005 m) and time (±0.005 s) using a meterstick and phone recording. The random error for the measurement of density resulted from the use of a graduated cylinder and triple beam balance, with error propagations of ±0.05 mL and ±0.005 g respectively. The overall error propagations for the average calculated bulk modulus is included in Table 1.

Discussion

The data did not match the textbook value for the bulk modulus of water by multiple orders of magnitude. The experimental value was found to be 38.3 Pa, while the textbook value is 1.96×10^9 Pa. The predominant speculation for this result is that the speed of sound model does not accurately reflect the speed of a wave through the water created by a mass being dropped into it. Experimentally, this could be resolved by measuring the wave speed of sound by using a speaker at a consistent frequency and volume and measuring the resulting wave speed instead of using a mass dropped into the solution. Analytically, there may be a different model that better represents the data.

In regards to consistencies in the data, the accuracy could be improved by utilizing additional trials, and the use of a sound generator instead of a mass being dropped would mean that waves would be more consistent. While energy does not affect wave speed, an increase in energy would result in more waves, which could make measuring the wave speed easier and more accurate overall.

Conclusion

The bulk modulus for the mediums was not able to be accurately determined given the experimental design. However, speculations were made regarding the difference between the model and the data, and future experimentation is recommended to include the use of a sound generator and a larger fluid vessel to increase the accuracy of the data and produce results that more accurately reflect the model in question.

References

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