

# 1140 Lab 1 and 1R: Pendulum

Last edited D 2022 – Feel free to add comments to this lab document if you have questions, or suggestions.

## Materials

- String
- Several weights, in lab we provide nuts
- Stopwatch<sup>1</sup>
- Ruler or computer ruler<sup>2</sup>

## Warm up questions<sup>3</sup>:

Use this PhET for the warm up questions: [Pendulum Lab](#) Please remember to integrate your warm up questions into your introduction // background of your lab report!

1. Which factors in this simulation (not including gravity or friction) change the period of the pendulum's swing?
2. Looking at high and medium gravity and friction, which factors change the period of the pendulum's swing?

## Theory

### Simple pendulum:

For a simple pendulum (a small mass on a long string), the restoring force  $F = -mg \sin(\theta)$ , which for small angles reduces to  $F = -mg \theta$  the period of oscillation is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration of gravity<sup>4</sup>. This equation is accurate only for small oscillation amplitude, where the approximation  $\theta = \sin(\theta)$  can be assumed to hold. For larger angles, the third-order approximation,  $\sin(\theta) = \theta - \frac{\theta^3}{6}$  must be used. This yields a correction factor for T, giving,

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<sup>1</sup> In the future remote students will want a phone camera and Tracker.

<sup>2</sup> [https://www.ginifab.com/feeds/cm\\_to\\_inch/actual\\_size\\_ruler.html](https://www.ginifab.com/feeds/cm_to_inch/actual_size_ruler.html)

<sup>3</sup> When writing up your lab report please integrate your warm up questions into the introduction of your paper.

<sup>4</sup>  $T = 2\pi \sqrt{\frac{L}{g}}$

$$T = 2\pi\sqrt{\frac{L}{g}} \left(1 + \frac{1}{16}\theta_0^2\right)^5$$

Note: this equation assumes angles are in radians.<sup>6</sup>

## Procedure

Please assemble your real-life pendulum. You do not need to know the exact length of the pendulum, but you should be able to approximate. Additionally, you do not need to know the exact mass of your attached weight, but you should be able to approximate<sup>7</sup>.

Test a statically significant number of trials testing all appropriate variables.<sup>8</sup>

What sort of error is inherent in this system?

You should include a picture or diagram of your setup in your lab report, with the materials you used labeled on that setup.

Please add your data to [the communal data list](#).<sup>9</sup>

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<sup>5</sup> $\mathrm{(Eq3)}\hspace{0.5in} T=2\pi\sqrt{\frac{L}{g}}\left(1+\frac{1}{16}\theta_0^2\right)$

<sup>6</sup> <https://www.andrews.edu/phys/wiki/PhysLab/doku.php?id=lab.9>

<sup>7</sup> Students can think about, and possibly write about, how precise their measurements of the length and mass should be.

<sup>8</sup>This reference helps explain when to know when you have enough data:  
<https://openbooks.library.umass.edu/p132-lab-manual/chapter/when-do-i-have-enough-data-also-fixed-references-in-spreadsheets/>

<sup>9</sup> We will be looking at this data next week!

## Data Analysis

- Have two plots of your data, of mass vs period and length vs period. Please plot using Python, Matlab or another coding software (not Excel). If you choose Python we have a Python Expert who can help you troubleshoot code. You can also find some example code on [the GitHub](#).
- Calculate an error on those periods.
  - **Statistical error:** Standard deviation of the different periods / sqrt(number of trials).
  - **Systematic error:** google human reaction time and use that number as your systematic error. What might be another example of systematic error in this system?
  - Include error bars of the systematic + the statistical error in your graphs <sup>10</sup>

See this resource if you need a refresher on statistical vs systematic uncertainty: [Systematic and Statistical Errors](#).

**Extra Credit:** Take a pendulum home and deploy it in 5 different locations. Does your period change based on the location? Please give a reason why or why not? Please include pictures of the 5 places or setups (we love pictures). Please return the pendulum when you are done. (+2)<sup>11</sup>

## Deliverable for lab 1R

Before your next lab session (please see Canvas) please submit a lab report with the following sections// components: [Generic Deliverables for this lab.docx](#)

**Stop here! The following documents are for future labs**

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<sup>10</sup> <https://github.com/Dana-physicsLabs/PH1120-1121/blob/master/Plotting%20with%20error%20bars.ipynb>

<sup>11</sup> Extra credit in lab only applies to lab. So if lab counts for 20% of your grade in this class, you can get a total of 20 points factored into your final grade. You cannot get over a 100% lab grade. Extra credit in lab is meant to be a challenging, interesting way to gain some points you may have lost for small errors. The extra credit assignments are often things we would love to include in the main experiment, but we just are not sure if there would be time in the two hours assigned to lab.

# 1140 Lab 2R: Damped Oscillator

D 2022 remote lab 3

## Lab Goals

- Introduce concept of theory and data matching.
- Work through an example of chi squared analysis.
- Take more pendulum data to try and confirm unconfirmed pendulum theories. How do they even work?

## Supplies

- String
- One lighter mass (Under 3 lb)<sup>12</sup>
- One heavier mass (Over 3lb if possible, but totally fine if you cannot)
- Camera<sup>13</sup>
- Tracker Software<sup>14</sup> or other similar video tracking software

## Warm Up Question

- Please download Tracker or another similar video tracking software to your computational device before class.

## Theory

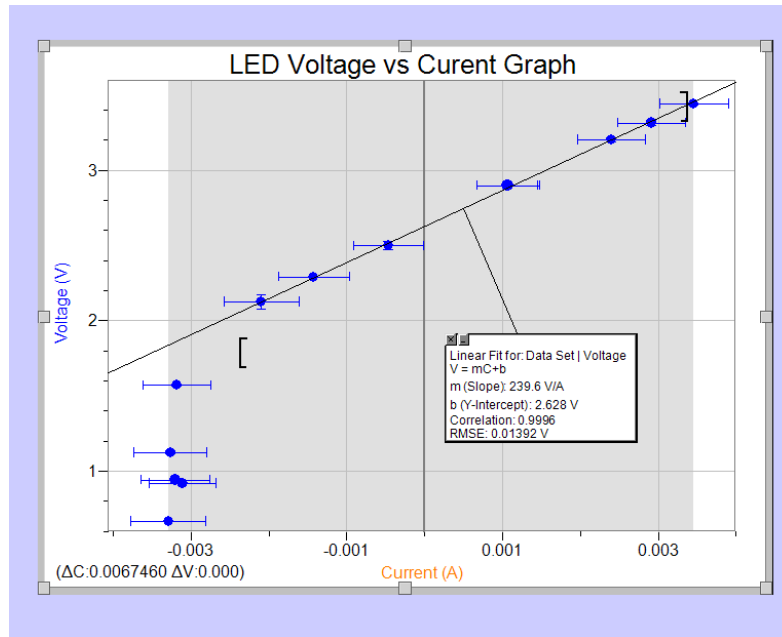
There are many types of fits for data, but not all of them belong on your data. Previously in General Mechanics or E&M your lab instructor may have told you to apply a linear fit to some data, such as the velocity of a falling ball, but clearly a linear fit would not be appropriate for the full current vs voltage curve of a LED, as seen below.

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<sup>12</sup> If you do not have access to a scale please let us know

<sup>13</sup> Camera phone works well

<sup>14</sup> <https://physlets.org/tracker/>



The learning goal of your lab professor was probably to have you fit your data to a theory as represented by an equation. For the velocity of a falling ball that theory is gravity accelerating the ball downwards, and for a LED that theory is that diodes have very little current flowing until the forward voltage is reached.

In order to test how closely data really matches a theory physicists often use a chi-squared test.

$$\chi^2_R = \frac{1}{N - f} \sum_i \frac{[y(x_i) - y_i]^2}{\sigma_i^2}$$

Where  $N$  = number of data points, and  $f$  = number of parameters in fit (for example,  $f = 4$  for a cubic fit).

## Warm up questions

Use this PhET for your warm up questions: [https://phet.colorado.edu/sims/html/curve-fitting/latest/curve-fitting\\_en.html](https://phet.colorado.edu/sims/html/curve-fitting/latest/curve-fitting_en.html)

- What is a bad chi-squared fit? Take a screenshot of a bad chi-squared fit and explain why it is a bad fit.
- What are residuals from a fit in this context?
- If you were to place your data from Lab 2R onto this chart what line would best fit it? Is that a good chi-squared fit line? Why or why not?

- Looking at [Anscombe's quartet, why](#) do you think it's important to plot your data?

## Procedure

Please download Tracker or another video analysis software onto your computer. Using the pendulum you made last week, record the oscillations of the pendulum until it comes to a complete stop. Please try three different lengths or masses and try objects at the end of your pendulum that have a lot of air resistance.

Using the video analysis software track the mass at the end of your pendulum through it's complete stop, and then export that data into Python or Matlab<sup>15</sup>.

### Analysis:

What equation should you fit to that data? How well does that data fit the predicted trendline? Apply a chi-squared fit to your data, and reflect on how well that data fits.<sup>16</sup>

Please include in your lab report a 'Lab Selfie' for your remote instructor that includes your pendulum setup.

<https://github.com/Dana-physicsLabs/PH1140/blob/main/Chi%20Squard%20test%20for%20oscillator.ipynb>

For more information <https://towardsdatascience.com/gentle-introduction-to-chi-square-test-for-independence-7182a7414a95>

## Deliverable for lab 2R

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<sup>15</sup> We encourage you to use whatever plotting software you're most comfortable with, except Excel or Google Sheets.

<sup>16</sup> If you cannot do a chi squared fit that is okay, just do your best.  
<https://www.youtube.com/watch?v=GCLfs8GKxpQ>

## Lab feedback

- Apply fits judiciously
  - Pay close attention to equations.  $\sqrt{L}$  is NOT Linear.
- - Proofreading
- Personal stories or interest are irrelevant
  - Use numbers in your analysis

Stop here! The following documents are for future labs

# 1140 Lab 3R option a: Double Pendulum

Last edited D 2022 – Feel free to add comments to this lab document if you have questions, or suggestions.

## Lab Goals

- Build and design your own double pendulum from items that are readily available
- Record video of your setup and transfer it into a programming language
- Plot your data and look for patterns

## Supplies

- String (try to find some string that is at least 1 meter)
- Two masses of different weights
- Camera or phone camera
- Tracker Software<sup>17</sup> or other similar video tracking software

## Warm up questions

Please consider watching this [video](#)<sup>18</sup> for an overview of a double pendulum. Professor Lee talks about the equations needed for a double pendulum in the first few minutes of this video, and a double pendulum is demonstrated around 7:30.

The assumption is that you will not have the ability to make a rigid double pendulum, but you can use string to make a double pendulum. In order to make a double pendulum with string your top mass needs to have a higher momentum than your lower mass. Why might that be?

## Procedure

Build your real-life double pendulum, measuring the lengths and getting a rough estimate of the masses. If the momentum of the top half needs to be higher than the bottom half this means the upper weight should be about twice as heavy as the lower weight. Experiment by moving some of the mass until they oscillate in an interesting way.<sup>19</sup>

Record the motion of your pendulum several times from different initial angles. Try also to record the motion of your pendulum multiple times from a similar large initial angle.

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<sup>17</sup> <https://physlets.org/tracker/>

<sup>18</sup> [https://ocw.mit.edu/courses/physics/8-03sc-physics-iii-vibrations-and-waves-fall-2016/part-i-mechanical-vibrations-and-waves/lecture-4/copy\\_of\\_lecture-4-video/](https://ocw.mit.edu/courses/physics/8-03sc-physics-iii-vibrations-and-waves-fall-2016/part-i-mechanical-vibrations-and-waves/lecture-4/copy_of_lecture-4-video/)

<sup>19</sup> <https://makezine.com/projects/string-up-simple-chaotic-double-pendulum-cat-toy/>



Import the video into Tracker and track the position (x and y) and velocity (x and y) of both masses. Export your data from Tracker into a CSV and load that CSV into the programming tool of your choice to construct your plots.

## Plotting your data

Please plot for each mass

- x vs y
- Your general phase spaces plots (eg: x vs x velocity)
- At least one random plot that you think no one else has done!

Experiment with different fitting functions that seem reasonable and then do a chi-squared test to determine if your fitting function is a good fit.

## Deliverable for lab 3R

Before your next lab session (Please see Canvas) please submit a lab report with the following sections// components: [Generic Deliverables for this lab.docx](#)

# 1140 Lab 3R option b: Pendulum Modeling

Last edited D 2022 – Feel free to add comments to this lab document if you have questions, or suggestions. Originally written by Ed Jarvis.

## Objectives:

- Write code to numerically solve the actual pendulum oscillator
- Write code to compare this actual solution to the small angle approximation
- Explore the limits (or constraints or variables or options but whatever sounds best to you) of friction, initial angle, time steps

## Theory:

You've been taught that the solution to the pendulum oscillator is:

$$\theta(t) = \theta_0 e^{-\gamma t} \cos(\omega t + \delta)$$

However, this solution is for the simplified differential equation:

$$\ddot{\theta}(t) = -\mu \dot{\theta} - \frac{g}{l} \theta$$

This simplification is from the Taylor series approximation of the sine function, the full differential equation is as follows:

$$\ddot{\theta}(t) = -\mu \dot{\theta} - \frac{g}{l} \sin \theta$$

This equation is solvable only by using functions called elliptic integrals. Rather than go through the laborious mathematics to solve this equation analytically, the goal of this lab is to solve the equation numerically.

## Procedure:

To solve the pendulum oscillator use python, with the packages `numpy` and `matplotlib.pyplot`. You will want to initialize the various parameters of your system: **gravity, length, friction, initial angle, and initial velocity**. You will want to establish the total time, time step (start small!), as well as define various other parameters (like the angular frequency) in terms of these parameters.

You will then want to define your differential equation as such in python:

```
def get_theta_double_dot(theta, theta_dot):  
    return -mu * theta_dot - (g/l) * np.sin(theta)
```

Next, you will want to solve the equation. Think about what it means to 'solve' a differential equation; you want to know all of the angles at any particular time. Since the initial angle was defined, you just need to **update that value for every time step**. The simplest way to update that angle is to add the amount it would have moved by in that time step. That amount is simply the angular velocity multiplied by that time step.

In addition, you'll need to update the angular velocity in the exact same way, by adding on the angular acceleration multiplied by the time step. You will calculate the angular acceleration using the differential equation. Then you will append the theta value to a list so you can construct a list of numbers for plotting.

Finally, you will plot your data. You will then plot the theoretical equation you learned in class. You will want to run your code and vary several different parameters. **What happens when friction is very high, very low, or zero? What happens when the initial angle is xvvery large, very small, or exactly pi radians? What happens if your time step is too large?**

## Deliverable for lab 3R

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We recommend the following as interesting discussions for your papers:

- Several graphs of angle vs time for each case (see end of procedure for more details)
- Discussion of differences between small angle approximation and the numeric solution
- Discussion of timesteps and errors that can arise
- Blooper graphs: trials that did not work or seemed to make graphs that were nonsense

If you'd like to try to model a double pendulum for some extra credit, see the github page:

<https://github.com/wojciechmo/double-pendulum>