Department of Physics

Lab 3 - Capacitance

Learning Objectives

- Reinforce parallel vs series circuits.
- Practice and cement propagating uncertainties.

Lab Equipment

- Power supply
- Vernier circuit board
- Vernier voltage probe
- A *22* $k\Omega$ resistor
- Capacitors
- Alligator-clip and banana wires

The behavior of a discharging capacitor

There are multiple ways to measure the capacitance of an unknown capacitor, one relatively easy way is to test capacitance indirectly by measuring the voltage of a discharging capacitor over time. When discharging through a resistor, the voltage across the capacitor $V(t)$ decays exponentially over time according to the equation

$$V(t) = V_0 \exp\left(-\frac{t}{RC}\right)$$

where V_0 is the voltage at the time $t = 0$, R is the resistor and C is the capacitance of the capacitor.

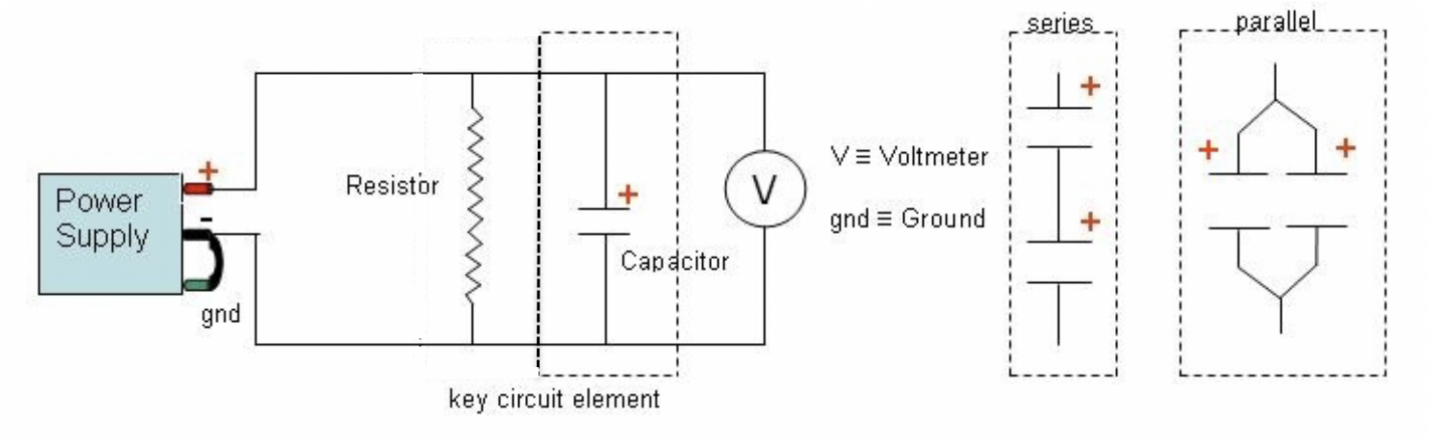


Figure 1: The first circuit shows a connection where a resistor and a capacitor is in parallel. Second and third circuits show series and parallel connections involving capacitors

Question 1 *Draw a circuit diagram to show how the two capacitors being tested in series should be situated into your test circuit. Remember that there should be your DC power supply, your resistor in parallel to the capacitors, a switch, and your voltmeter also in parallel to your capacitors. Use Figure 1 to help visualize the circuit.*

Setting up the circuit and obtaining the data

You should use the **22kΩ resistor** in parallel to the capacitors.

Set up the circuit with one of the two capacitors as shown in Figure 1. Note that the capacitor is in parallel with the resistor, and the voltmeter is measuring the voltage across the capacitor. It is important to know that some capacitors have a polarity and must be hooked up in the correct orientation or else there is a chance that they will be damaged. There are a few ways to determine the orientation of the capacitor:

- The longer leg of the capacitor is the positive leg, and this must be closer to the positive terminal of the power supply.
- The negative side is marked with a black line, sometimes in the shape of an arrow pointing in the direction of current flow.
- The stripe of arrows on the side of the capacitor indicates the direction of positive current through the capacitor. For example, Fig. 2
- Some capacitors, such as the one sometimes soldered to the Vernier circuit board, are not polarized and you may hook them up in any direction.

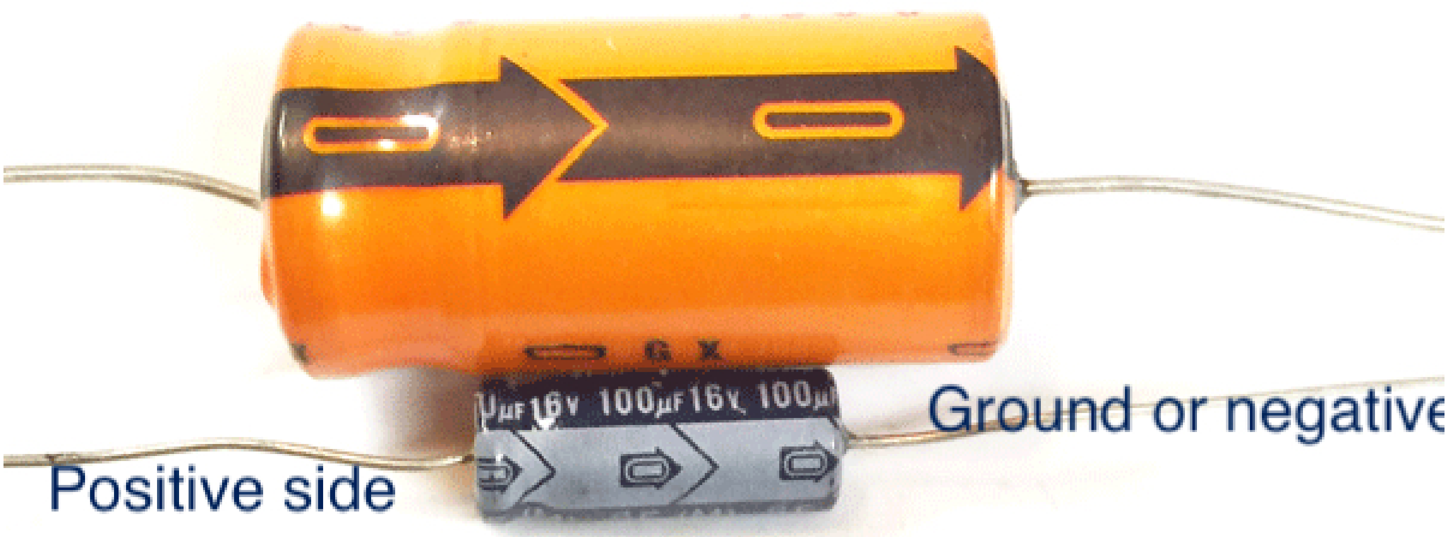


Figure 2: A sample capacitor, similar to a capacitor you might see in lab. The arrow points towards the end that should go towards ground. Picture found at Search Autoparts.com [2]

Trial	LoggerPro C Value	Uncertainty (C Value)	Measured Capacitance	Uncertainty (measured)	Theoretical Capacitance	Uncertainty (theoretical)
Capacitor 1	LoggerPro		Eq. 1	Eq. 6	Capacitor sleeve	Appendix
Capacitor 2						
Series					Eq. 3	Eq. 4
Parallel					Eq. 2	Eq. 5

Figure 3: Example table for taking measurements. **For python code please see this link in Github**.

Data Collection

1. Open Logger Pro on the computer and make sure that the voltage probe is recognized. Connect the leads of the voltage probe together and zero the sensors in the Logger Pro software, this procedure will help minimize any systematic error. You should repeat this process often, certainly if you notice any odd measurements.

2. With the power supply off, connect the circuit as shown in Figure 1 using the first configuration.
3. **Make sure that you have the capacitor(s) in the correct orientation!**
4. Turn down the voltage and current knobs on the power supply, then turn on the supply. While watching the display on the power supply, slowly adjust the supply so you are outputting *5 volts*, you will have to adjust both the voltage and the current control as the output will be limited by the lowest setting.
5. When you are ready to collect data, press the **Collect** button in Logger Pro, and then disconnect the power supply from the circuit, either by removing the positive wire or moving the inline switch to the battery (3v) position. You can verify from the circuit diagram that this will result in a closed loop where the capacitor discharges all of its stored charge through the resistor. You should see an exponential decay in Logger Pro as the voltage drops from *5.0 V* back to *0.0 V*.

   Do not turn off the power supply as a substitute for disconnecting the supply from the circuit this will lead to erroneous data.

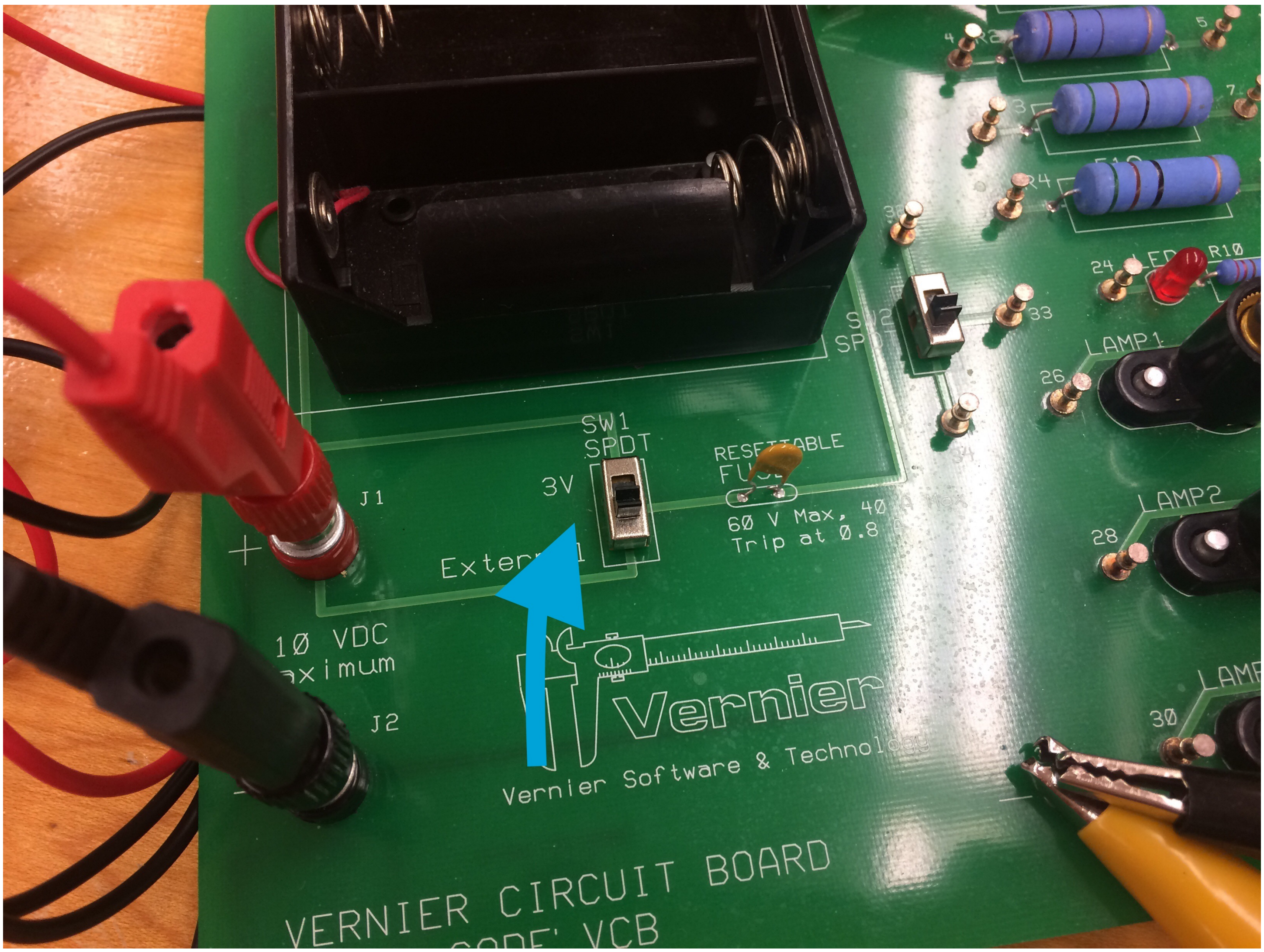


Figure 4: When you are ready to collect data, press the **Collect** button in Logger Pro, and then disconnect the power supply from the circuit by moving the inline switch to the battery (3v) position. That inline switch is pictured here.

Calculate the capacitance and uncertainty of this capacitor using Logger Pro, as described in the next section. Once you have obtained the capacitance value, redo the experiment using:

1. Another capacitor
2. Both capacitors in series
3. Both capacitors in parallel

Calculating the capacitance

Once you have a plot of voltage versus time in Logger Pro, you can calculate capacitance. Recall that the equation that governs the discharge of a capacitor through a resistor is

$$V(t) = V_0 \exp\left(-\frac{t}{RC}\right)$$

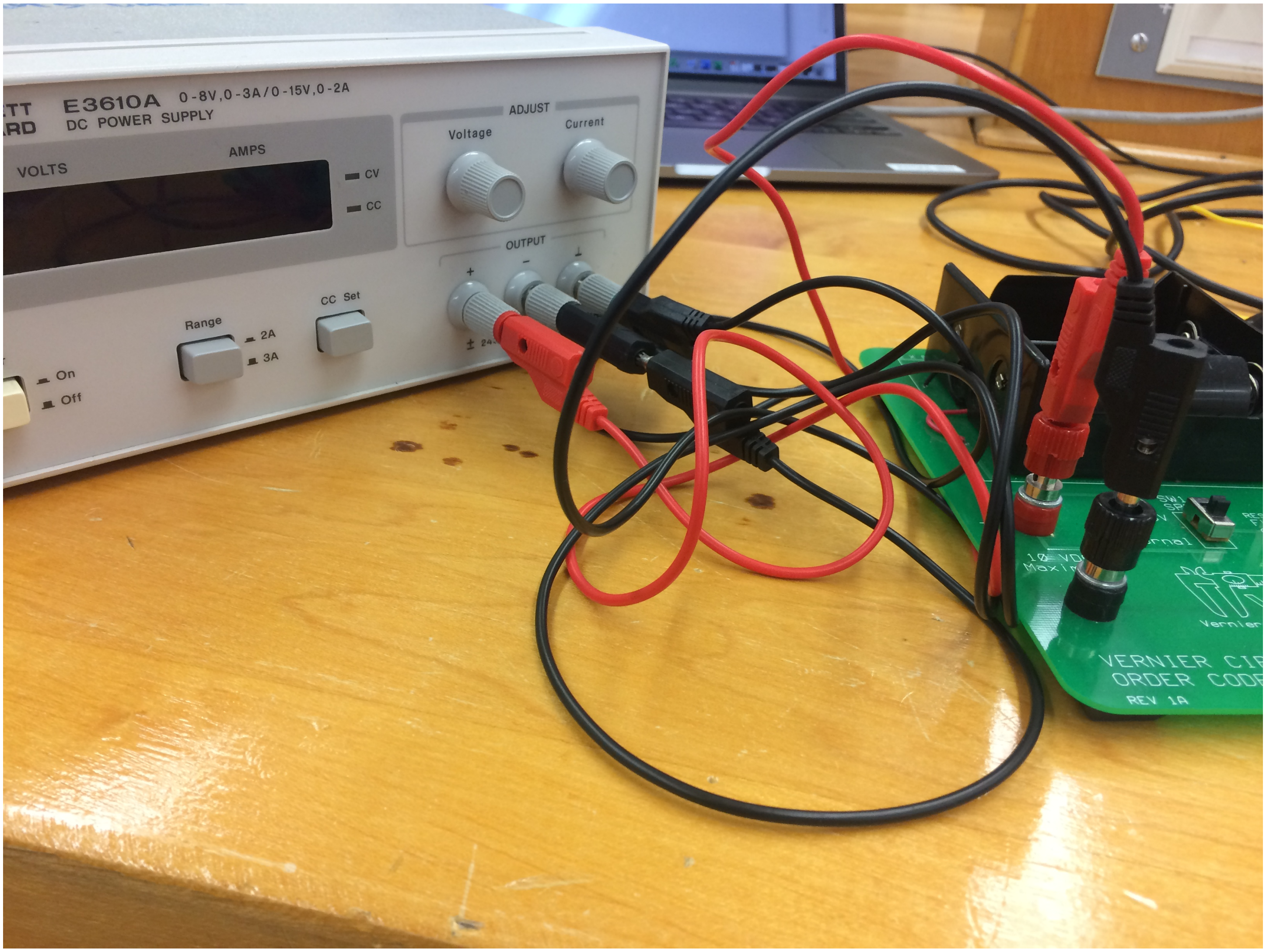


Figure 5: If your power supply is failing, please connect your negative output to your ground output on your power supplies. That will increase stability of your circuit.

Perform a curve fit on the position graph using the equation that best fits the data. The dialog box is shown in Figure 6, once you chose the appropriate equation form, you must use the `Try Fit` button to see the fit on your data. Once you are satisfied that you have the correct equation, you can click `Ok` to return to your graph with the new fit overlaid.

As long as you chose the "Natural Exponent" equation to fit your data, Logger pro will return 3 parameters with uncertainties that correspond with the fit, A, C and B. A is equal to V_0, B represents any vertical (DC) offset in your circuit and C is equal to $\frac{1}{rc}$ (somewhat unfortunately for labeling reasons). **Performing a small amount of algebra, we can see that,**

$$Capacitance(c) = \frac{1}{Resistance(r) * Parameter(C)} \tag{1}$$

You can then use the uncertainty propagation equation for multiplication and division, Equation 4, to find the uncertainty in capacitance using your uncertainties in resistance and the parameter C. To find the uncertainty of the value for the 22kΩ resistor, use the color code and calculate the uncertainty based on its rated tolerance.

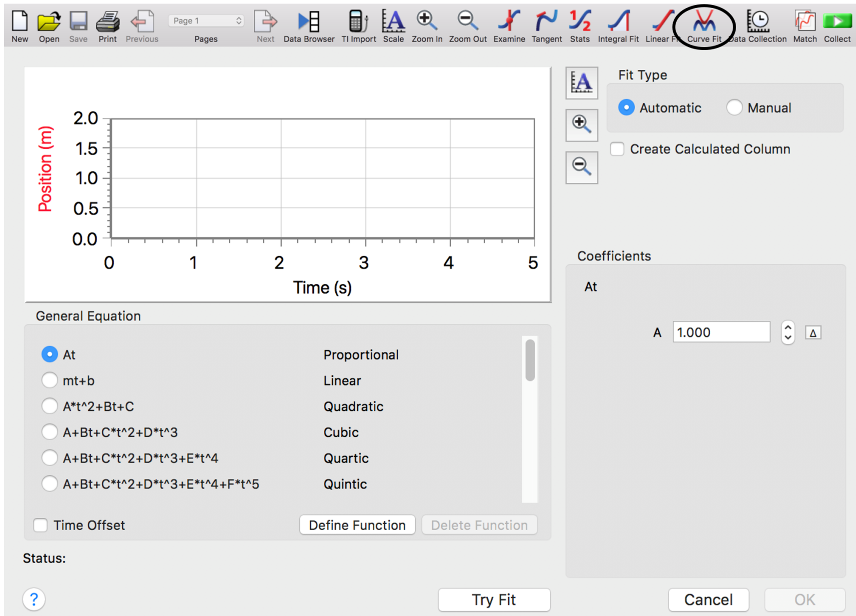


Figure 6: Dialog Box for the Curve Fitting feature.

The equations for calculating the equivalent capacitance for series and parallel are as follows:

$$C_{parallel} = c_1 + c_2 \tag{2}$$

and

$$C_{series} = \frac{c_1 c_2}{c_1 + c_2} \tag{3}$$

While the propagation of uncertainty for the case of parallel capacitors is simply the method shown in Equation 5, the formula for series capacitors is much more difficult. As such we have included it here,

$$\delta C_{series} = \frac{(c_2)^2}{(c_1 + c_2)^2} * \delta c_1 + \frac{(c_1)^2}{(c_1 + c_2)^2} * \delta c_2 \tag{4}$$

Propagation of Uncertainties

For python code please see this link in Github.

Addition and Subtraction

In the case of addition and subtraction, the equation for combining uncertainties is

$$\delta x = \delta x_1 + \delta x_2 + \delta x_3 \tag{5}$$

where δx is the total uncertainty of your calculation and δx_1, δx_2, and δx_3 are the uncertainties of your individual measurements.

Multiplication and Division

This method is valid for both multiplication and division of measurements with uncertainties.

$$\frac{\delta A}{|A|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|} \tag{6}$$

where A is the area , x is the length, y is the width ,δx and δy are the uncertainties associated with these measurements, and δA is the propagated uncertainty of the product or quotient.

To translate,

$$\frac{\delta C_{ap}}{|C_{ap}|} = \frac{\delta C_{parameter}}{|C_{parameter}|} + \frac{\delta R}{|R|} \tag{7}$$

Where δC_{ap} is the uncertainty on the capacitor that you want to calculate for, C_{ap} is the capacitance, $C_{parameter}$ is the C parameter from Logger Pro, then the $\delta C_{parameter}$ is the +/- value on that C parameter (again to be found on Logger Pro. Then R is the resistor value, and δR is the uncertainty on that resistance.

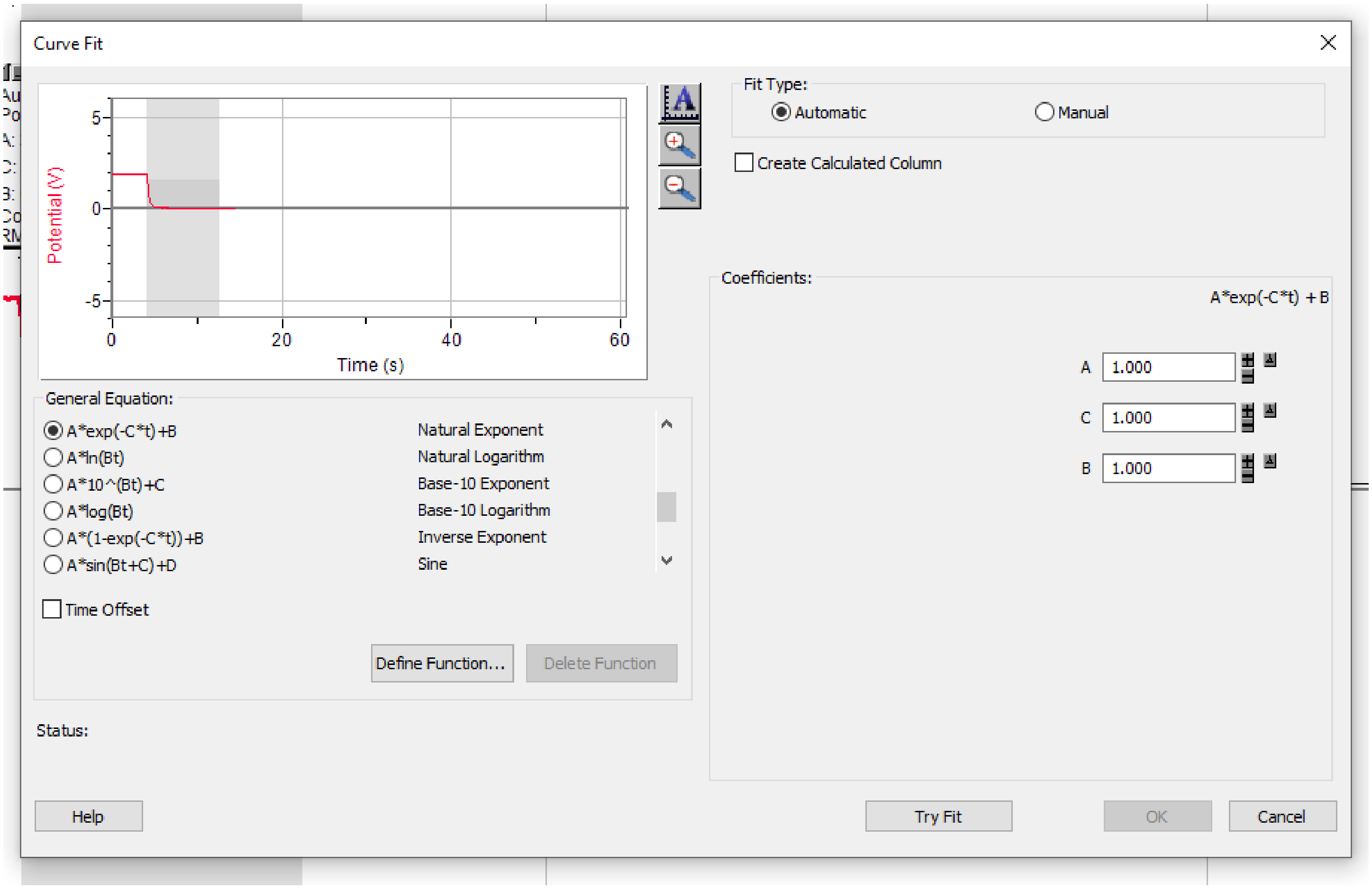


Figure 7: Try using the Natural Exponent fit for the best fit line.

Question 2 **Lab Modus Operandi**

Communicate the steps that you took when collecting and analyzing your data.

Pretend you are writing this so a fellow student that missed this lab could take and analyze the data using only this section. For example, you do not need to tell them to press start in Logger Pro or open the program, but you would want to tell them what sensors you used to collect data and if there are any special settings that you used. (3-4 sentences)

Question 3 **Data Analysis**

Show your calculations here for the theoretical series and parallel equivalent capacitances, include the propagation of uncertainty calculations.

If you used Python, include your working and commented python code for the propagation of uncertainty. You should copy and paste the code into the answer sheet. If you do it by hand please write down your work or take a photo of your work. [a]

[a]Lab Partners can share code.

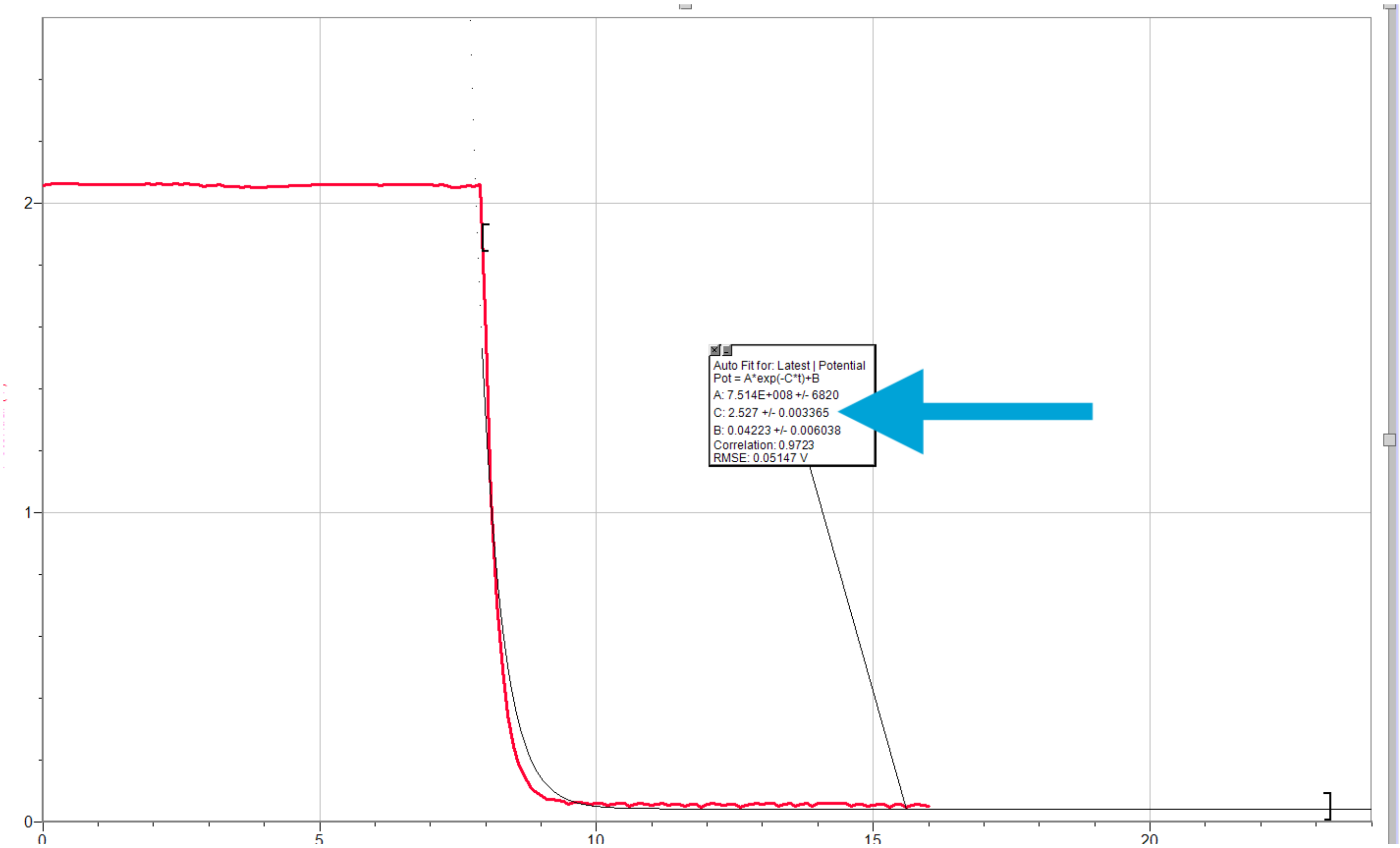


Figure 8: Use the uncertainty on C as your measured uncertainty input.

Question 4 **Results**

- *Create a table below that lists your measured and calculated results for capacitance, include units, uncertainties and check for proper formatting. See Fig. 3 for an example of what your table should look like.*
- *Attach the four graphs of the data you collected today. Each graph should follow the figure guidelines (below) and include the curve fit with uncertainties.*

  The four graphs you need today are:

  - *Capacitor on your vernier circuit board*
  - *Loose Capacitor*
  - *These two capacitors in series*
  - *These two capacitors in parallel*

Question 5 **Conclusion**

Based on your results listed above, did the capacitance of the series and parallel circuits match with the calculated value of total capacitance within uncertainty? What further questions do you have about this lab? How would you test those questions if you had the ability to do so?

Graph and Data Checklist:

You should have answered all of the questions highlighted by the gray boxes, you should have 4 graphs with captions and fits and at least 1 table with your data in it. Make sure that you used complete sentences and that your conclusions match your results.
The four graphs you need today are:

- Capacitor on your vernier circuit board
- Loose Capacitor
- These two capacitors in series
- These two capacitors in parallel

Clean Up Procedure

Once you have collected all your data, and before you leave for the day please make sure to

1. **Turn off power supplies.** You may leave them plugged into the walls, but make sure they are off.
2. Leave Voltage and Current probes (Ammeter and Voltmeter's) plugged in to the logger pro, but please make sure none of the wires are touching the ground. Please also leave your green box plugged into your computer and at your station.
3. Unhook your wires from your circuit boards, and place the circuit board back in their white boxes.
4. Put wires back in the white box, or if there is no room in the white box then put them in the communal wire box that is near the front of each room.
5. If you used a capacitor and attached it to the board **please do not remove it from the board after connecting it**. Too much connecting and unconnecting will weaken or break the wires eventually.

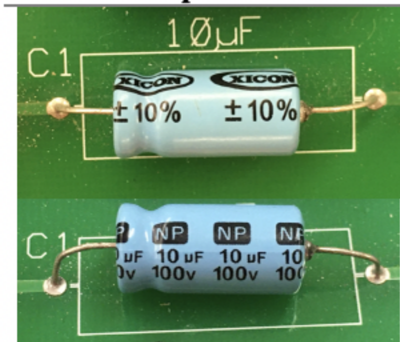
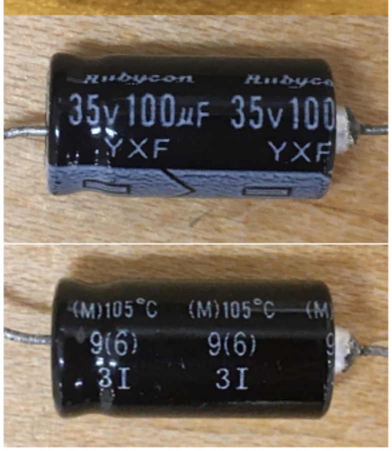
Capacitor	Tolerance	References
	±10%	Sleeve marking
	±10%	Lelon Part Numbering System
	±20%	Nichicon Datasheet Digi-Key Electronics
	±20%	Rubycon Datasheet Digi-Key Electronics

Figure 9: Here you can see an example of the different types of capacitors we will use in this experiment. You can find their tolerances, or error as we sometimes call it, also in this note.

Appendix

Once you have collected all your data, and before you leave for the day please make sure to

1. **Turn off power supplies.** You may leave them plugged into the walls, but make sure they are off.
2. Leave Voltage and Current probes (Ammeter and Voltmeter's) plugged in to the logger pro, but please make sure none of the wires are touching the ground. Please also leave your green box plugged into your computer and at your station.
3. Unhook your wires from your circuit boards, and place the circuit board back in their white boxes.
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5. If you used a capacitor and attached it to the board **please do not remove it from the board after connecting it**. Too much connecting and unconnecting will weaken or break the wires eventually.

The Figures and Caption Rules

There are a few very important aspects to creating a proper figure and caption. If you follow these rules, not only will you get points on your physics lab grades, you will impress your instructors and peers in the future.

The Caption

- The caption should start with a label so you can reference the figure from other places in your paper/report. For this course you should use "Figure 1", "Figure 2", etc.
- The caption should allow the figure to be standalone, that is to say, by reading the caption and looking at the figure, it should be clear what the figure is about and why it was included without reading the whole paper.
- The caption should contain complete sentences and be as brief as possible while still conveying your information clearly (this is not always easy).
- Please add captions to your tables as well, otherwise we will not know what we are looking at.

The Figure

- **Make sure that the resolution is high enough to not be pixelated at its final size.**
- Check that any text is readable at the final size (Using a smaller graph in Logger Pro will cause the text to be larger in relation to the graph when inserted into another program).
- For graphs, ensure that the axes are labeled (including units) and that there is a legend if you have multiple data sets on the same graphs.

Tables

- The first row of the table should be a header, where each item is labeled with what is contained in that row. If it is a physical measurement it should have the correct units.
- For tables include a short caption of what is contained in the table, or what was examined.
- The caption should start with a label so you can reference the figure from other places in your paper/report. For this course you should use "Table 1", "Table 2", etc. For an example of a good table caption please see Figure 10.

Table 1. Baseline characteristics of study participants

Variables	Intervention group (n=14)	Control group (n=15)
Women (no [%])	7 (50)	5 (33)
Median age (range)	22.0 (19 – 58)	21.0 (18 – 70)
First winter in icy conditions (no [%])	—	1 (7)
Previous falls on ice (no [%])	8 (57)	11 (73)
$\geq$ 1 fall this winter (no [%])	4 (29)	7 (50)
Injury from fall this winter (no [%])	1 (7)	—
Time been walking this route (no [%]):		
<6 months	3 (21)	2 (13)
6–12 months	9 (64)	9 (60)
>12 months	2 (14)	4 (26)

Figure 10: An example table from the paper Lianne Parkin, Sheila M Williams, and Patricia Priest, "Preventing Winter Falls: A Randomised Controlled Trial of a Novel Intervention" 122, no. 1298 (2009): 9.

Error

Humans can often be a source of error, but describing one's total error as 'human error' does little to illuminate the subject. Humans can contribute error to a system, but it is not their mere presence, often, that causes that error. That error is contributed by a specific action, or lack of action of the operator and you should always be specific. If we ask for why there might be error in a system, and someone responses with just human error without explaining what, specifically, that answer will not receive credit.

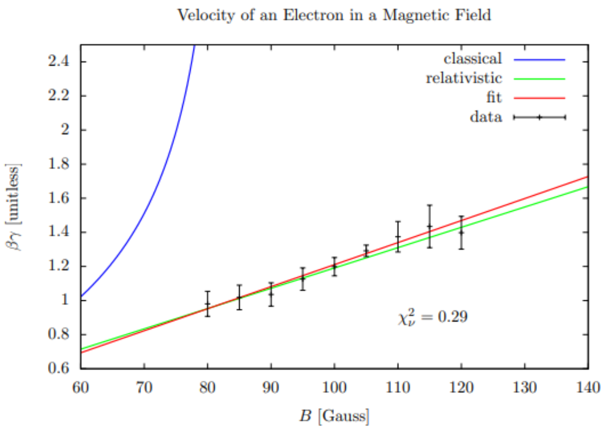

Figure 11: Example of a good figure with excellent error bars and a label. Figure from Philip Ilten of University College Dublin [?].

Python

According to IEEE Spectrum, Python is the most popular programming languages. Python is a free, general purpose, cross discipline programming language that has moved to the forefront of many disciplines.

If you decide to use Python your TA's will help you troubleshoot your code. While they might be able to help you troubleshoot when you use a different program or code, be aware of the fact that they are not familiar with all programming codes. There are many languages (R, Matlab, Opal, Julia, etc.) out there that are just as useful as Python, but we have chosen to use Python here. You may use any programming language you wish, but not Excel or Google Sheets.

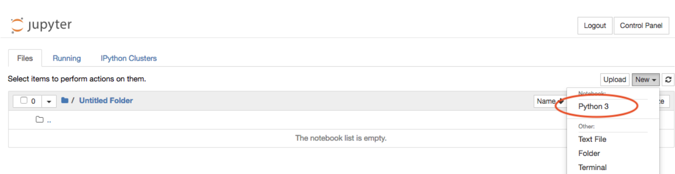

Figure 12: Navigate to `jupyterhub.wpi.edu/hub/login` and sign in with your WPI email address and password, choose an instance to spawn (either is fine) and create a new Python 3 file as shown here

We have set up a Jupyter notebook you may use. The website is https://jupyterhub.wpi.edu/hub/login.[1] There are many ways to learn Python, including reading a book, asking a friend, working through examples, or googling furiously when problems arise. We encourage you to discover which approach works best for you. Going forward this class will provide basic Python examples, but feel free to iterate upon the template we provide. What we provide is a stripped down version, and elaboration is encouraged. See this Github repository for our examples. We hope at the end of this term you will be able to add to your resume "Proficient in Python".

Jupyter uses a cell based system and evaluated variables carry over to the next cell. There are a few different types of cells, Figure 13 shows 2 kinds, the code cell, which we will be using most of the time, and the markdown cell, which you can use to add nicely formatted notes to you file.

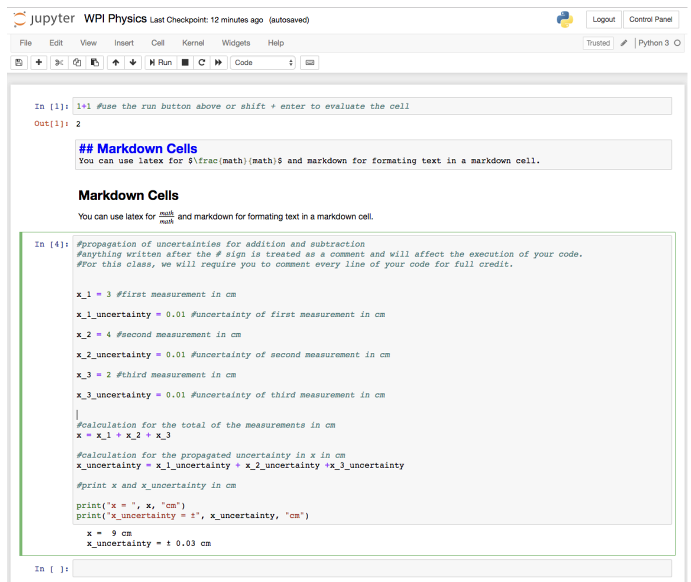

Figure 13: Above is the code that you could use use to propagate uncertainty for values that are added or subtracted. Always remember to comment your code.

If you prefer to work through a book or examples we recommend Mark Newman's book, which is available for free on his website [1]. Chapter Two is a basic introduction to the syntax for Python. Chapter Three covers graphs and visualizations, and we hope you will look into it if you learn best from a book. If you wish to get a head start in this class we recommend reading this book.

If you wish for a more advanced textbook there is a compilation of free online computational physics books here.

[1] If you cannot log in please email WPI's IT department, and they will be happy to help polite students. The first thing they will tell you, however, is to check to make sure you don't have to change your password and try a VPN if you are off campus.

References

[1] Mark Newman. *Computational Physics with Python*. CreateSpace Independent Publishing Platform, 2012.

[2] How to deal with electrical concerns related to EMI.