WPI | Department of Physics

Lab 4 Kirchhoff's Loops Law

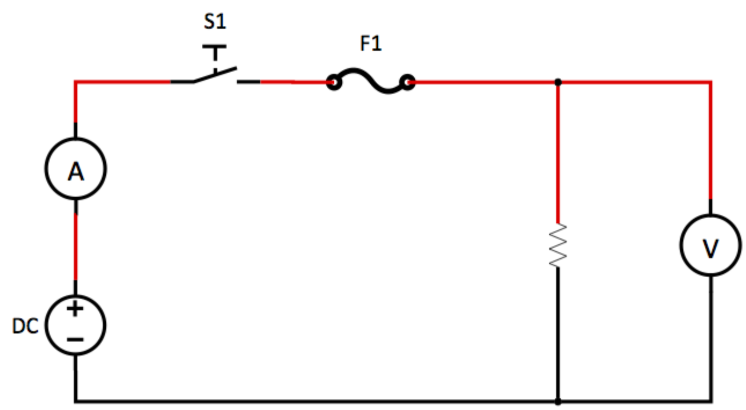

Figure 1: A simple resistor test circuit

Lab Objectives

- Construct a complex electric circuit using various circuit components.
- Measure voltages in various parts of the circuit and find the relations between them.

Overview

In this lab you will use Ohm's law to calculate the resistance of 4 difference resistors from measurements of voltage and current and measure the resistors in series and parallel configurations.

Theory of Resistance

I Theory

I.1. Conservation of Charge

The total charge in a closed system remains conserved. This means that charge can neither be created nor destroyed, it can simply be transferred from one object to another. This is called the law of conservation of charge.

In Lab, when you constructed an electric circuit, and connected a DC power supply to it, you observed a flow of charge in the circuit. What causes the flow of charge in an electric circuit? What is the role of battery in the circuit? You measured

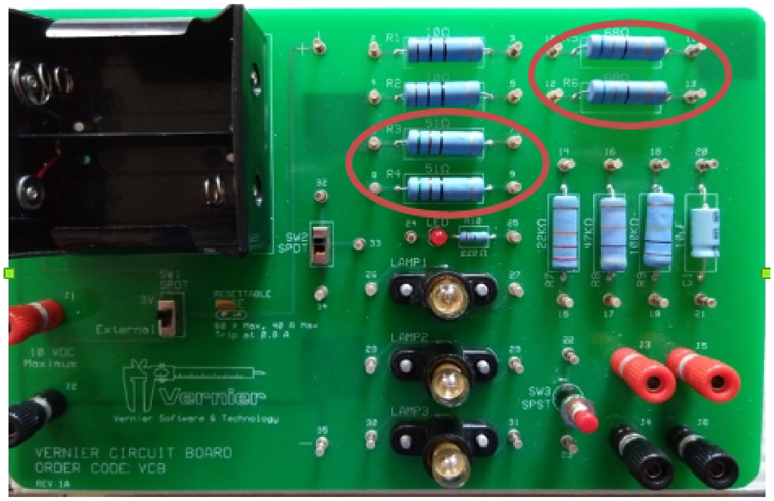

Figure 2: The resistors we will be using today

potential differences across the resistor in the circuit. How are the potential differences related to the battery voltage or the electromotive force (emf) of the battery or DC power supply?

The total energy in a closed system remains conserved. This means that energy can neither be created nor destroyed, it can simply be transformed from one form to another. This is called the law of conservation of energy.

I.2. Kirchhoff's rules

In this lab, you will be constructing complex electric circuits with multiple resistors and batteries, and investigating the relations between currents and voltages in such circuits.

I.2.1 Kirchhoff's junction rule

The algebraic sum of all currents at a junction is zero. A junction in an electric circuit is a point where at least three currents come into or go out of it. Fig. 3 shows a simple junction in an electric circuit.

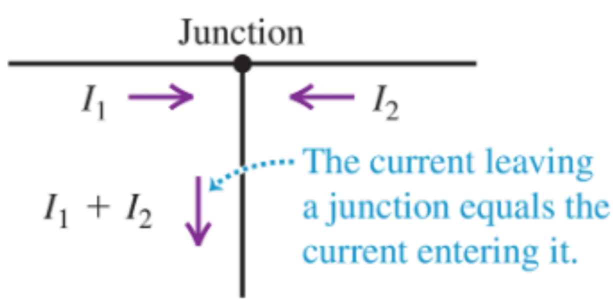

Figure 3: A simple junction in an electric circuit.

Mathematically Kirchhoff's junction rule can be written as:

$$\Sigma I = 0 \tag{1}$$

When you write Kirchhoff's junction rule, take the currents flowing toward a junction to be positive and the currents flowing out of junction to be negative. For example, in Fig. 3, the currents I_1 and I_2 are positive and the current (I_1+ I_2) is negative, i.e.,

$$\Sigma I = I_1 + I_2 - (I_1 + I_2) = 0 \tag{2}$$

Kirchhoff's junction rule is based on the conservation of electric charge.

I.2.2 Kirchhoff's loop rule

The algebraic sum of emfs and potential differences in a loop is zero. A loop in an electric circuit is a closed path containing sources (batteries, generators, etc.,) which drive charges and currents, and sinks (resistors, bulbs, motors, etc.,) which dissipate energy in heat, light or some other forms. Fig. 4 shows multiple junctions and loops in a complex electric circuit.

Mathematically, Kirchhoff's loop rule can be written as follows:

$$\Sigma V = 0 \tag{3}$$

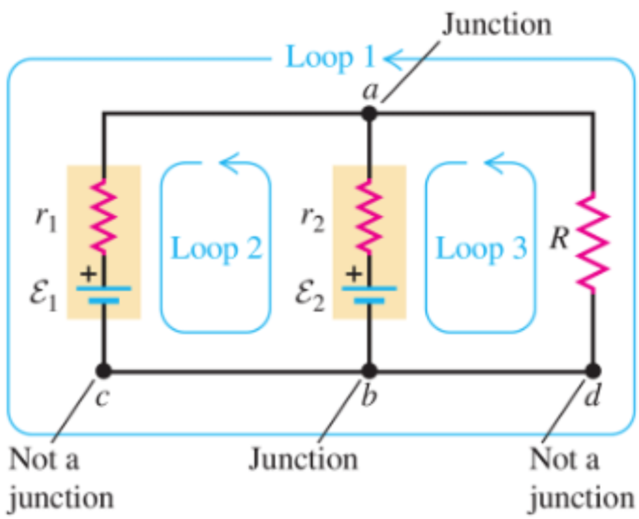

Figure 4

For emfs, the sign of emf is taken positive in traveling from the negative terminal of a battery to its positive terminal, while going around a loop, and the sign of emf is taken negative in traveling from the positive terminal of a battery to its negative terminal, while going around the loop. Fig. 5 shows a sign convention for emfs in applying Kirchhoff's rule.

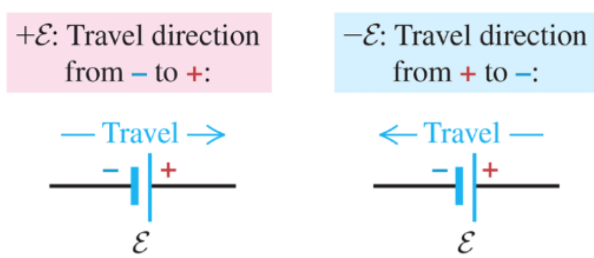

Figure 5: A sign convention for emfs in Kirchhoff's loop rule

For potential differences, the sign of the product (IR) is taken positive if the direction of the travel is against the direction of current flow, and the sign of the product (IR) is taken negative if the direction of the travel is in the direction of current flow. Fig. 6 shows a sign convention for potential differences in applying Kirchhoff's rule.

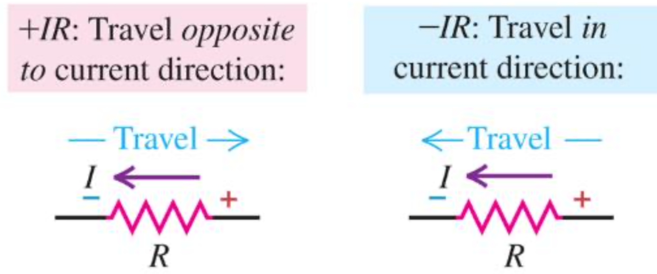

Figure 6: A sign convention for emfs in Kirchhoff's loop rule

Kirchhoff's loop rule is based on the conservation of energy. Using PhET simulation platform, you will be constructing a complex circuit as shown in Fig. 7.

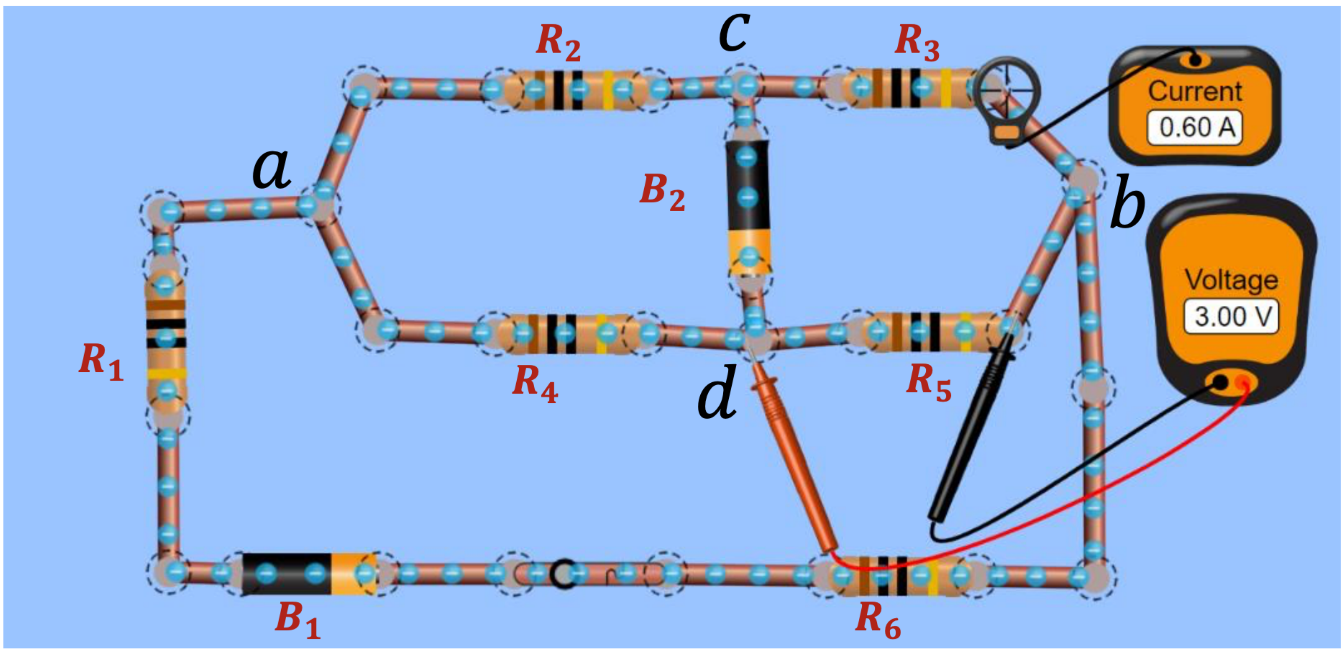

Figure 7: A complex electric circuit with multiple batteries and multiple resistors. The blue spheres represent the electrons flowing through the circuit.

We can re-draw the circuit shown in Fig. 7 using electric symbols as shown in Fig. 8, where the red arrows represent the direction of the conventional current.

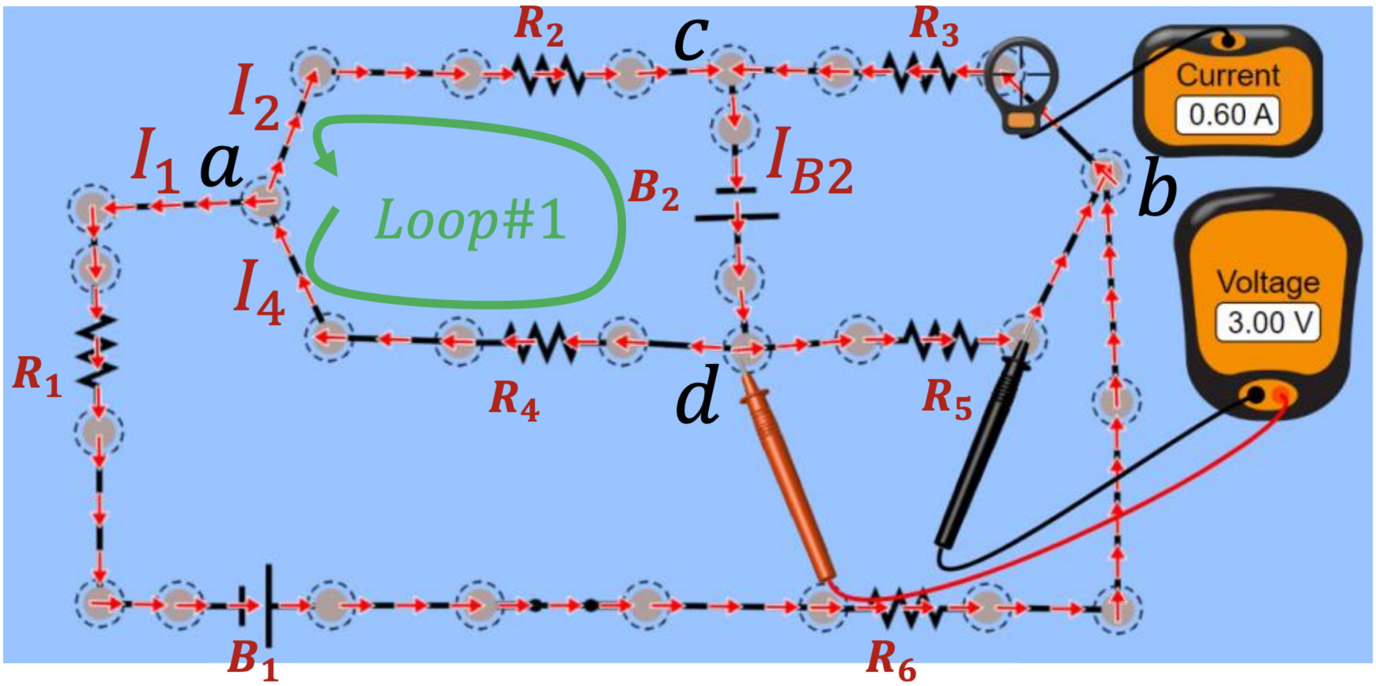

Figure 8: A complex electric circuit with multiple batteries and multiple resistors. The red arrows represent the direction of conventional current flowing through the circuit.

We can write equations relating currents, emfs, and potential differences by using Kirchhoff's rules at various junctions and loops with proper sign conventions. Those equations can be solved for unknown currents, emfs and potential differences. For example, at Junction a (Fig. 8),

$$I_4 - I_1 - I_2 = 0 \tag{4}$$

Then, in Loop #1 (travel the loop in counterclockwise (CCW) direction from starting from point a)

$$+I_4R_4 - \epsilon_{B_2} + I_2R_2 = 0, \tag{5}$$

Where ϵ_{B_2} is the emf of the battery B_2. For another example of this calculation please see here

The Circuit Board

There are number of resistors and other components on the Vernier circuit board that you each will have at your lab station. We will be using 5 of the resistors today. They are the **10 ohm**, two **51 ohm** and the two **68 ohm** resistors.

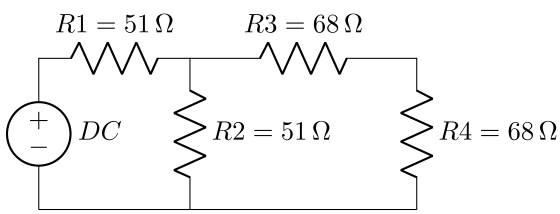

Figure 9: This is the circuit diagram of your first circuit for today. There are three loops in this circuit.

Solving for the voltage across each resistor

To find the voltages across these resistors you will need these three systems of equations. We will provide those for your first circuit, but you will need to set up your own systems of equations to solve for the second circuit

$$I_{tot} = I_2 + I_3 \tag{6}$$

If loop arrow matches current arrow, it's a voltage drop. So for loop 1:

$$emf - R1I_{tot} - R2I_2 = 0 \tag{7}$$

Loop 2:

$$R2I_2 - R3I_3 - R4I_3 = 0 \tag{8}$$

Please use these three equations above to solve for I_t, I_2, I_3, then you can use those values to solve for the voltage across the resistors with Ohm's Law. In your first table you are also asked for the theoretical voltage uncertainty across that component. You have to propagate forward the uncertainty on the resistor, as given by the resistor uncertainty band. That means you should do your found voltage, V, and multiply it by the uncertainty on the resistor given by the resistor uncertainty band, δR to find your uncertainty on the theoretical voltage, δV.

$$V * \delta R = \delta V \tag{9}$$

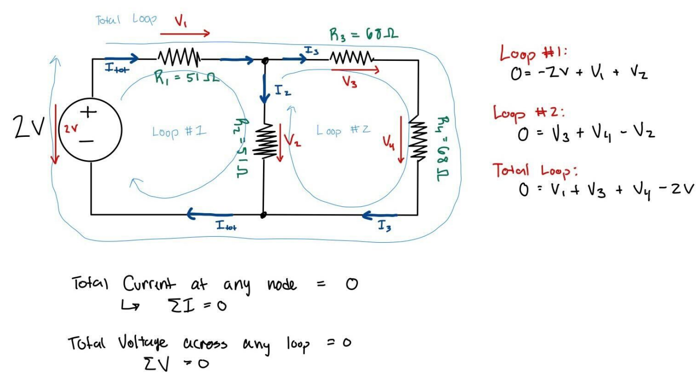

Figure 10: A second way of looking at our first circuit, annotated by CW, who is great.

Question 1 **Data Analysis part 1**

Before you begin data collection fill out the table 1 below. If you wish to use a software to solve your system of equations please consider checking out the Matlab or Python help at the bottom of the lab.

Show your calculations for the voltages across each resistors.

Table 1: Fill out this theoretical voltage for the first circuit.

Resistor	Resistor uncertainty band	Theoretical voltage across that component	Theoretical voltage uncertainty across that component
2V	Nothing needs to be entered here	Nothing needs to be entered here	Nothing needs to be entered here
R1 = 51 Ω			
R2 = 51 Ω			
R3 = 68 Ω			
R4 = 68 Ω			

Data Collection

1. Redraw Figure 9 for yourself, such that you can keep track of the voltage across the resistors that we will be measuring. We recommend marking which direction your voltmeter is measuring.
2. Open Logger Pro on the computer and make sure that both the voltage probe is recognized.
3. With the power supply off, connect the circuit as shown above in the Figure 9.
4. Turn down the voltage and current knobs on the power supply, then turn on the supply. While watching the display on both the computer and the power supply, slowly adjust the supply so you are outputting *2.0 volts*, as measured across your DC power supply on LoggerPro. You will have to adjust both the voltage and the current control as the output will be limited by the lowest setting.

5. Take your LoggerPro Voltmeter and measure the voltage across your first resistor, R1, 51 Ω, for 10 seconds. Include the mean and standard deviation of that data in your table. For an example table please see Table 2. Pay special attention to what direction the voltage is flowing. The sign of your voltage will be important later.
6. Then move your LoggerPro Voltmeter, being very careful not to touch the metal as we have not turned the DC power supply off, and measure the voltage across the center resistor, R2, also 51 Ω.
7. Measure the voltage across the other resistor, R3, 68 Ω.
8. Finally, please measure the voltage across your final resistor, R4, 68 Ω. After you have recorded these values onto your chart please move onto your second circuit.
9. You will repeat this process with a second circuit, pictured in Fig. 11.

Question 2 *Please fill out the table 2 below with your results from measuring the voltage across the resistors and DC power supply.*

Table 2: Data collection example for your first circuit.

Resistor	Resistor uncertainty band	Measured voltage across that component	Measured voltage uncertainty across that component
Voltage input	Nothing needs to be entered here		
R1 = 51 Ω			
R2 = 51 Ω			
R3 = 68 Ω			
R4 = 68 Ω			

Check your Data! You just used Kirchhoff's Law to calculate the expected voltage in table 1. Are the values on the same order of magnitude? Do they match your understanding of Kirchhoff's voltage loop law?

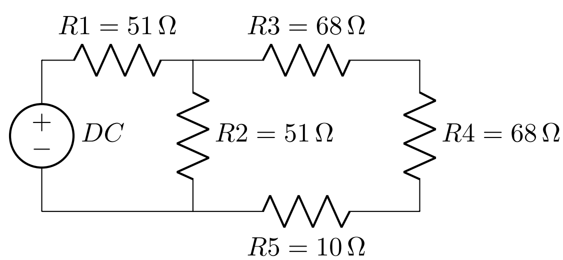

Figure 11: This is the circuit diagram of your second circuit for today. There are three loops in this circuit.

Question 3 **Data Analysis part 2**

Before you begin data collection on your second circuit fill out the table 3 below. If you wish to use a software to solve your system of equations please consider checking out the Matlab or Python help at the bottom of the lab.

Show your calculations for the voltages across each resistors.

Table 3: Fill out and include in your lab writeup this theoretical voltage for the second circuit.

Resistor	Resistor uncertainty band	Theoretical voltage across that component	Theoretical voltage uncertainty across that component
2V	Nothing needs to be entered here	Nothing needs to be entered here	Nothing needs to be entered here
R1 = 51 Ω			
R2 = 51 Ω			
R3 = 68 Ω			
R4 = 68 Ω			
R5 = 10 Ω			

Question 4 *Please fill out the table 4 below with your results from measuring the voltage across the resistors and DC power supply.*

Table 4: Data collection example for your second circuit.

Resistor	Resistor uncertainty band	Measured voltage across that resistor	Measured voltage uncertainty across that resistor
Voltage across the DC power supply	Nothing needs to be entered here		
R1 = 51 Ω			
R2 = 51 Ω			
R3 = 68 Ω			
R4 = 68 Ω			
R5 = 10 Ω			

Question 5 **Lab Modus Operandi**

Communicate the steps that you took when collecting and analyzing your data.

Pretend you are writing this so a fellow student that missed this lab could take and analyze the data using only this section. For example, you do not need to tell them to press start in Logger Pro or open the program, but you would want to tell them what sensors you used to collect data and if there are any special settings that you used. (3-4 sentences)

Question 6 **Results**

Write a sentence or two for each question asked below, use the actual numbers from your data analysis.

1. Numerically compare your theoretical calculations for each loop in your first circuit with what you know about Kierchoff's Loops Laws.
Then please individually compare your theoretical voltages against your corresponding measured values and examine if they fall within each others uncertainty.

2. Numerically compare your theoretical calculations for each loop in your second circuit with what you know about Kierchoff's Loops Laws.
Then please individually compare your theoretical voltages with your corresponding measured values and see if they fall within each others uncertainty.

Question 7 **Conclusion**

Write 2-4 sentences for each question asked below based on your results. If your results do not agree please speculate on some possible reasons why they do or don't. If they do agree make sure to re-write the necessary numbers to support that.

1. Do the theoretical equations for your loop 1, 2 and 3 for each of your circuits today agree, within uncertainties, of your measured values?

2. What further questions do you have about this lab? How would you test those questions if you had the ability to do so?

Question 8 *Table and Data Checklist: You should have answered all of the questions highlighted by the gray boxes. Make sure that you used complete sentences and that your conclusions match your results.*
Please include your ***4 tables*** *that covers the 2 configurations (Theoretical voltages across the resistors in circuit 1 and 2, and then experimental voltages across circuit 1 and 2). Include a proper caption and label for your table.*

Extra Credit

Test out the theory of Kirchhoff's Junction rule on the intersection of R1, R2 and R3 for your second circuit. Please include a table of the currents for all of those sections, as well as a discussion of the error inherent in the system, and whether or not that error is tolerable. (Up to 4 points of extra credit)

Appendix

Matlab to Solve System of Equations

** DANA PLEASE FILL THIS OUT **

Once you have collected all your data, and before you leave for the day please make sure to

1. **Turn off power supplies.** You may leave them plugged into the walls, but make sure they are off.

2. Leave Voltage and Current probes (Ammeter and Voltmeter's) plugged in to the logger pro, but please make sure none of the wires are touching the ground. Please also leave your green box plugged into your computer and at your station.

3. Unhook your wires from your circuit boards, and place the circuit board back in their white boxes.

4. Put wires back in the white box, or if there is no room in the white box then put them in the communal wire box that is near the front of each room.

5. If you used a capacitor and attached it to the board **please do not remove it from the board after connecting it**. Too much connecting and unconnecting will weaken or break the wires eventually.

The Figures and Caption Rules

There are a few very important aspects to creating a proper figure and caption. If you follow these rules, not only will you get points on your physics lab grades, you will impress your instructors and peers in the future.

The Caption

- The caption should start with a label so you can reference the figure from other places in your paper/report. For this course you should use "Figure 1", "Figure 2", etc.

- The caption should allow the figure to be standalone, that is to say, by reading the caption and looking at the figure, it should be clear what the figure is about and why it was included without reading the whole paper.

- The caption should contain complete sentences and be as brief as possible while still conveying your information clearly (this is not always easy).

- Please add captions to your tables as well, otherwise we will not know what we are looking at.

The Figure

- **Make sure that the resolution is high enough to not be pixelated at its final size.**

- Check that any text is readable at the final size (Using a smaller graph in Logger Pro will cause the text to be larger in relation to the graph when inserted into another program).

- For graphs, ensure that the axes are labeled (including units) and that there is a legend if you have multiple data sets on the same graphs.

Tables

- The first row of the table should be a header, where each item is labeled with what is contained in that row. If it is a physical measurement it should have the correct units.

- For tables include a short caption of what is contained in the table, or what was examined.

- The caption should start with a label so you can reference the figure from other places in your paper/report. For this course you should use "Table 1", "Table 2", etc. For an example of a good table caption please see Figure 12.

Table 1. Baseline characteristics of study participants

Variables	Intervention group (n=14)	Control group (n=15)
Women (no [%])	7 (50)	5 (33)
Median age (range)	22.0 (19 – 58)	21.0 (18 – 70)
First winter in icy conditions (no [%])	—	1 (7)
Previous falls on ice (no [%])	8 (57)	11 (73)
≥ 1 fall this winter (no [%])	4 (29)	7 (50)
Injury from fall this winter (no [%])	1 (7)	—
Time been walking this route (no [%]):		
<6 months	3 (21)	2 (13)
6–12 months	9 (64)	9 (60)
>12 months	2 (14)	4 (26)

Figure 12: An example table from the paper Lianne Parkin, Sheila M Williams, and Patricia Priest, "Preventing Winter Falls: A Randomised Controlled Trial of a Novel Intervention" 122, no. 1298 (2009): 9.

Human Error

Humans can often be a source of error, but describing one's total error as 'human error' does little to illuminate the subject. Humans can contribute error to a system, but it is not their mere presence, often, that causes that error. That error is contributed by a specific action, or lack of action of the operator and you should always be specific. If we ask for why there might be error in a system, and someone responses with just human error without explaining what, specifically, that answer will not receive credit.

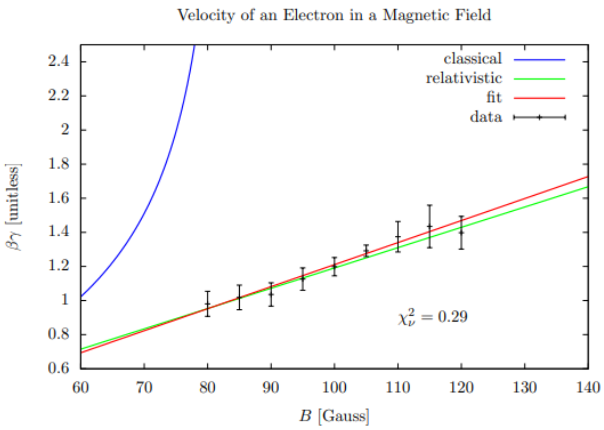

Figure 13: Example of a good figure with excellent error bars and a label. Figure from Philip Ilten of University College Dublin [?].

Python

According to IEEE Spectrum, Python is the most popular programming languages. Python is a free, general purpose, cross discipline programming language that has moved to the forefront of many disciplines.

If you decide to use Python your TA's will help you troubleshoot your code. While they might be able to help you troubleshoot when you use a different program or code, be aware of the fact that they are not familiar with all programming codes. There are many languages (R, Matlab, Opal, Julia, etc.) out there that are just as useful as Python, but we have chosen to use Python here. You may use any programming language you wish, but not Excel or Google Sheets.

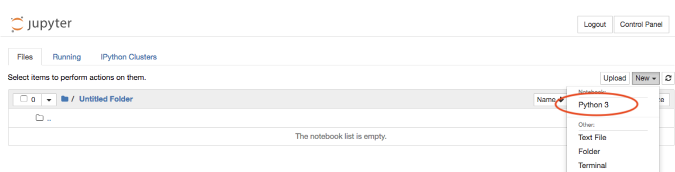

Figure 14: Navigate to `jupyterhub.wpi.edu/hub/login` and sign in with your WPI email address and password, choose an instance to spawn (either is fine) and create a new Python 3 file as shown here

We have set up a Jupyter notebook you may use. The website is https://jupyterhub.wpi.edu/hub/login.[1] There are many ways to learn Python, including reading a book, asking a friend, working through examples, or googling furiously when problems arise. We encourage you to discover which approach works best for you. Going forward this class will provide basic Python examples, but feel free to iterate upon the template we provide. What we provide is a stripped down version, and elaboration is encouraged. See this Github repository for our examples. We hope at the end of this term you will be able to add to your resume "Proficient in Python".

Jupyter uses a cell based system and evaluated variables carry over to the next cell. There are a few different types of cells, Figure 15 shows 2 kinds, the code cell, which we will be using most of the time, and the markdown cell, which you can use to add nicely formatted notes to you file.

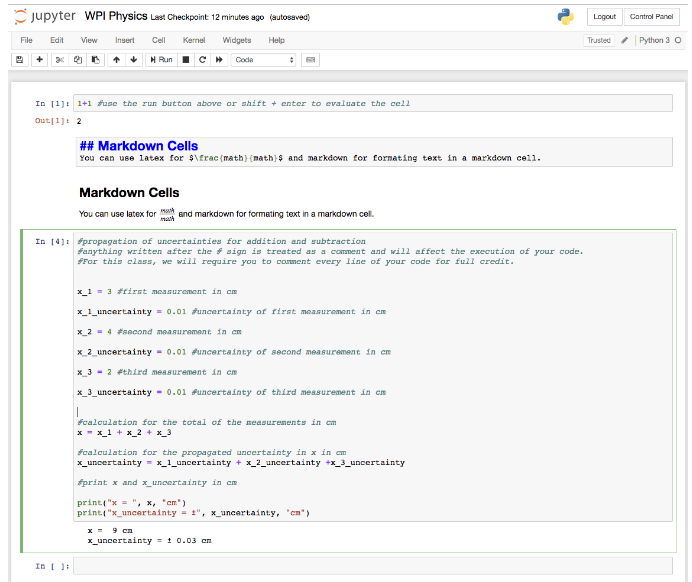

Figure 15: Above is the code that you could use use to propagate uncertainty for values that are added or subtracted. Always remember to comment your code.

If you prefer to work through a book or examples we recommend Mark Newman's book, which is available for free on his website [?]. Chapter Two is a basic introduction to the syntax for Python. Chapter Three covers graphs and visualizations, and we hope you will look into it if you learn best from a book. If you wish to get a head start in this class we recommend reading this book.

If you wish for a more advanced textbook there is a compilation of free online computational physics books here.

Propagation of uncertainties for addition and subtraction

Adding a measurement is trivial, but what about adding the uncertainties? In the case of addition and subtraction, the equation for combining uncertainties is

$$\delta x = \delta x_1 + \delta x_2 + \delta x_3 \,, \tag{10}$$

where δx is the total uncertainty in your length calculation and δx_1, δx_2, and δx_3 are the uncertainties of your individual measurements.

This is called error propagation. Notice that the error in your final measurement is much larger than the error of any of your individual measurements.

[1] If you cannot log in please email WPI's IT department, and they will be happy to help polite students. The first thing they will tell you, however, is to check to make sure you don't have to change your password and try a VPN if you are off campus.

Propagation of uncertainties for multiplication and division

Again, multiplying two numbers is trivial, but what about the uncertainties? This time we will use a slightly different method of error propagation from our one above for addition because we are multiplying them instead of adding them. This method is valid for both multiplication and division of measurements with uncertainties. The formula is

$$\frac{\delta A}{|A|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|} \,, \tag{11}$$

where A is the area, x is the length, y is the width, δx and δy are the uncertainties associated with these measurements, and δA is the propagated uncertainty of the area.

Therefore if we were to do the uncertainty of, say two numbers divided by each other, let's give them the relationship $\frac{V}{I} = R$. Then the error on R would be

$$\delta R_u = [\frac{\delta V_u}{V} + \frac{\delta I_u}{I}] * \frac{V}{I}, \tag{12}$$

where δV_u is the uncertainty on V, δI_u is the uncertainty on I, and δR_u is the uncertainty on R.

Where does this derive from?