Department of Physics

Complex circuits and Kirchhoff's rules

1120 Lab 4R

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I Lab Objectives

Construct a complex electric circuit and investigate how currents and voltages vary in various parts of the circuit.

- Construct a complex electric circuit using various circuit components.
- Measure currents and voltages in various parts of the circuit and find the relations between them.

Lab Materials

- Computer with internet access

Links for this lab

- PhET Construction Kit

II Theory

II.1. Conservation of Charge

In Lab #1, when you brought a neutral balloon close to a sweater with free charge (free electrons), the balloon picked up some charge (electrons from the sweater were transferred to the balloon) and became negative. On the other hand, the sweater lost some charge (electrons were lost to the balloon) and it became positive. In such charge transfer process, what can you say about the total charge contained in the system (sweater + balloon) before and after the process?

The total charge in a closed system remains conserved. This means that charge can neither be created nor destroyed, it can simply be transferred from one object to another. This is called the law of conservation of charge.

In Lab #2, when you constructed an electric circuit, and connected a battery to it, you observed a flow of charge in the circuit. What causes the flow of charge in an electric circuit? What is the role of battery in the circuit? You measured potential differences across the resistor in the circuit. How are the potential differences related to the battery voltage or the electromotive force (emf) of the battery?

The total energy in a closed system remains conserved. This means that energy can neither be created nor destroyed, it can simply be transformed from one form to another. This is called the law of conservation of energy.

II.2. Kirchhoff's rules

In this lab, you will be constructing complex electric circuits with multiple resistors and batteries, and investigating the relations between currents and voltages in such circuits.

II.2.1 Kirchhoff's junction rule

The algebraic sum of all currents at a junction is zero. A junction in an electric circuit is a point where at least three currents come into or go out of it. Fig. 1 shows a simple junction in an electric circuit.

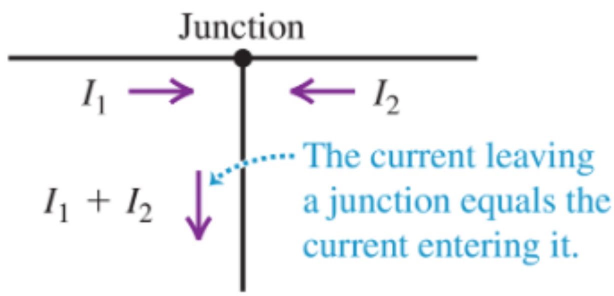

Figure 1: A simple junction in an electric circuit.

Mathematically Kirchhoff's junction rule can be written as:

$$\Sigma I = 0 \tag{1}$$

When you write Kirchhoff's junction rule, take the currents flowing toward a junction to be positive and the currents flowing out of junction to be negative. For example, in Fig. 1, the currents I_1 and I_2 are positive and the current $(I_1 + I_2)$ is negative, i.e.,

$$\Sigma I = I_1 + I_2 - (I_1 + I_2) = 0 \tag{2}$$

Kirchhoff's junction rule is based on the conservation of electric charge.

II.2.2 Kirchhoff's loop rule

The algebraic sum of emfs and potential differences in a loop is zero. A loop in an electric circuit is a closed path containing sources (batteries, generators, etc.,) which drive charges and currents, and sinks (resistors, bulbs, motors, etc.,) which dissipate energy in heat, light or some other forms. Fig. 2 shows multiple junctions and loops in a complex electric circuit.

Mathematically, Kirchhoff's loop rule can be written as follows:

$$\Sigma V = 0 \tag{3}$$

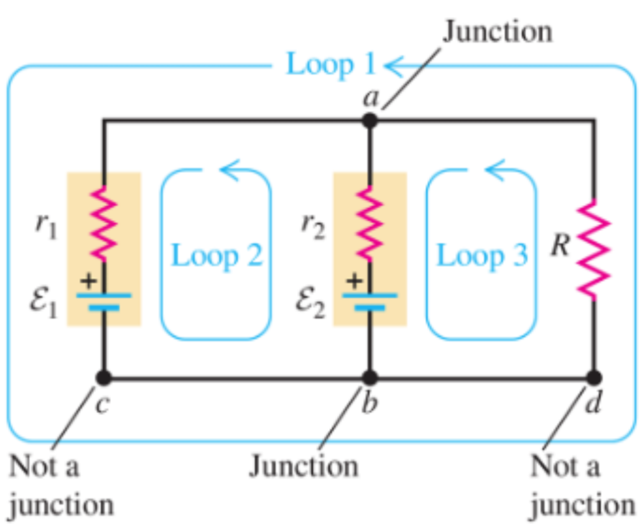

Figure 2:

For emfs, the sign of emf is taken positive in traveling from the negative terminal of a battery to its positive terminal, while going around a loop, and the sign of emf is taken negative in traveling from the positive terminal of a battery to its negative terminal, while going around the loop. Fig. 3 shows a sign convention for emfs in applying Kirchhoff's rule.

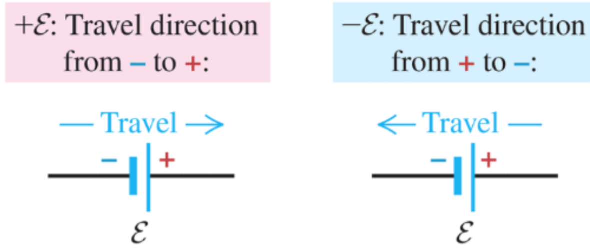

Figure 3: A sign convention for emfs in Kirchhoff's loop rule

For potential differences, the sign of the product (IR) is taken positive if the direction of the travel is against the direction of current flow, and the sign of the product (IR) is taken negative if the direction of the travel is in the direction of current flow. Fig. 4 shows a sign convention for potential differences in applying Kirchhoff's rule.

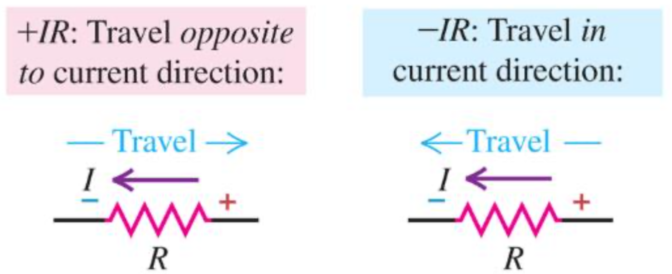

Figure 4: A sign convention for emfs in Kirchhoff's loop rule

Kirchhoff's loop rule is based on the conservation of energy. Using PhET simulation platform, you will be constructing a complex circuit as shown in Fig. 5.

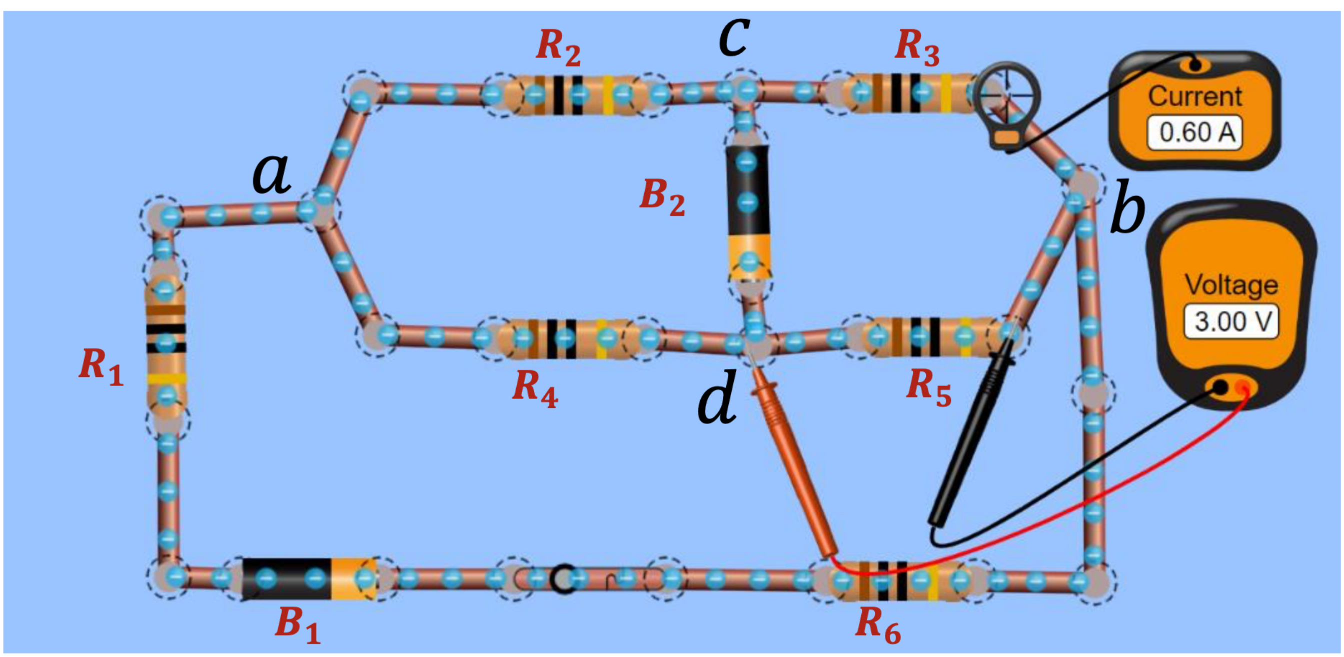

Figure 5: A complex electric circuit with multiple batteries and multiple resistors. The blue spheres represent the electrons flowing through the circuit.

We can re-draw the circuit shown in Fig. 5 using electric symbols as shown in Fig. 6, where the red arrows represent the direction of the conventional current.

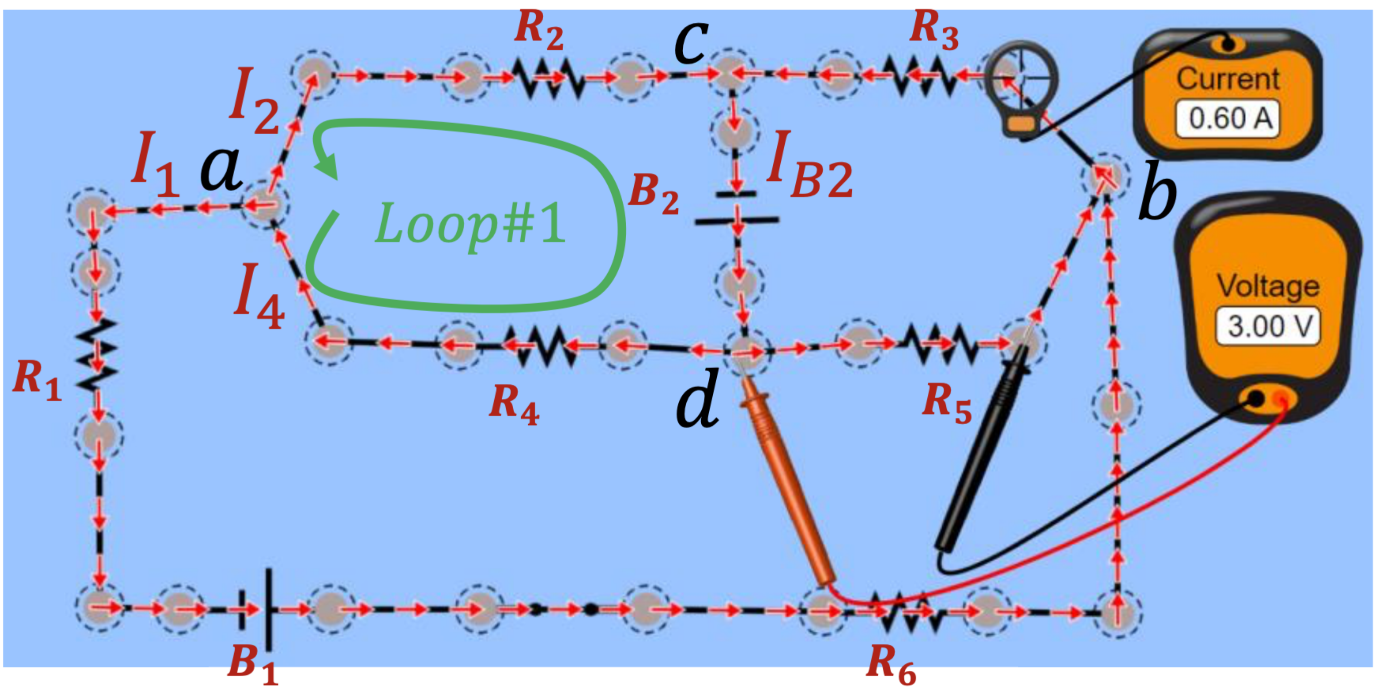

Figure 6: A complex electric circuit with multiple batteries and multiple resistors. The red arrows represent the direction of conventional current flowing through the circuit.

We can write equations relating currents, emfs, and potential differences by using Kirchhoff's rules at various junctions and loops with proper sign conventions. Those equations can be solved for unknown currents, emfs and potential differences. For example, at Junction a (Fig. 6),

$$I_4 - I_1 - I_2 = 0 \tag{4}$$

Then, in Loop #1 (travel the loop in counterclockwise (CCW) direction from starting from point a)

$$+I_4 R_4 - \epsilon_{B_2} + I_2 R_2 = 0, \tag{5}$$

Where ϵ_{B_2} is the emf of the battery B_2. For another example of this calculation please see here

How many junctions and loops are possible in the circuit shown in Fig. 6? Can you write equations like Eq. 4 and Eq. 5 for those junctions and loops? Try to write them out, and show them to your lab instructor! Note: Ideally, the potential difference across a connecting wire is zero. Consider an ideal connecting wire in your calculations. Also, consider batteries, voltmeters and ammeters to be ideal unless stated otherwise.

Question 1 Introduction

Please write an introduction section based on the theories you will work with today. For reference on what it should look like please consult the Generic Deliverables at this link.

III Procedure

We strongly recommend you watch these short videos in the following links (before or after the lab hour). These videos show how currents and voltages are measured in complex circuits in a real lab. First Video Second Video

Open the PhET simulation 'circuit-construction-kit' window by clicking on the following link: PhET Construction Kit.

Construct a complex circuit as shows in Fig. 7. Set your batteries **B1= 10V and B2= 3V** and all the resistors to the value 10 ohm.

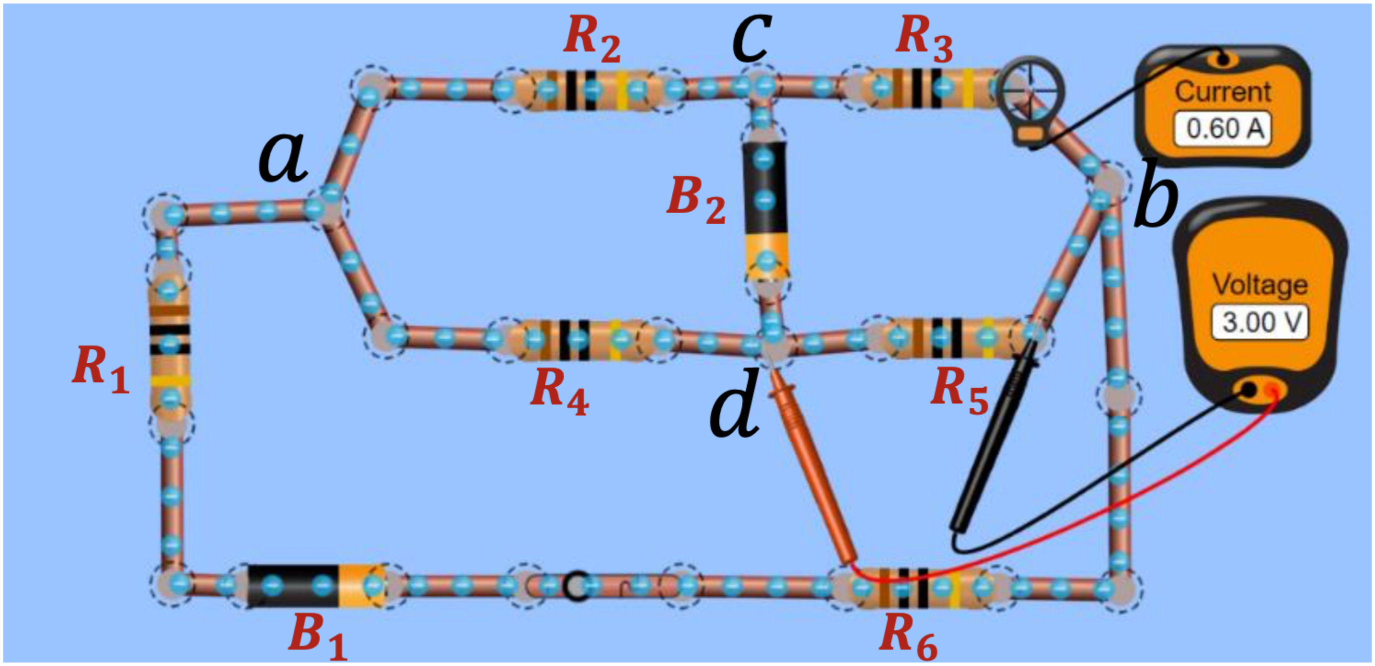

Figure 7: A complex electric circuit, showing the flow of electrons. If you want to check and see if your circuit matches this one in terms of components the batteries are 9V and resistors are 10 Ω, then your Ammeter should read 0.60A and Voltmeter should read 3.00 V. For this experiment, however, you will use different values for the batteries.

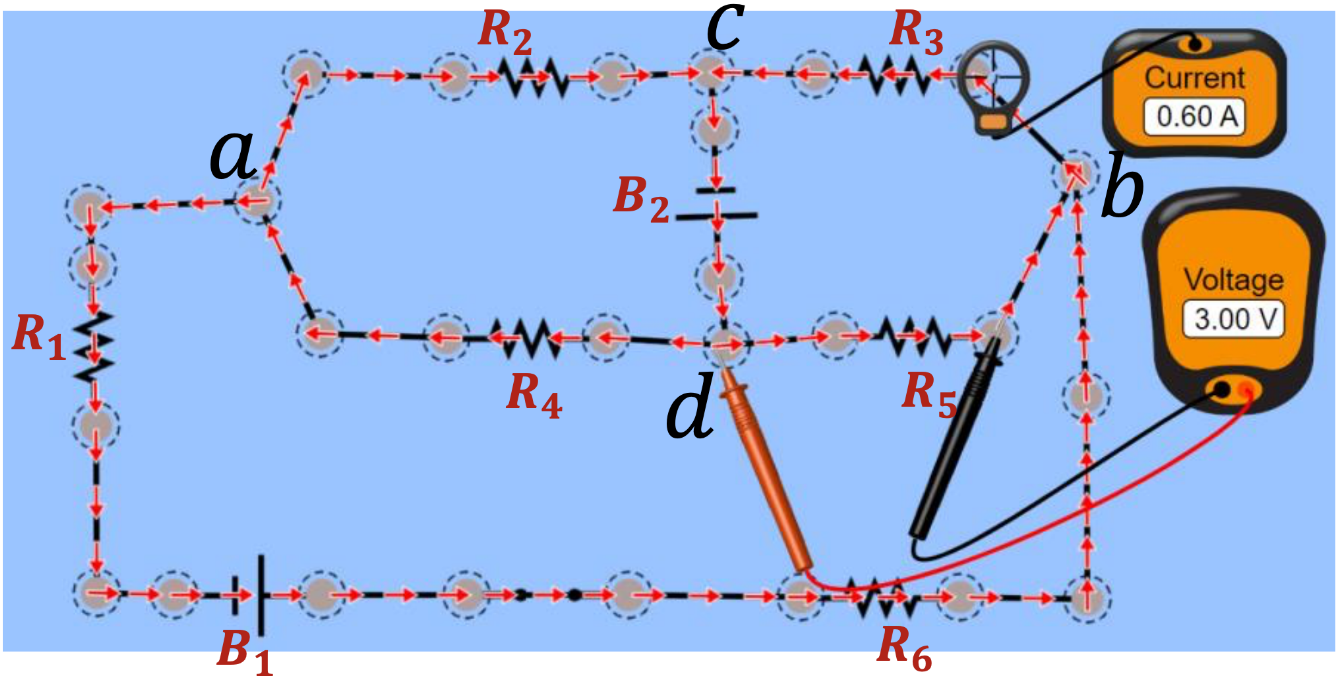

Figure 8: The complex circuit shown in Fig. 7 with circuit symbols and conventional current flow. The red arrows show the directions of flow of the conventional current in each branch of the circuit.

Question 2 **Results, part 1**

Please write a Results section based on the results you have collected today. For reference on what it should look like please consult the Generic Deliverables at this link. For guidance on what questions should be addressed please see below.

Using a voltmeter and an ammeter, measure the potential differences (voltages), V's across the resistors (R_1, R_2, etc.), and currents I's passing through them. Record the values you measured in Table 1. Calculate the ratio of the voltage to current in each resistor. Record the calculated values of the ratios in the last column of Table 1.

Table for question 1			
Resistors	Voltage (V)	Current (I)	Ratio of V to I (V/I)
R_1	$V_1 =$	$I_1 =$	
R_2	$V_2 =$	$I_2 =$	
R_3	$V_3 =$	$I_3 =$	
R_4	$V_4 =$	$I_4 =$	
R_5	$V_5 =$	$I_5 =$	
R_6	$V_6 =$	$I_6 =$	

Question 3 **Results, part 2**

Find Σ I for the currents I_3, I_5, and I_6 using proper sign convention for currents. Show your calculation, and explain how you got there (2-3 sentences)

Question 4 **Results, part 3**

Measure the emf of battery B_1 and that of B_2, and record the results in your worksheet.

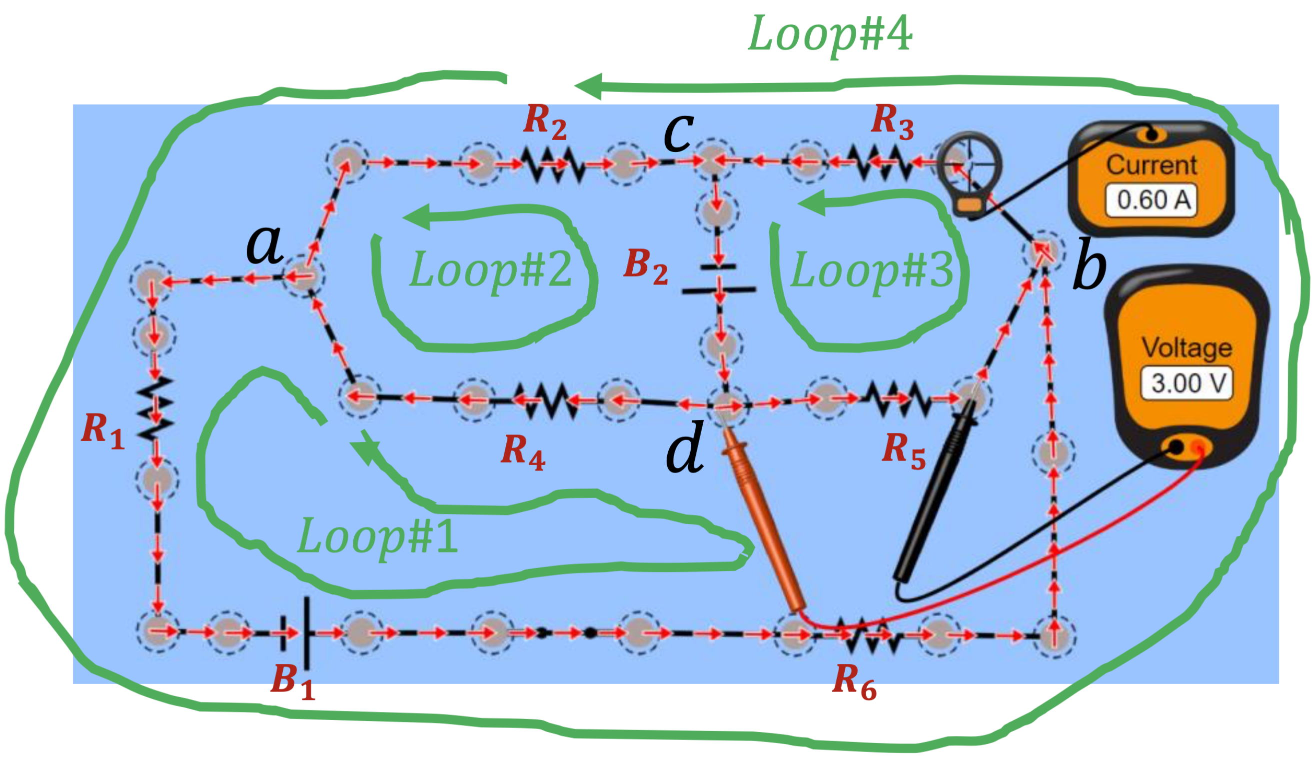

Figure 9: Some current loops in the complex circuit. Four closed loops are marked (Loop 1, 2, 3 and 4) in this complex circuit. Please continue to use the same resistor (10 Ohms), however the battery (B1= 10V and B2= 5V here) values may be different.

Question 5 **Results, part 4**

Calculate ΣV in these loops, using proper sign convention for the potential differences and emfs. Then record the results in the last column of the next table. Please show your work.

Table for question 4		
Loops	V, ϵ (V)	ΣV
1	V_1 = V_4 = V_5 = V_6 = ϵ_{B_1} =	
2	V_2 = V_4 = ϵ_{B_2} =	
3	V_3 = V_5 = ϵ_{B_2} =	
4	V_1 = V_2 = V_3 = V_6 = ϵ_{B_1} =	

Question 6 **Results, part 5**

Measure the potential differences between point A and B and record on your worksheet. There are several ways that you can calculate V_{ab} from the voltages and emf's that you measured in Question 1 and Question 4. Show at least two calculations to get V_{ab} in your results by following two different paths through the circuit.

Question 7 **Conclusion**

Please write a conclusion section based on the results you have collected today. For reference on what it should look like please consult the Generic Deliverables at this link.

IV Acknowledgement

This lab was first developed by Dr. Rudra Kafle to replace the on-site labs for D term, 2020 due to the COVID-19 Emergency. They have been iterated upon by Dana, who takes full credit for any mistakes therein.