Adding Integers

Adding Integers with the Same Signs:

1. **Add** the absolute values of the numbers (without their signs).
2. **Keep the sign** (either positive or negative) of both numbers.

Adding Integers with Different Signs:

1. **Subtract** the absolute value of the numbers (without their signs) having the largest number on top.
2. Keep the **sign of the largest absolute value**. (larger number determines the sign)

EXAMPLES:

Same Signs:	Different Signs:
7 + 10 = 17	4 + (-9) = -5
-6 + (-5) = -11	-7 + 18 = 11

Find each sum.

1. 21 + 15	2. -11 + 81
3. -1 + 39	4. -8 + (-24)
5. 90 + (-79)	6. 31 + 96
7. 25 + (-90)	8. 15 + 31 + (-20)
9. 8 + 41 + 35	10. 18 + (-80) + (-45)

Subtracting Integers

SAME, CHANGE, CHANGE

When Subtracting ANY Numbers:

1. Change any minus sign to a plus.
2. Change the sign of the number immediately **after** each minus to its opposite (change a positive number to a negative and vice-versa).
3. Follow the rules for adding integers.

EXAMPLES:

Two Numbers:	More Than Two:
-4 – 1 →	6 – 1 – (-3) →
-4 **+** (**-1**) = -5	6 **+** (**-1**) **+** **3** = 8

Find each difference.

1. 39 – 18	2. 65 – 72
3. -85 – (-42)	4. -15 – (-86)
5. -21 – 24	6. -15 – (-57)
7. 652 – (-57)	8. 346 – 865
9. -8 – (-4) – (-6)	10. 90 – (-26) – (-48)

Multiplying/Dividing Integers

When Multiplying ANY Numbers:

1. **Multiply** or **divide** the absolute values of the numbers.
2. For the **sign of the product/quotient**, follow the rules below.
   - Positive x Positive = Positive — Positive ÷ Positive = Positive
   - Negative x Negative = Positive — Negative ÷ Negative = Positive
   - Positive x Negative = Negative — Positive ÷ Negative = Negative
   - Negative x Positive = Negative — Negative ÷ Positive = Negative
   - If there are an even number of negative integers being multiplied/divided, the product will be positive.
   - If there are an odd number of negative integers being multiplied, the product will be negative.

EXAMPLES:

2(8) = 16	-8 · 6 = -48	16 ÷ -8 = -2
-10 × -10 = 100	2(-5) = -10	$\frac{-28}{-4} = 7$

Find each product/quotient.

1. -8(6)	2. -10 · -10
3. -24 ÷8	4. $\frac{-21}{7}$
5. -14(-4)	6. -96 ÷ -4
7. $\frac{48}{16}$	8. -15 ÷ -15
9. 5(11)(-3)	10. 10(-8)(-2)

Adding/Subtracting Rational Numbers

When Adding/Subtracting ANY fraction:

1. Use GCF to get common denominators.
   - Add/Subtract numerators.
   - Denominators stay the same.
2. Add/subtract the whole numbers if needed.
   __When subtracting, the largest absolute value goes on top__
3. Reduce to lowest terms.
4. Use the sign of the number with the larger absolute value.

When Adding/Subtracting ANY numbers in decimal form:

1. Line up the place values.
2. Use zeros as place holders.
3. Integer rules apply.

EXAMPLES:

$\frac{-2}{3}+\frac{5}{9}=\frac{-6}{9}+\frac{5}{9}=\frac{-1}{9}$

$-2\frac{3}{5}-5\frac{4}{9}=-2\frac{27}{45}+-5\frac{20}{45}=-7\frac{47}{45}=-8\frac{2}{45}$

$$\begin{array}{r} 43.29 + 3.127 \\ 43.290 \\ +\ \ 3.127 \\ \hline 46.417 \end{array}$$

Find each sum or difference.

1. $8\frac{5}{12}-2\frac{7}{12}$	2. $\frac{14}{21}+\frac{-2}{7}$
3. $\frac{5}{8}-\frac{2}{3}$	4. $-1\frac{3}{4}+\frac{-3}{16}$
5. $\frac{4}{7}+\frac{-2}{7}$	6. $\frac{14}{25}+\frac{2}{5}$
7. 85.3 – 37.07	8. 27 + 5.19
9. -34.1 + (-17.63)	10. -18.21 – (-7.3)

Multiplying/Dividing Rational Numbers

When Multiplying ANY fractions:

1. Rewrite all numbers (whole numbers, mixed numbers, integers) as a fraction.
2. Reduce by simplifying a numerator with a denominator.
3. Multiply numerators. Multiply denominators.
4. Integer rules apply for the sign.

When Dividing ANY fractions:

1. Rewrite all numbers (whole numbers, mixed numbers, integers) as a fraction.
2. Change the division sign to multiplication and take the reciprocal of the fraction immediately **after** the division sign.
2. Reduce by simplifying a numerator with a denominator.
3. *Follow rules for multiplying fractions.*

When Multiplying ANY numbers:

1. Multiply the numbers.
2. Count how many total numbers **after** the decimal.
3. Put the decimal in so that there are the same amount of numbers after the decimal.
4. Integer rules apply for the sign.

When Dividing ANY numbers:

1. Move the decimal out of the divisor and then that many times in the dividend.
2. Use zeros as place holders.
3. Divide and bring decimal straight up in the quotient.
4. Integer rules apply for the sign.

EXAMPLES:

$\frac{1}{2} \bullet \frac{-2}{7} = \frac{-1}{7}$

$-1\frac{1}{9} \div \frac{2}{3} = \frac{-10}{9} \bullet \frac{3}{2} = \frac{-5}{3}$

$0.63 \div 0.9 = .9\overline{)}\,.63$ (.7; - 63; 0)

Find each product or quotient.

1. $-\frac{5}{6}\left(-\frac{2}{5}\right)$	2. $2\frac{5}{6} \bullet 3\frac{1}{3}$
3. $-10 \div \frac{3}{8}$	4. $\frac{-16}{7} \div \left(-\frac{12}{35}\right)$
5. 85(0.07)	6. **$-0.104 \div (-0.13)$**
7. $13.42 \div 67.1$	8. 2.001(0.05)

Evaluating Expressions

When Evaluating ANY Expression:

1. **Substitute** each variable with its assigned value.
2. **Simplify** the expression using order of operations.

★ Be careful! When replacing a variable with a **negative value**, put **parentheses** around the value in the expression.

EXAMPLES:

Evaluate the expression 4xy, if x = -5 and y = -6.

4**xy** → 4(**-5**)(**-6**) → 180

Evaluate each expression.

1. Evaluate 3x when x = -6.	2. Evaluate -8x when x = -5.
3. Evaluate 0 ÷ y when y = -12.	4. Evaluate $\frac{x}{4}$ when x = -8.
5. Evaluate $\frac{-144}{y}$ when y = -12.	6. Evaluate -2(x + y) when x= -1 and y = 4.
7. Evaluate 3(y + x) when x = 6 and y = 1.	8. Evaluate (a + c) – b when a = 0.4, b = 3.5, and c = 15.61.
9. Evaluate c – d – a when a = 0.4, c =15.61, and d = 0.03.	10. Evaluate x + y when x = $\frac{3}{8}$ and y = $\frac{3}{4}$.

Translating Into Expressions

To Translate Sentences into Algebraic Expressions:

1. Identify the variable by telling what phrase the variable stands for in the sentence. (This could be the phrase "a number" or it could be the unknown information in the sentence).
2. Translate the sentence into related numbers, operations, and variable(s). Usually, the order of the translation will mimic the order of the sentence. (It is helpful to know what words and phrases represent the four main operations, addition, subtraction, multiplication, and division.)

EXAMPLES:

"**Seven less than some number**" Let n = some number → **n – 7**

"**Thirteen** dollars **plus** the **cost of food**" Let f = cost of food → **13 + f**

Identify the variable. Then, translate into an expression.

1. A number more than seven	2. The product of some number and six
3. Some number decreased by twelve	4. The quotient of ninety and a number
5. Eight less than some number	6. Twice the number
7. Half of some number	8. Seventeen more than a number
9. Brian is triple his nephew's age	10. Maria ran 4½ miles more than Amy

Two-Step Equations

To Solve Two–Step Equations:

1. **Isolate** the **variable** by using inverse operations
2. **Check** your solution by replacing the variable with the integer.

Examples:

$2x - 10 = 12$	$7x + 9 = -12$	$-3x + 4 = 19$	$\frac{x}{3} + 7 = 10$
$+10 \quad +10$	$-9 \quad -9$	$-4 \quad -4$	$-7 \quad -7$
$2x = 22$	$7x = -21$	$-3x = 15$	$\frac{x}{3} = 3$
$\div 2 \quad \div 2$	$\div 7 \quad \div 7$	$\div -3 \quad \div -3$	$\bullet 3 \quad \bullet 3$
$x = 11$	$x = -3$	$x = -5$	$x = 9$

Solve and check. Show all of your work.

1. $6m + 1 = -23$	2. $5 + 4d = 37$
3. $3 - 7y = -25$	4. $6 - 5b = -14$
5. $\frac{11}{12}e + 25 = 47$	6. $15 - \frac{1}{7}w = -3$
7. $8(x + 3) = 72$	8. $-7(z - 6) = -70$
9. $-0.6(r + 0.2) = 1.8$	10. $\frac{-2}{3}(w - \frac{4}{9}) = -\frac{4}{5}$

Writing Equations

To Write an Equation:

1. **Identify** a variable. Ex: Let x = the number
2. Look for **key words**: Ex decrease (-), increase (+), is (=)

Example:

A **number** increased by 6 is 24.
Let x = the number

$x + 6 = 24$

Five less than a **number** times three is -25
Let x = the number

$3x - 5 = -25$

Identify the variable. Translate into an equation.

1. Twice a number decreased by 7 is 19.	2. Six times a number increased by 8 Is -84.
3. Four minus one-fifth a number is -6.	4. Eight plus two-thirds a number is 12.
5. A company charges $2 for each balloon in an arrangement and a $3 delivery fee. You have $9 to spend. Write an equation for this situation.	6. It costs $7.50 to enter a petting zoo. Each cup of food to feed the animals is $2.50. If you have $12.50, how many cups can you buy?
7. Jamal and two cousins received the same amount of money to go to a movie. Each boy spent $15. Afterward, the boys had $30 altogether. How much money did each boy receive?	8. Mr. Singh had three sheets of stickers. He gave 20 stickers from each sheet to his students and has 12 total stickers left. How many stickers were originally on each sheet?

Solving and Graphing Inequalities

To solve and graph inequalities:

1. **Isolate** the variable by using inverse operations.
2. **Graph** the solution on the number line.
   - A closed circle is used when the point is included. ($\leq$, $\geq$)
   - An open circle is used when the point is not included. (<,>)

Example:

$$\begin{array}{r} X + 9 < 16 \\ -9 \quad -9 \\ \hline x < 7 \end{array}$$

Since x is less than 7, shade all values that are less than 7. Use an open circle.

Solve and graph each inequality:

1. $x - 4 < 10$	2. $x + 9 > 12$
3. $4x \leq 2.4$	4. $0.7x \geq 56$
5. $\frac{x}{4} \leq 8$	6. $\frac{x}{9} \geq 1.5$
7. $x - 7.3 > 24$	8. $x + 5\frac{2}{3} < 31$

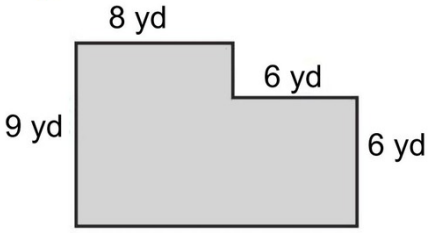
Geometry

Area of square: s^2	Area of trapezoid: $\frac{b_1+b_2}{2} \bullet h$	perimeter: add all sides
Area of rectangle: $b \bullet h$	Area of circle: $\pi \bullet r^2$	circumference: $2 \bullet \pi \bullet r$ or $\pi \bullet d$
Area of parallelogram: $b \bullet h$	Area of triangle: $\frac{b \bullet h}{2}$	Volume of prism: Area of the base $\bullet$ h or lwh

Solve. Show all work!

1. Find the circumference of a circle whose radius is $2\frac{3}{4}$ feet. Round to the nearest tenth.

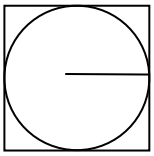
2. Find the perimeter of the composite shape.



3. Find the area of a triangle whose base is 7.5 cm and whose height is 11 cm. Round to the nearest tenth.

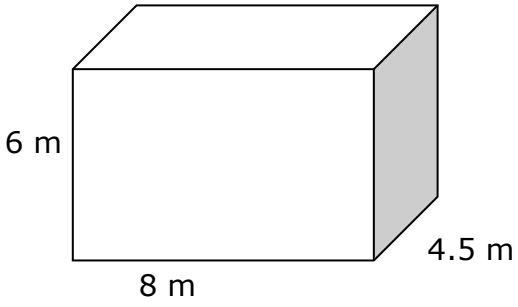
4. Find the area of a square whose side is $5\frac{2}{3}$ m.

5. Find the area of the square given a radius of 5 cm.

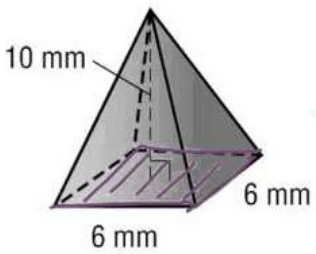


6. Find the volume of a cube whose side is 14 meters.

7. Find the volume of a rectangular prism.



8. Find the volume of the square pyramid.



Proportions

Setting up:

1. Create two equal ratios.
2. **Label** each numerator and denominator so that they **match.**

Solving:

1. **Cross multiply** by multiplying the two diagonals and set them equal to one another.
2. **Solve.**

$$\frac{3}{5} = \frac{x}{10} \qquad 3 \bullet 10 = 5 \bullet x$$

$$30 = 5x$$

$$6 = x$$

Solve. Show All Work!

1. $\frac{x}{2\frac{1}{3}} = \frac{8}{3}$	2. $\frac{6}{2} = \frac{4}{x}$
3. $\frac{5.1}{1.7} = \frac{7.5}{x}$	4. $\frac{6.4}{0.8} = \frac{8.1}{x}$
5. Given the scale 2 cm = 3 m, how long is the scale drawing of a basketball rim that is 16 m. tall?	6. To tie dye a shirt orange, you need 2 parts red to 5 parts yellow. How much yellow do you need if you have 13 parts red?
7. Find the unit price of a case (12 cans) of soda for $2.25.	8. A rectangle is 11.4 in tall and 5.4 in wide. If it is reduced to a height of 5.7 in then how wide will it be?

Constant of Proportionality

To Find *k*:

1. Find the **unit rate**, *y/x*.

To Determine if Proportional or Not:

1. Check that every *y/x* is the same or "constant."

Cans of Paint (x)	5	10	6	9	2
Bird Houses Painted (y)	15	30	18	27	6

For every can of paint you could paint 3 bird houses.

15/5 = 3; k = 3

Use the tables below to find the constant of proportionality, *k*, or to determine whether the relationship is proportional or not.

1. Find the constant of proportionality.

x(hours)	y(cookies)
1	4
2	8
3	12
4	16

2. Find the constant of proportionality.

x(weeks)	y(years)
2	4
3	6
5	10
9	18

3. Find the constant of proportionality.

x(hours)	y(miles)
1	5
2	10
3	15
4	20

4. Find the constant of proportionality.

x(dollars in sales)	y(dollars in commissions)
$30	$3
$50	$5
$80	$8
$90	$9

5. Do *x* and *y* have a proportional relationship? Explain your answer.

x(minutes)	y(gallons)
2	30
3	45
5	60
6	90

6. Do *x* and *y* have a proportional relationship? Explain your answer.

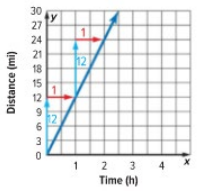
x(months)	y(dollars)
2	$48
3	$72
4	$88
5	$120

Graphing Relationships

To Graph and Determine Proportionality:

1. Plot each ordered pair, *(x, y)*, by moving right *x* units and up *y* units from the origin.
2. Check that the points are in a **line** and would include the **origin**, (0, 0).
3. If so, the relationship is proportional; if not, then it is not proportional.

Hours, x	$y = 12x$	Miles, y
0	$y = 12(0)$	0
1	$y = 12(1)$	12
2	$y = 12(2)$	24

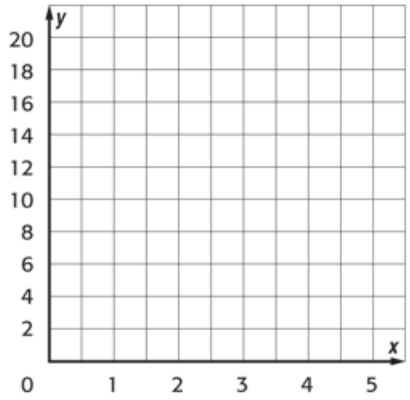


Proportional; points align; (0, 0)

Determine whether the relationship between the two quantities shown in each table is proportional by graphing on the plane.

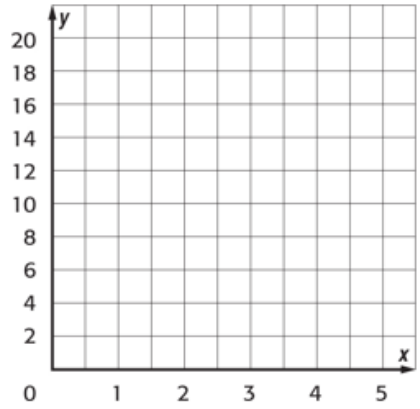
1.

x(number of tennis balls sold)	3	4	5	6
y(total cost)	6	8	10	12



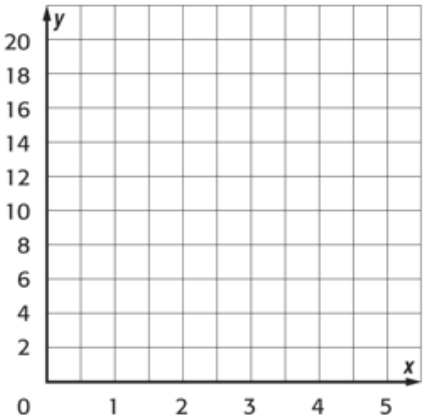
2.

x(gallons)	1	2	3	4
y(quarts)	4	8	12	16



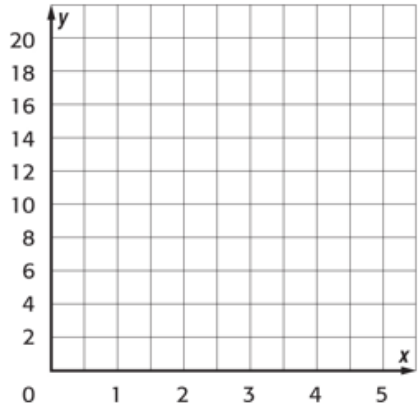
3.

x(number of video games rented)	1	2	3	4
y(cost)	3	5	7	9



4.

x(number of minutes)	2	4	6	8
y(number of calories)	4	8	12	16



Percents

Setting Up:

1. Set up a **proportion** $\frac{\%}{100} = \frac{part(is)}{whole(of)}$
2. Labels must match vertically or horizontally.

Solve:

1. Solve by cross multiplying.

Sydney completed 3 out of her 5 skill sheets, what percent did she complete?

$$\frac{3completed}{5total} = \frac{\%completed}{100total}$$

Solve. Show All Work!

1. Express 30% as a fraction in simplest form.	2. Express $18\frac{4}{5}$% as a decimal.
3. If 125% of a number is 20, what is the number?	4. $33\frac{1}{3}$% of 150 is what number?
5. What percent of 52 is 6.24?	6. A bike that is originally $240 is on sale for 20% off. What is the sales price?
7. In your class, there are 8 girls and 14 boys. What percent of your class is girls?	8. 272 out of 320 students in your school were surveyed about their favorite soda. What percent of the school population was surveyed?
9. Diane's allowance is $20 per week. She saves 30% of her allowance. How much does she save each week?	10. Your Dad bought a concert ticket for $126. He said that you have to pay 75% of the cost of the ticket. How much did you have to give your Dad?